A-level Physics Tutor Guides

A-level Physics COURSE NOTES

ELECTRICITY

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CONTENTS

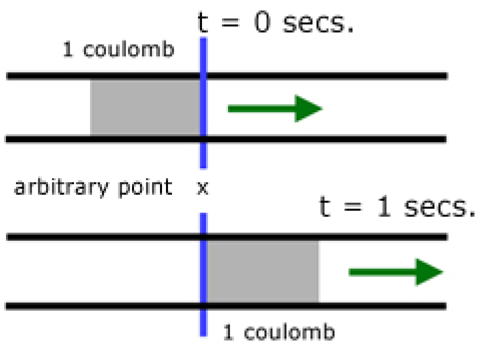
Electrical Concepts & Units - Part 1

Charge (symbol **Q**)

On the sub-atomic level we have a unit of charge of 'e' , the charge on the electron. This has the value -1.602×10^{-19}C. Similarly, a proton has a charge of $+1.602\times10^{-19}$C .

The macro unit of charge in electricity is the **Coulomb** (C). It is quite a large unit and has approximately the same charge as 6.2×10^{18} electrons(or protons).

By definition the Coulomb is defined as: **the charge that passes an arbitrary fixed point when a current of 1 Ampere flows for 1 second.**



Current (symbol **I**)

Electric current can be thought of as a flow of charged particles. In the normal case, when charge flows through wires these particles are electrons. However, when current flows in a vacuum, a solution or a melt, the charge carriers are ions. In semiconductors the charge carriers are exotic particles called 'holes'.

By definition the Ampere (A) is defined as: **the current that passes an arbitrary fixed point when a charge of 1 Coulomb flows for 1 second.**

Since, amount of fluid = rate of flow x time

by analogy, charge = electric current x time

(Coulombs) = (Amperes) x (seconds)

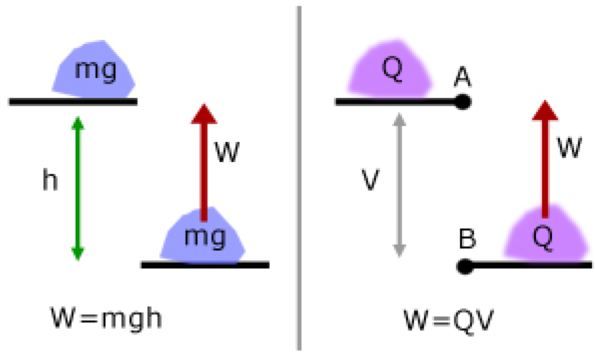
$$\mathbf{Q = It}$$

<u>Potential Difference</u> (symbol **V**)

Potential difference is short for **potential energy** difference. Our definition of the volt relates charge and the work needed to move that charge between two points.

In our definition of the volt, the charge is 1 Coulomb and the work done is 1 Joule.

Hence the definition : **Two points A & B are at a potential difference of 1 volt if the work required to move 1 coulomb of charge between them is 1 Joule.**



A simple analogy is gravity. Compare the energy difference between a rock at the bottom of a cliff and the energy in moving it to the top of the cliff. An energy difference exists between the top of the cliff and the bottom. The amount of energy difference depends on the size of the rock and the height of the cliff (P.E. = mgh) .
In our analogy, height relates to potential difference and rock weight relates to the amount of charge. Work is done on the rock against the force of gravity. Work is done on the charge in moving it against an electrostatic force.

The volt has unit of joules per coulomb (JC^{-1}).

We now have an equation linking work/energy **W**, charge **Q**and potential difference **V** :

$$V = \frac{W}{Q} \qquad\qquad W = QV$$

work = charge x potential difference

(Joules) = (Coulombs) x (volts)

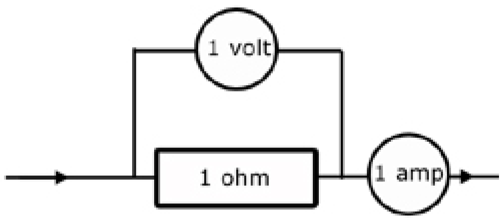
<u>Resistance</u> (symbol **R**)

By definition, the electrical resistance (R) of a conductor is the ratio of the p.d. (V) across it to the current (I) passing through it.

$$R = \frac{V}{I}$$

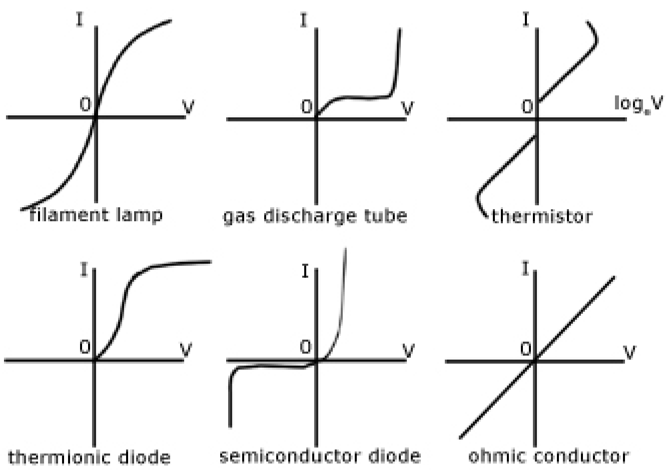
From this equation, by making each quantity unity, we define the Ohm:

The resistance of a conductor through which a current of 1 ampere flows when a p.d. of 1 volt exists across it.



