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MA 131101

Roll No. of candidate | | | | | | | | | | |

2017

B.Tech. 1st Semester End-Term Examination

MATHEMATICS — I

(Old Regulation)

Full Marks – 100 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *six* from the rest.

PART A

1. Answer *all* questions : (10 × 1 = 10)

(a) If $u = f(x/y)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is ______.

(b) If $f(x)$ has derivatives of every order in the neighborhood of zero, under what conditions $f(x)$ can be expanded in an infinite series.

(c) Define homogenous function.

(d) The integrating factor for $\frac{dy}{dx} + Py = Q$ is ______.

(e) $X = r\cos\theta$, $Y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$?

[Turn over

(f) Write down the reduction formula for $\int_0^{\pi/} \sin^n x\,dx$.

(g) The value of $\int_0^{\pi/2} \cos^6 x\,dx$ is ________.

(h) Under what condition, the equation $M(x,y)\,dx + N(x,y)\,dy = 0$ become exact?

(i) What is the solution of $y = px + f(p)$?

(j) The complementary function for $\frac{d^2y}{dx^2} - y = e^x$ is ________.

PART B

2. Answer the following questions :

(a) Find y^n, if $y = \cos^2(5x+4)$. (3)

(b) If $y = \tan^{-1} x$, then prove that

$(1+x^2)Y_{n+1} + 2nxY_n + n(n-1)Y_{n-1} = 0$ and also find Y_n at $x = 0$. (4 + 1 = 5)

(c) Expand, $\log(1+x)$ in power of x. (3)

(d) If $v = f(2x-3y, 3y-4z, 4z-2x)$, then prove that $6v_x + 4v_y + 3v_z = 0$. (4)

3. Answer the following questions :

(a) Solve : (3)

$$(y\cos x+\sin y+y)\,dx+(\sin x+x\cos y+x)\,dy=0.$$

(b) Find the n^{th} derivative of $Y=\dfrac{x^2}{(x-a)(x-b)}$. (4)

(c) If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$, then prove that, $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=r^2\sin\theta$. (5)

(d) If, $u=\dfrac{5xy^2}{z^3}$ and error in x,yz are 0.001, compute percentage error when $x=y=z=1$. (3)

4. Answer the following questions :

(a) Find the point upon the plane $ax+by+cz=p$ at which the function $f=x^2+y^2+z^2$ has a minimum and find the minimum. (5)

(b) Evaluate $\int_0^{\pi/4}\tan^6 x\,dx$. (3)

(c) If, $f(x)=f(0)+xf'(0)+\dfrac{x^2}{2}f''(\theta)$, find θ when, $x\to 1$, where $f(x)=(1-x)^{\frac{5}{2}}$. (4)

(d) Sketch the polar curve $r=a\sin 3\theta$. (3)

5. Answer the following questions :

(a) If $x^3 + y^3 - 3axy = 0$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (4)

(b) Deduce the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$, where m and n are both positive integers greater than 1. (5)

(c) Find the entire area of the cardioid $r = a(1+\cos\theta)$. (3)

(d) Solve : $(x + \sin y)\,dx + (x\cos y - 2y)\,dy = 0$. (3)

6. Answer the following questions :

(a) Evaluate, $\int_0^{\infty} \frac{e^{-ax}\sin x}{x} dx$ by applying differentiation under integral sign and hence, evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. (5)

(b) Find the area of the region bounded by the parabolas $y = 6x - x^2$ and $y = x^2 - 2x$. (4)

(c) Find the integrating factor of $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$. (2)

(d) Solve : $y - 2px = \tan^{-1}(xp^2)$. (4)

7. Answer the following questions :

(a) Prove that, $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$. (4)

(b) Compute, $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ if the region of integration is bounded by the coordinate planes and the plane $x + y + z = 1$. (4)

(c) Solve : $(D^2 - 5D + 6)\,y = e^x \cos 2x$. (4)

(d) Evaluate, $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$. (3)

8. Answer the following questions :

(a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$, using triple integration. (5)

(b) Prove that $\beta(m,n) = \beta(n,m)$. (3)

(c) Solve by method of variation of parameters the equation

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$ (4)

(d) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major axis. (3)

9. Answer the following questions :

(a) If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

(3)

(b) Solve the simultaneous differential equations

(5)

$$\frac{dx}{dt} - y = t$$

$$\frac{dy}{dt} + x = t^2.$$

Solve the differential equation $(D^2 - 2D)y = e^x \sin x$ by the method of undermined coefficient.

(c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that, $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{9}{(x + y + z)^2}$. (5)

(d) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^5 x \, dx$. (2)