Total No. of printed pages = 8

MA 131602

Roll No. of candidate | | | | | | | | | | |

2017

B. Tech 6th Semester End-Term Examination

COMPUTER SCIENCE AND ENGINEERING
FUZZY MATHEMATICS

Full Marks–100 Pass Marks–35 Time– Three hours

The figures in the margin indicate full marks for the questions.

I. Answer *all* questions : 3×10=30

1. Define fuzzy set with two suitable examples.

2. What is meant by membership function of a fuzzy set ? How is membership function different from characteristic function of crisp set ?

3. Define product and division of intervals. Find product and division of the intervals
$S_1 = [2, 5]$ $S_2 = [-1, 2]$

[Turn over

4. Write a short note on convex fuzzy set.

5. Define α-cuts of fuzzy sets. Find strong α-cuts of the following fuzzy set
$A = \{(1, .2), (2, 0.5), (3, 0.8), (4, 1), (5, 0.7), (6, 0.3)\}$ for all α-level set.

6. Define complement of a fuzzy set. A fuzzy set A is given by
$A = \{(2, 0), (3, 0.5), (4, 1), (5, 0.5), (6, 0)\}$, prove that, $A \cap A^c \neq \varnothing$

7. Explain linguistic hedges and linguistic modifier with a suitable example.

8. Write the conditions for an equivalence relation.

9. Define Type-1, Type-2 and Type-m fuzzy sets.

10. Is the following fuzzy relation transitive ?

$$\begin{bmatrix} 1 & .4 \\ .5 & .6 \end{bmatrix}$$

II. Answer any *eight* questions : 8×5=40

11. What is meant by cardinality of fuzzy set ? Explain different types of cardinality of fuzzy set with a suitable example.

12. Define : (a) fuzzy measure and

(b) possibility measure.

13. Define transitive closure of fuzzy relation. Find transitive closure for the following fuzzy relation

$$R = \begin{bmatrix} .7 & .5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & .4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}$$

14. If two fuzzy sets A, B are described by the membership functions as given below :

$$\mu_A = \left\{\frac{0.3}{1}, \frac{0.4}{2}, \frac{0.5}{3}, \frac{0.9}{4}, \frac{1}{5}\right\}$$

$$\mu_B = \left\{\frac{0}{1}, \frac{1}{2}, \frac{0.9}{3}, \frac{0.2}{4}, \frac{0}{5}\right\}$$

Write the relation matrix.

15. Membership function for the linguistic variable *"HOT"* and *"COLD"* are defined as follows :

$$"HOT" = \left\{\frac{0.2}{5}, \frac{0.4}{15}, \frac{0.6}{25}, \frac{0.8}{35}, \frac{1}{45}\right\}$$

$$"COLD" = \left\{\frac{0.9}{5}, \quad \frac{0.6}{15}, \quad \frac{0.3}{25}, \quad \frac{0.1}{35}, \quad \frac{0}{45}\right\}$$

Develop membership functions for the following linguistic phrases

(i) Very HOT

(ii) Slightly COLD

(iii) Very very COLD

16. A fuzzy relation is given below :

$$\mu_R = \begin{bmatrix} 1.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.8 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 1.0 & 0.8 & 0.7 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.8 & 1.0 & 0.7 & 0.5 & 0.7 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.7 & 0.7 & 1.0 & 0.4 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.5 & 0.4 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.7 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

(a) Is it a tolerance relation ?

(b) Find complete α-covers of the relation using partition tree.

17. Explain Mamdani implication relations with suitable example.

18. Define max-min composition of fuzzy relations. Two fuzzy relations R and S with membership grades μ_R and μ_S respectively are given below :

$$\mu_R = \begin{bmatrix} 1.0 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.3 \\ 1.0 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.3 \end{bmatrix}$$

$$\mu_S = \begin{bmatrix} 0.3 & 0.6 & 0.0 & 1.0 \\ 0.7 & 0.0 & 1.0 & 0.5 \\ 0.5 & 0.0 & 0.0 & 0.2 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

Obtain Max-Min composition.

19. Write five differences between probability theory and possibility theory.

20. Define disjunctive sum of crisp sets. Find simple disjunctive sum and disjoint sum for the following fuzzy sets

$A = \{(a, 0.2), (b, 0.5), (c, 0.2), (d, 1)\}$

$B = \{(a, 0.8), (b, 0), (c, 0.5), (d, 0.9)\}$

III. Answer any *three* questions : 3×10=30

21. For following the Fuzzy Relation Equation :

$$p \circ \begin{bmatrix} .5 & .7 & .2 \\ .4 & .6 & 0 \\ .2 & .4 & .6 \end{bmatrix} = [.5 \quad .5 \quad .2]$$

(a) Verify the existence of the solution 3

(b) If solution exists, find complete solution of the equation. 7

22. (a) Explain extension principle of fuzzy set. 4

(b) Two fuzzy sets defined on universal set $X = \{-1, 0, 1, 2\}$ are as 6

$A_1 = \{(-1, 0.4), (0, 0.5), (1, 0.2), (2, 0.5)\}$

$A_1 = \{(-1, 1), (0, 0.05), (1, 0.5), (2, 0.4)\}$

A mapping $f: X \rightarrow Y$ defined by $y = x_1^2 + x_2^2$. Derive fuzzy set B on Y using extension principle.

23. (a) Describe Zadeh implication relations. 3

(b) Given 7

Rule 1: IF height is TALL , THEN speed is HIGH

Rule 2 : IF height is MEDIUM, THEN speed is MODERATE

Where, fuzzy set for height (in feet) and speed (m/s) is given below :

$$H_1 \text{ (Height)} = \text{TALL} = \left\{\frac{0.3}{5}, \frac{0.8}{6}, \frac{1}{7}\right\}$$

$$H_2 \text{ (Height)} = \text{MEDIUM} = \left\{\frac{0.8}{5}, \frac{0.7}{6}, \frac{.6}{7}\right\}$$

$$S_1 \text{ (Speed)} = \text{HIGH} = \left\{\frac{0.4}{5}, \frac{0.7}{7}, \frac{.9}{9}\right\}$$

$$S_2 \text{ (Speed)} = \text{MODERATE} = \left\{\frac{0.6}{5}, \frac{.8}{7}, \frac{.7}{9}\right\}$$

Given H' (Height) = height is ABOVE AVERAGE = $\left\{\frac{0.5}{5}, \frac{0.9}{6}, \frac{.8}{7}\right\}$

Use Zadeh implication relation and fuzzy max-min composition, to infer S' = Speed is ABOVE NORMAL.

24. Write short notes (any *two*) : 5+5=10

(i) Evidence measure

(ii) Max Product and Max Average composition with examples

(iii) t-norm and s-norm

(iv) Fuzzy Union and Intersection as functions.