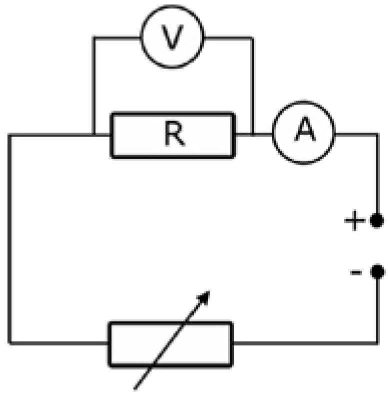
Note in the diagram, the voltmeter is in parallel with the resistance, while the ammeter is in series.

Some conductors/devices have variable resistances which depend on the currents flowing through them.



These are all **non-ohmic** conductors except for the bottom right graph. **Ohmic** conductors are all metals and follow Ohm's Law.

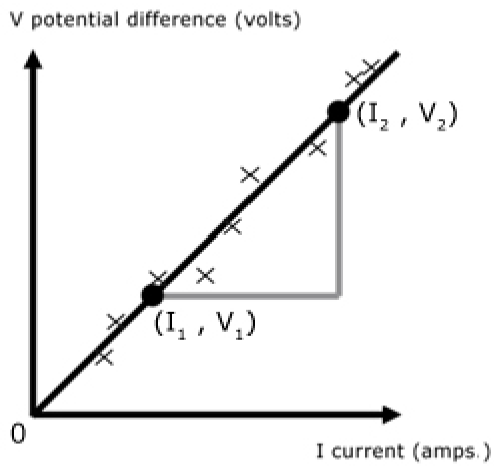
Ohm's Law



The current through a resistor is varied, while the p.d. across it is measured. The graph of V against I is a straight line through the origin.

Hence,

$$V \propto I \qquad \frac{V}{I} = \text{a constant}$$



$$\text{gradient (R)} = \frac{V_2 - V_1}{I_2 - I_1}$$

Ohm's law states: **the current through a conductor is directly proportional to the p.d across it, provided physical conditions* are constant.**
*eg temperature

Electrical Concepts & Units - Part 2

Resistivity **ρ** (rho)

By experiment it has been found that the resistance R of a material is directly proportional to its length l and inversely proportional to its cross-sectional area A.

$$R \propto l \qquad R \propto \frac{1}{A} \qquad R \propto \frac{l}{A}$$

Making the proportionality an equation, the constant of proportionality ρ (rho) is called the resistivity.

$$R = \rho \frac{l}{A}$$

Rearranging the equation and making the length and area unity, we can form a definition for the quantity.

$$\rho = \frac{RA}{l} \qquad \rho = \frac{R(1)}{(1)}$$

The resistivity of a material is the resistance of a cube of side 1 m, across opposite faces.

The units of resistivity can be found by substituting the units for l , A and R in the resistivity equation.

$$\rho = \frac{(\Omega)(m^2)}{(m)} = (\Omega)(m) = \Omega.m$$

Resistivity is measured in units of ohm-metres(Ω.m) .
Typical values of resistivity are:

metals	$\sim 10^{-8}$
semi-conductors	~ 0.5
glass/alumina	$\sim 10^{12}$

Conductance **G**

By definition, the conductance of a material is the reciprocal of its resistance. The unit of conductance is the Seimens S.

$$G = \frac{1}{R}$$

Conductivity **σ** (sigma)

By definition, the conductivity of a material is the reciprocal of its resistivity. The units of conductivity are Sm^{-1} or $\Omega^{-1}m^{-1}$.

$$\sigma = \frac{1}{\rho}$$

Current density **J**

Current density at a point along a conductor, is defined as the current per unit cross-sectional area.

$$J = \frac{I}{A}$$

The units of current density J are amps per m^2 or Am^{-2} .

Conduction in metals

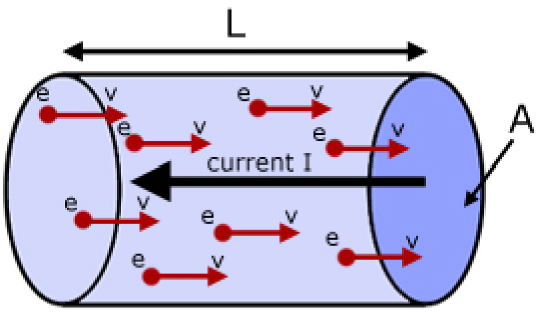
Metal atoms in crystal lattices each tend to lose one of their outer electrons. These **free electrons** wander randomly inside the crystal.

If a p.d. is applied across the metal, electrons drift in the direction of the positive contact. Since electrostatic field direction is from positive to negative, the direction is against the field.

The drift velocity, as it is called, is of the order of 10^{-3} ms^{-1} . When the electrons move through the crystal lattice they collide with metal ions. Each collision imparts kinetic energy from an electron to an ion. The ions thus gain vibrational energy and as a consequence the temperature rises. This phenomenon is called **ohmic heating**.

The potential difference produces a steady current through the metal. On their journey electrons gain and lose KE. The total amount of energy involved per m^3 is a measure of the resistivity of the particular metal.

