Total No. of printed pages = 5

MA 131401

Roll No. of candidate

2017

B. Tech 4th Semester End-Term Examination

NUMERICAL METHODS AND COMPUTATION

Full Marks–100 Pass Marks–35 Time–Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *ten* of the following questions : 3×10=30

(i) The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. If l is increased by 2% and g is increased by 2%, find the percentage error in T.

(ii) Round off the numbers 3·26425, 35·46735, 4985561, 0·70045, 1·000375 to four significant digits.

[Turn over

(iii) Evaluate $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$ taking interval of difference as 2.

(iv) If $\pi = 3{\cdot}141593$ is written as $\frac{22}{7}$, find its relative error.

(v) With usual notation, prove that $\Delta\nabla = \Delta - \nabla$.

(vi) Find the polynomial f(x) if f(0) = 2, f(1) = 3, f(3) = 6 and f(5) = 8.

(vii) Write down the error formula in Simpson's $\frac{1}{3}$rd rule for the function f(x) in the interval [a, b]. Hence find the error of the function $\frac{x}{1+x}$ in [0, 1].

(viii) Use Newton-Raphson formula to deduce the iterative formula to find $\frac{1}{\sqrt{a}}$.

(ix) Find a real root of the equation $x^3 - x - 11 = 0$ correct to three decimal places by bisection method computing upto third iteration.

(x) Solve $\frac{dy}{dx} = y - \frac{2x}{y}$, y(0) = 1 in the interval [0, 0·2] taking h = 0·1 by Euler's method.

(xi) Prove that $\Delta^3 y_2 = \nabla^3 y_5$.

2. Answer any *five* of the following questions : 8×5=40

(i) Use Newton's interpolation formula to evaluate sin 16° and sin 64° from the following table :

x :	15°	25°	35°	45°	55°	65°
sin x :	0·2588	0·4226	0·5736	0·7071	0·8192	0·9131

(ii) Use Lagrange's interpolation formula to fit a polynomial to the following data :

x :	−4	−1	0	2	5
f(x) :	1245	33	5	9	1335

Hence find f(1) and f(3).

(iii) Evaluate $\int_0^1 \frac{x}{1+x}\,dx$ by trapezoidal rule and estimate error in your result. Also find the value of $\log_e(2)$ from your result.

(iv) Solve by Gauss elimination method :

$$6x_1 - x_2 - x_3 = 19$$
$$3x_1 + 4x_2 + x_3 = 26$$
$$x_1 + 2x_2 + 6x_3 = 22$$

(v) Solve by matrix factorization method :

$$x_1 + x_2 + x_3 = 3$$
$$2x_1 - x_2 + 3x_3 = 16$$
$$3x_1 + x_2 - x_3 = -3$$

(vi) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places.

(vii) Using Newton's divided difference formula, find the missing value from the table :

x :	1	2	4	5	6
y :	14	15	5	—	9

3. Answer any *three* of the following questions : 10×3=30

(i) Solve by Gauss-Seidel method :

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

(ii) Use Runge-Kutta method of fourth order to find $y(0{\cdot}1)$, $y(0{\cdot}2)$; given that

$$\frac{dy}{dx} = y - x, \quad y(0) = 2.$$

(iii) (a) Prove that

$$y_1 = y_3 - 0{\cdot}3(y_5 - y_{-3}) + 0{\cdot}2\,(y_{-3} - y_{-5})$$

(b) Compute x when y = 8 by Lagrange's inverse interpolation formula :

x :	− 2	−1	1	2
y :	− 7	2	0	11

(iv) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ correct to four decimal places using Simpson's three-eighth rule considering 10 sub-intervals. Also estimate the error in your result.