Total No. of printed pages = 7

EC 131405

Roll No. of candidate | | | | | | | | | | |

2017

B. Tech 4th Semester End-Term Examination

RANDOM VARIABLES AND STOCHASTIC PROCESSES

Full Marks–100 Pass Marks–35 Time–Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *ten* questions : 3×10=30

   (i) What is De-Morgan's law ?

   (ii) What is duality principle ?

   (iii) Let A be an arbitrary event in S and let φ be a null set. Show that

   (a) $A \cup \varphi = A$

   (b) $A \cap \varphi = \varphi$

   (iv) A card is drawn at random from an ordinary deck of 52 playing cards.

   (a) What the probability that the card is jack ?

[Turn over

(b) What the probability that the card is red 10 ?

(c) What the probability that the card will be 5 or smaller ?

(v) Show that for a zero mean random variable its variance is equal to its mean square value.

(vi) Let X and Y are two random variables with joint pdf $f_{xy}(xy)$. Show that if X and Y are uncorrelated then

$$E[XY] = E[X] \cdot E[Y]$$

(vii) Sketch the typical plots of pdf and edf of a random variable.

(viii) Distinguish between continuous and discrete random variables with examples.

(ix) Given that a random variable X has the following values :

(a) $\{-5 < x < 10\}$

(b) $\{5, 6, 10 < x < 15\}$

(c) $\{2, 4, 6, 8, 9, 10\}$

State whether X is discrete, continuous or mixed type.

(x) Distinguish between ensemble average and time average.

(xi) Distinguish between wide sense stationary and strict sense stationary processes.

(xii) Distinguish between deterministic and random signals.

(xiii) Show that second central moment of a random variable is equal to its variance.

2. Answer any *eight* questions : 5×8=40

(i) A fair die is tossed. Find the probabilities of the events :

(a) A = {odd number shows up}

(b) B = {number larger than 3 shows up}

(c) $A \cup B$ and $A \cap B$.

(ii) Find the constant b > 0 such that the function

$$f_x(x) = \begin{cases} e^{3x/4}, & 0 \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

is a valid probability density function.

(iii) The probability density function of a random variable X is given by

$f_x(x) = K(1 - 2x^2),\ 0 < x < 1$

Find :

(a) the value of K

(b) commutative distribution function $F_x(x)$.

(iv) The Rayleigh density function is given by

$$f_x(x) = \begin{cases} xe^{-x^2/4}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Prove that $f_x(x)$ satisfies the properties of density function,

(a) $f_x(x) \geq 0$ for all x,

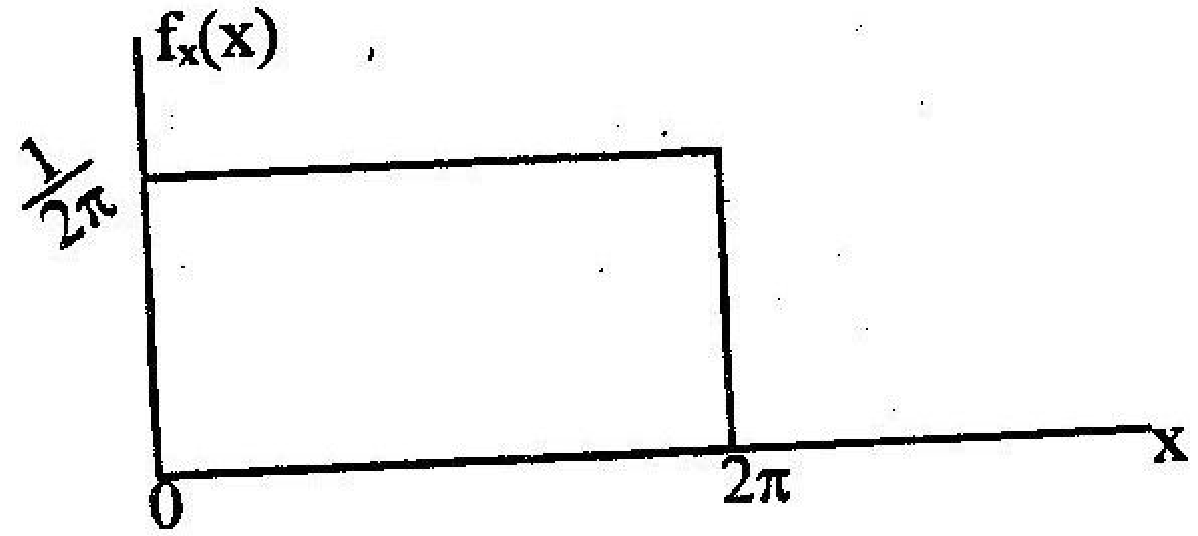
(b) $\int_{-\alpha}^{\alpha} f_X(x)\,dx = 1$

(v) With respect to a random process, prove that

(a) $R_{xx}(0) \geq |R_{xx}(\tau)|$

(b) $R_{xx}(-\tau) = R_{xx}(\tau)$

(vi) Probability density function of uniformly distributed random variable X is shown below :



Find

(a) $E[X]$

(b) $E[X^2]$

(c) $E[(X - m_x)^2]$

(vii) Write short note on Gaussian distribution with special reference to central limit theorem.

(viii) Define psd of a WSS random process X(t). How is it related to the acf of X(t) ? List the properties of psd.

(ix) Consider a random process X(t) given by,

$X(t) = A\cos(\omega_c t + \varphi)$.

Where ω_c and φ are constants and A is a random variable. Determine whether X(t) is WSS.

(x) A random process given by a randomly phased sinusoid,

$X(t) = A \cos(\omega_c t + \phi)$. Where A and ω_c are constants while ϕ is a random variable that is uniformly distributed over the range $(0, 2\pi)$. Find the acf and average power of X(t).

3. Answer any *three* questions : 10×3=30

(i) Given the square law transformation

$Y = aX^2, \; a > 0$

X and Y are two random variables. Find the cdf and pdf of the random variable Y.

(ii) The joint density function of two variables X and Y is given by

$$f_{XY}(x,y) = \begin{cases} \dfrac{xy}{96}, & 0 < x < 4,\; 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

Determine :

(a) $P(1 < X \leq 2,\; 2 < Y < 3)$

(b) The marginal distribution function of X and Y

(c) Joint distribution function of X and Y, if X and Y are independent.

(iii) A random process is defined by

$Y(t) = X(t) \cos(\omega t + \phi)$

where X(t) is a wide sense stationary random process that amplitude modulates a carrier of angular frequency ω with a random phase of ϕ independent of X(t) and uniformly distributed on $(-\pi, \pi)$.

Find

(a) Find the mean E[X] of Y(t).

(b) Find the autocorrelation function $R_{YY}(t)$ of Y(t).

(c) Is Y(t) is a wide sense stationary ?

(iv) Given an LTI system where

h(t) = impulse response of the system

X(t) = input random process

Y(t) = output random process

$m_x(t)$ = mean of X(t)

$R_x(\tau)$ = acf of X(t).

Show that if the input is WSS, then the output is also WSS. Also, find the expression of mean and acf of the output process.