Expressions for the current **I** through a conductor and the current density **J** can be formed using the concept of drift velocity ***v*** .



where:

- **I** - current flowing through conductor
- **L** - length of conductor
- **A** - cross-sectional area
- **n** - no. free electrons/unit vol.
- **e** - charge on the electron
- ***v*** - average electron drift velocity

From the diagram the following can be implied:

volume of the cylindrical section = LA

no. free electrons in the section = nLA

quantity of mobile charge **Q** in the section = **nLAe**

The time ***t*** for all the electrons in the section to travel from one face to the another is the time for one electron (on the far right) to travel the whole length.

$$t = \frac{L}{v}$$

since,

$$I = \frac{Q}{t}$$

substituting for Q and t,

$$I = \frac{nLAe}{\frac{L}{v}}$$

$$= \frac{nLAve}{L}$$

$$\underline{I = nAve}$$

since current density J is defined as:

$$J = \frac{I}{A}$$

it follows that,

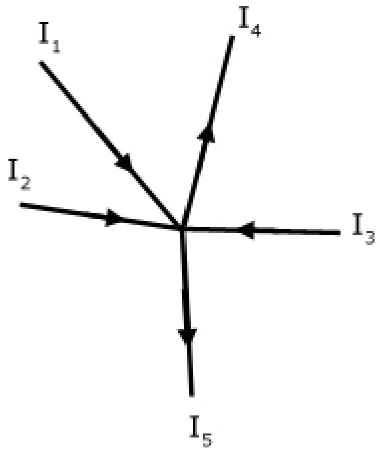
$$J = \frac{nAve}{A}$$

$$\underline{J = nve}$$

Simple Circuits

Kirchhoff's Laws

1st Law - The sum of the currents entering a node/junction equals the sum of the currents leaving.



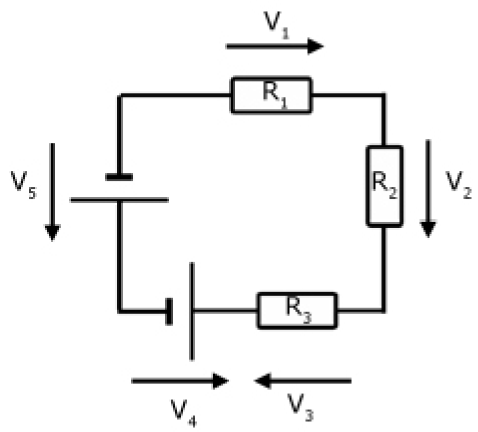
$$I_1 + I_2 + I_3 = I_4 + I_5$$

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

This can also be expressed as the algebraic sum: $\Sigma I = 0$

2nd Law - **Around any closed loop in a circuit, the algebraic sum of the individual p.d.'s is zero.**

This can also be described as:
Around any closed loop in a circuit, the sum of the emf's equals the sum of the p.d.'s across resistive elements.

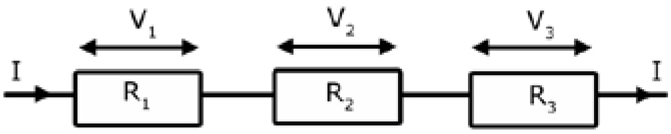


$$V_1 + V_2 + V_3 - V_4 - V_5 = 0$$

The convention is that clockwise p.d.'s are positive.

Simply, emf's cause a rise in pd. Other circuit elements(eg resistors) cause falls in pd. In a closed circuit, the sum of the rises in pd equals the sum of the falls.

Resistors in series



Consider three resistors, R_1 R_2 R_3 with the same current flowing through each.

If the p.d. across each one respectively is, V_1 V_2 V_3 . Then the total p.d. V_{total} across the arrangement is:

$$V_{total} = V_1 + V_2 + V_3$$

By Ohm's law, V=IR, therefore:

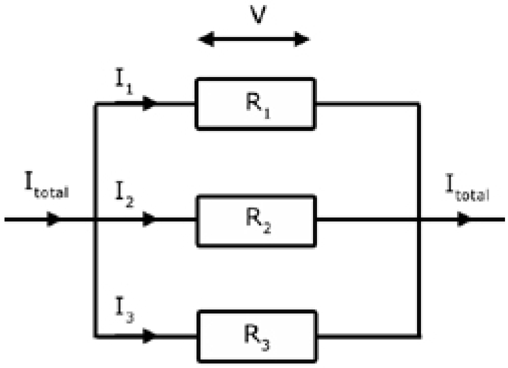
$$V_{total} = IR_{total} \qquad V_1 = IR_1$$
$$V_2 = IR_2 \qquad V_3 = IR_3$$

substituting for V_{total} V_1 V_2 V_3 into the equation for p.d.,

$$IR_{total} = IR_1 + IR_2 + IR_3$$

$$\underline{R_{total} = R_1 + R_2 + R_3}$$

Resistors in parallel



Consider three resistors R_1 R_2 R_3 with the same p.d. (V) across each of them.

Using Kirchhoff's 2nd law, we can write:

$$I_{total} = I_1 + I_2 + I_3$$

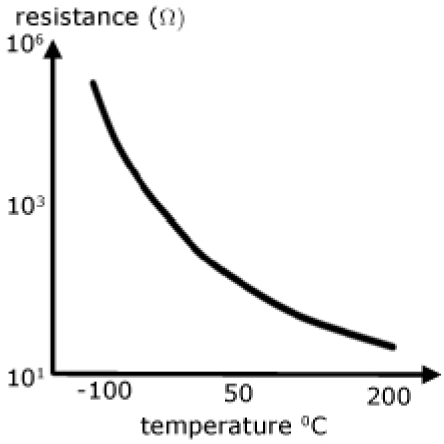
By Ohm's law, V=IR and I=V/R , therefore :

$$\frac{V_{total}}{I_{total}} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3}$$

$$\underline{\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

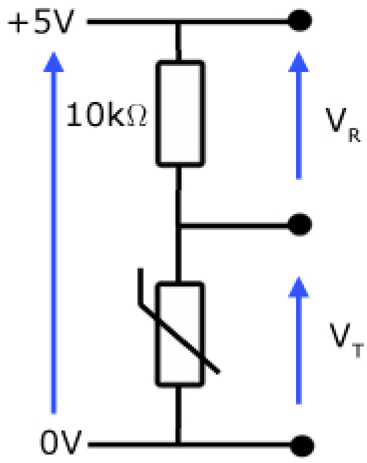
The Thermistor

A thermistor is a bipolar* semiconductor circuit element. It is in effect a temperature dependent resistor.
*contacts can be connected + - or - +



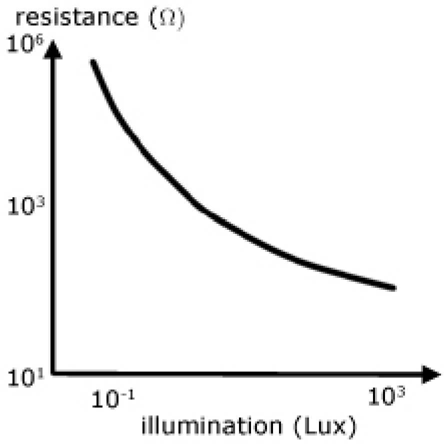
The arrangement below is called a potential divider. The p.d.'s V_R and V_T are in the ratio of the resistors they appear across.

When the thermistor is hot its resistance is low and of the order of 100's of ohms. In this case, most of the 5V p.d. falls across the 10kΩ resistor. As the temperature decreases, the resistance of the thermistor increases. When its resistance reaches 10kΩ the p.d. is shared equallybetween it and the series resistor. At really cold temperatures the resistance increases to the order of MΩ's, when most of the p.d. falls across it and not the series resistor.



The Light Dependent Resistor (LDR)

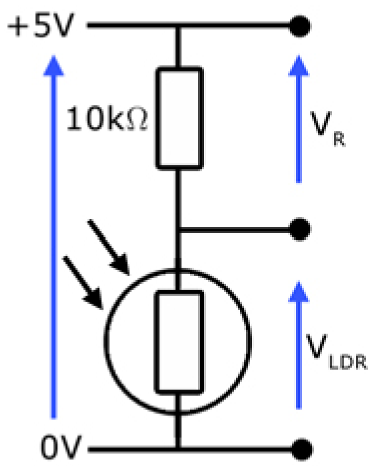
Like the thermistor, the LDR is also a bipolar semiconductor circuit element. LDR's are made from high resistance semiconductor material, whose resistance decreases with increasing incident light intensity.



Typically effect of light on a LDR is to reduce its resistance from ~ 10^6 Ω to ~ 10^2 Ω.

The arrangement below is called a potential divider. The p.d.'s V_R and V_{LDR} are in the ratio of the resistors they appear across.

In the dark, the resistance of the LDR is of the order of MΩ's. So most of the 5V p.d. falls across it and not the series resistor. With more illumination, the resistance of the LDR decreases. When it reaches 10kΩ the p.d. is shared equally with the series resistor. In bright light, its resistance is of the order of 100's of Ω's. Then, most of the p.d. falls across the series resistor.



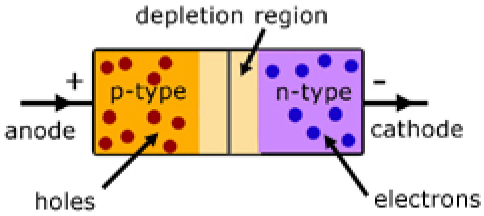
Light Emitting Diode (LED)

An LED is essentially a modified **junction diode** (or **p-n diode**) so that it gives out light when current flows through it.

Junction diodes are made from two types of semiconductor material, which have been 'doped' to alter their properties.

p-type: rich in charge carriers called 'holes' (missing electrons)

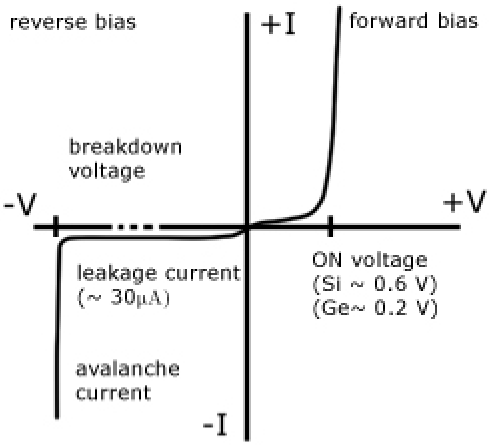
n-type: rich in free electrons



The two types of semiconductor meet in the middle at what is called the **p-n junction**. Here, with no p.d. applied, the holes from the p-type meet up with the free electrons from the n-type and cancel each other out.

However, when a p.d. is applied, with n-type '-' and p-type '+' (called **forward biased**), a current flows. This current is made up of free electrons moving across to the '+' terminal and holes moving towards the '-' terminal.

When the polarity is reversed(**reverse bias**), with n-type made '+' and p-type made '-' , no current flows. So we have a device that only allows current to flow in one direction.

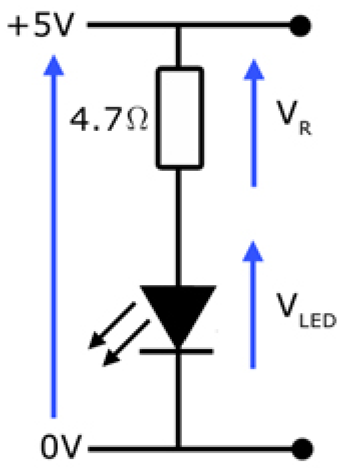


On a V-I graph the top right quadrant shows how a very small forward p.d. causes the diode to conduct. Notice the high current for a small p.d. increase.

The bottom left quadrant shows what happens when the diode is reverse biased ('+' contact connected to '-' supply and vice versa). Notice for increasing p.d. there is a constant '**leakage current**' . This is very small, being of the order of micro-amps. There comes a point when the p.d. is so high that '**breakdown**' occurs. A large current passes and the diode is destroyed.

it is essential in LED circuits that the exact p.d. falls across the device. If the p.d. is too high the LED will allow too much current to flow through it. The result will be overheating and failure.

To avoid this, an LED always has a 'limiting resistor' placed in series to limit the current. The level of current designed for is just enough to trigger light from the device.



Example: Find the limiting resistor for an LED, where:

- i) the max. LED current required is 100mA
- ii) the forward LED voltage is 0.65V
- iii)the supply p.d. is 5V

If 0.65V falls across the LED, then 4.35V must fall across the limiting resistor.

The current through both the LED and the limiting resistor is 100mA.

Therefore the limiting resistance R is given by R = V/I .

$$R = 4.35/0.1 = 43.5\ \Omega$$

The closest commercial resistor value is 47 Ω

E.M.F. & Internal Resistance

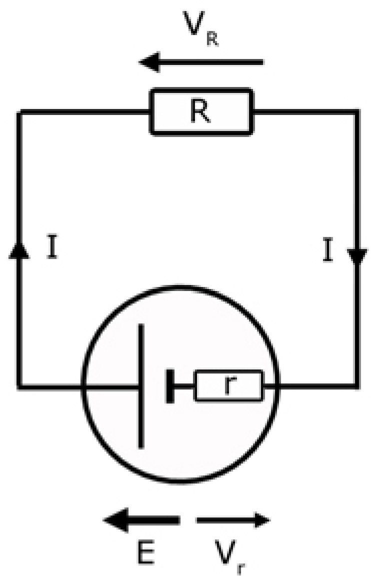
The single cell

E.M.F.(E) is the p.d. across a cell when it delivers no current.

It can also be thought of as the energy converted into electrical energy, when 1 Coulomb of charge passes through it.

The **internal resistance**(r) of a cell is a very small resistance. For a 'lead-acid' cell it is of the order of 0.01 Ω and for a 'dry' cell it is about 1 Ω.
This means that a lead-acid cell will deliver a higher current than a dry cell.

We can obtain important equations for E and r by considering a cell with a resistance in a circuit.



The total resistance R_{total} is the sum of the series resistor and the internal resistance of the cell.

$$R_{total} = R + r$$

by summing p.d. around the circuit ,

$$E = IR_{total}$$

substituting for R_{total}

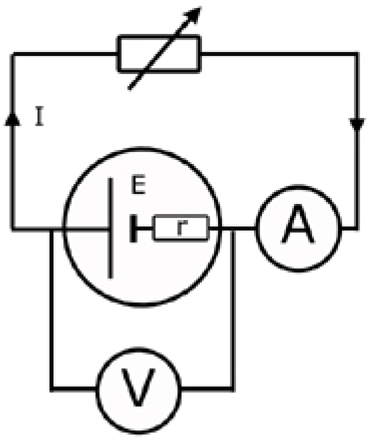
$$E = IR + Ir$$

by Ohm's law, substituting $IR = V_R$

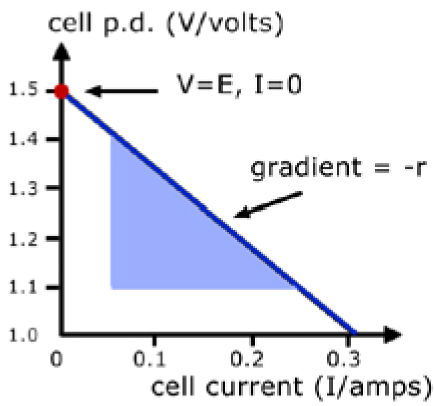
$$\underline{E = V_R + Ir}$$

Note, V_R is called the **terminal p.d.** . That is the p.d. across the cell when it is delivering current.

Measurement of E & r



After taking readings of terminal p.d. (V_R) and current (I), a graph is drawn.



Information can be obtained from the graph by manipulating the equation obtained for E and r:

$$E = V_R + Ir$$

$$E - Ir = V_R$$

transposing the I and r, turning the equation around,

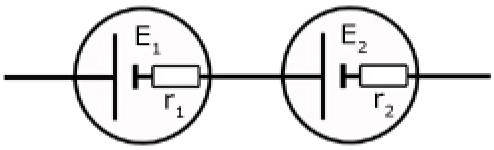
$$V_R = -rI + E$$

comparing with the equation of a straight line,

$$y = mx + c$$

Therefore the gradient is '**- r**' and the intercept on the vertical axis is '**E**' .

Cells in series



$$\underline{E = E_1 + E_2}$$

but

$$E_1 = Ir_1 \qquad E_2 = Ir_2$$

and

$$E = Ir$$

where ***r*** is the internal resistance of the combination

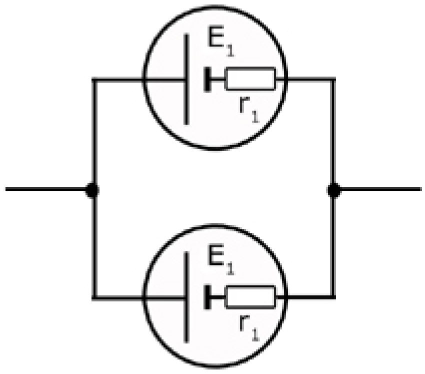
therefore,

$$Ir = Ir_1 + Ir_2$$
$$\underline{r = r_1 + r_2}$$

So to sum up, two cells in series are equivalent to one cell with an EMF equal to the sum of the two cells. The internal resistance of the combination is the sum of the internal resistances of the two cells.

Cells in parallel

The arrangement dealt with here is only for cells that are similar. For dissimilar cells the relationship is complicated, but can be resolved using Kirchhoff's Laws.



For similar cells the EMF's are equal and the internal resistances are also equal.

Therefore the combined EMF, E is given by,

$$\underline{E = E_1}$$

and the internal resistance of the combination is:

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_1}$$

$$\frac{1}{r} = \frac{2}{r_1}$$

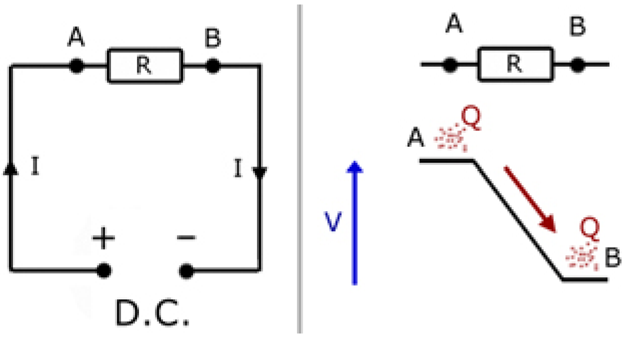
$$2r = r_1$$

$$\underline{r = \frac{r_1}{2}}$$

Power & Energy

Energy

To understand how energy is converted consider a simple circuit with a 'load' * resistor and a D.C. source.
* this could be a heater element, a motor or indeed any component with resistance



Charge loses potential energy(PE) as it moves through the resistor. This electrical PE is transformed mostly into heat energy dissipated in the resistor.

The PE is defined as*:

work = charge x potential difference

$$W = QV$$

(Joules) = (Coulombs) x (Volts)

*for more information see the definition of the volt

Since,

charge = (current) x (time current flows)

$$Q = It$$

(Coulombs) = (Amps) x (seconds)

therefore, substituting for Q in the work equation,

$$W = (It)V$$

$$\underline{W = VIt}$$

Power

By definition, 'power' is the rate of working and is equal to the work done divided by the time taken.

$$P = \frac{W}{t}$$

substituting for **W**

$$P = \frac{VIt}{t}$$

cancelling the '**t**'

$$\underline{P = VI}$$

(Watts) = (Volts) x (Amps)

note: 1 Watt is a rate of working of 1 Joule per second

The equation for power can be modified if we make substitutions using Ohm's Law.

$$P = VI$$

$$V = IR \qquad I = \frac{V}{R}$$

substituting in the power equation for **V** ,

$$P = (IR)I$$

$$\underline{P = I^2R}$$

substituting in the power equation for **I**,

$$P = V\left(\frac{V}{R}\right)$$

$$\underline{P = \frac{V^2}{R}}$$

The kilowatt-hour (kWh)

A kilowatt-hour is a unit of energy.

By definition, a kilo-watt hour is the amount of energy consumed when a rate of working(power) of 1 kilowatt is used for 1 hour.

conversion of 1 kWh to Joules:

$$1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ h} = 1000 \text{ W} \times 3600 \text{ s} = 3600000 \text{ J}$$

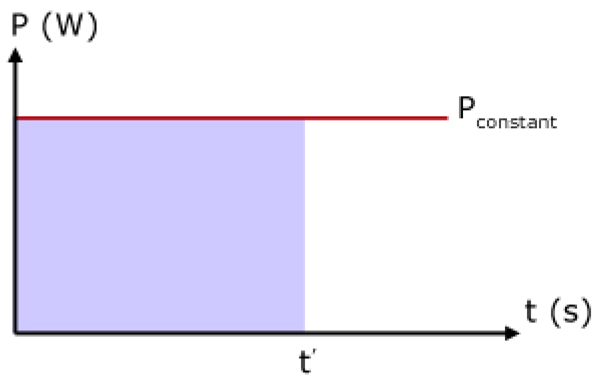
$$\mathbf{1\ kWh = 3.6 \times 10^{6}\ J}$$

Energy for direct current(D.C) & alternating current (A.C.)

Direct current does not vary with time and it is always in one direction. On a plot of power against time, D.C. is a horizontal line.

The area under the plot gives the work done/energy used.

This is simply the product of the constant power($\mathbf{P_{const.}}$)and the time interval that the power is used for(t') .



$$W = P_{const.}t'$$

However, for A.C. the situation is more complex.

Here not only does the current value vary, but its direction varies too.

The power through the resistor is given by:

$$P = I^2R$$

But we must average of this power over time 't' to calculate the energy/work.

so the energy/work done is given by,

$$W = P_{average}\, t$$
$$= \left(I^2R\right)_{average}\, t$$
$$= \left(I^2\right)_{average}\, Rt$$

The root mean square (RMS) current is defined as:

$$I_{RMS} = \sqrt{\left(I^2\right)_{average}}$$

I_{RMS} is the square root of the average of the current squared.

therefore energy/work done is given by,

$$W = \left(I_{RMS}\right)^2 Rt$$

I_{RMS} is the equivalent D.C. current having the same effect on a resistor as the A.C.

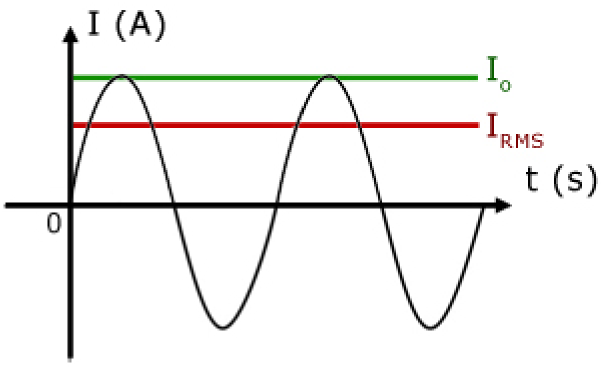
Here is a graph of an A.C. sinusoidal waveform:

$$I = I_o \sin(\omega t)$$

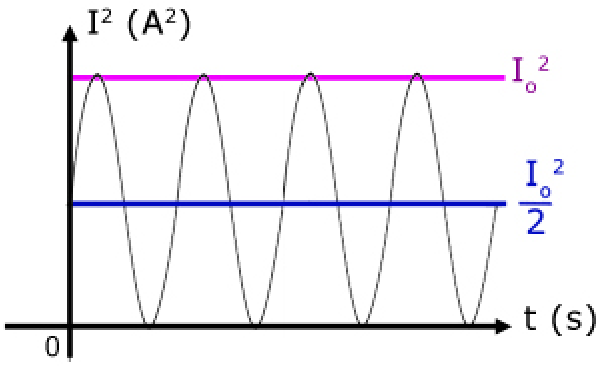
where,

I_o is the maximum current

ω is the angular frequency, $\omega = 2\pi f$ (π pi , f frequency)



$$I_{RMS} = \frac{I_0}{\sqrt{2}} = 0.707\ I_0$$



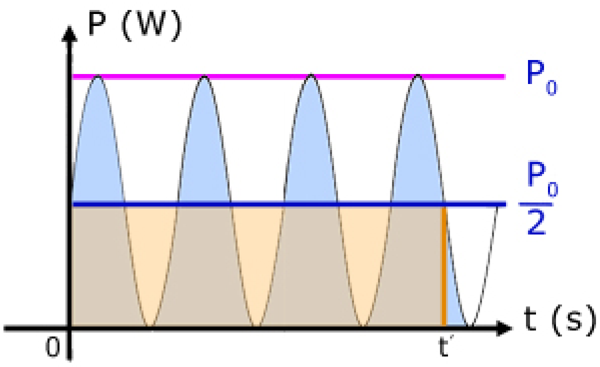
Recalling the A.C. energy/work done equation,

$$W = (I_{RMS})^2 Rt$$

and substituting for I_{RMS}

$$W = \left(\frac{I_0}{\sqrt{2}}\right)^2 Rt$$

$$W = \frac{I_0^2}{2} Rt$$



Therefore at time t' the energy W dissipated in resistor R is given by:

$$W = \frac{P_0}{2} t'$$

note: to avoid confusion between **W** in equations and **W** on the graph

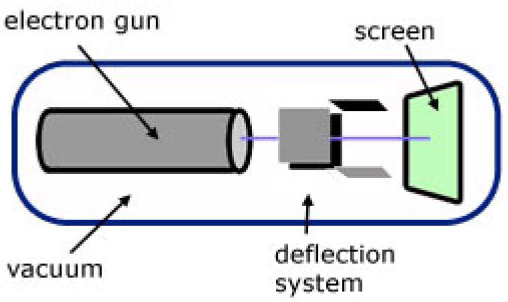
P (W) on the graph means power **P** in watts.

W in equations is the energy/work done

The Cathode Ray Oscilloscope (C.R.O)

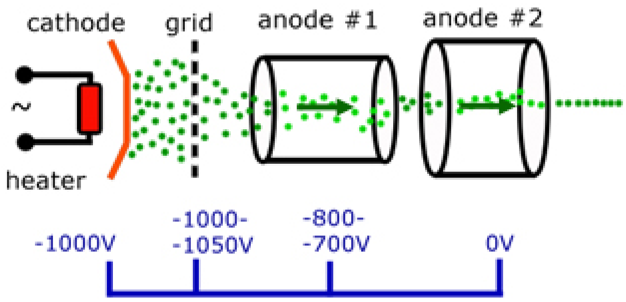
Construction

The cathode ray oscilloscope consists of three main elements:



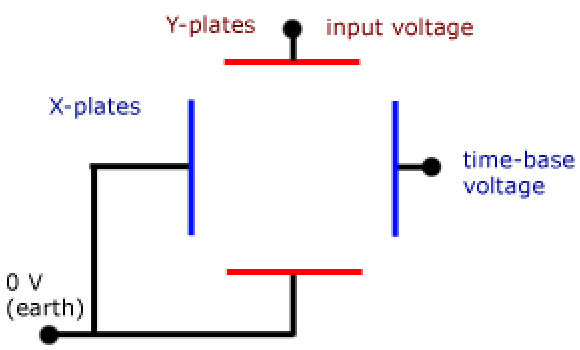
electron gun

Electrons are produced by **thermionic emission**. Essentially a cathode(negative electrode) is heated and electrons boil off the surface to be attracted by a series of anodes (positive electrodes).
The anodes accelerate the electrons and collimate them into a narrow beam.

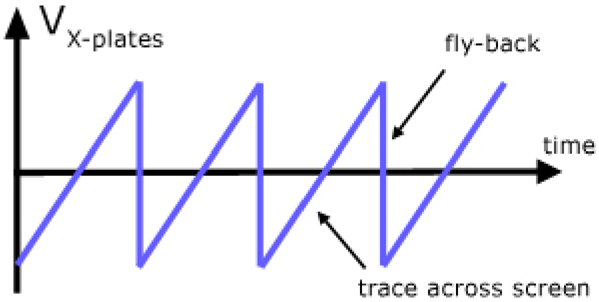


deflection system

The deflection system consists of two pairs of parallel plates called the **X-plates** and the **Y-plates**.



To display a waveform, a repetitive reversing voltage is applied to the X-plates.
This causes the electron beam to be slowly repelled from the left-hand plate and attracted towards the right-hand plate. On the CRO screen this translates as an illuminated dot moving from left to right.

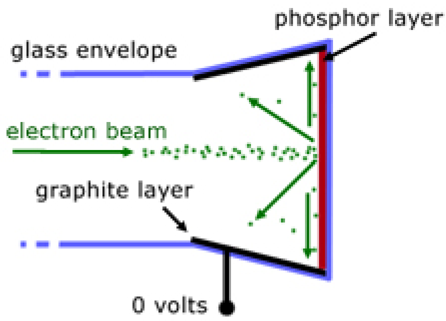


The voltage is then reversed and increased rapidly. The effect is to move the dot very quickly from right to left (fly-back).
The applied voltage is called the **time-base**. The curve has the general shape of a '**saw-tooth**' and is often referred to by this name.

The p.d. applied to the Y-plates is the signal to be examined. With the p.d. across the X-plates (the time-base) switched off, a sinusoidal signal makes the dot go up and down, executing simple harmonic motion. With the time-base on, a sine waves is displayed.

display

The display screen is coated on the inside with a very thin layer of a **phosphor** called **cadmium sulphide**. This fluoresces (gives out green light) when electrons impact its surface.

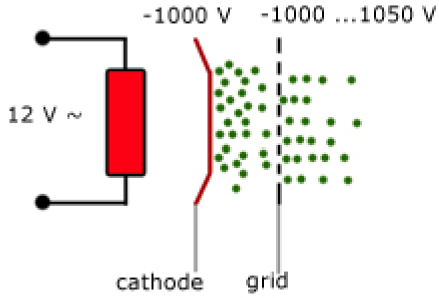


A layer of graphite is painted on the inside of the vacuum tube close to the fluorescent screen. As a result of impacting electrons, the screen acquires a negative charge. To reduce this effect, the graphite is electrically connected to 'earth' (zero volts). This allows excess charge to drain away.
Without this feature, the accumulated charge would reduce the numbers of electrons arriving at the screen, reducing brightness.
The graphite layer also catches rebounding electrons that are back scattered from impact with the screen.

Controls

brightness

The brightness of a CRO display is a measure of the numbers of electrons impacting the screen. The heating element heats the cathode in the 'electron gun' to produce electrons forming a beam current.



This beam current is controlled by having a grid over the cathode. The p.d. of the grid can then be varied to accelerate electrons through it or repel some back to the cathode.

focus

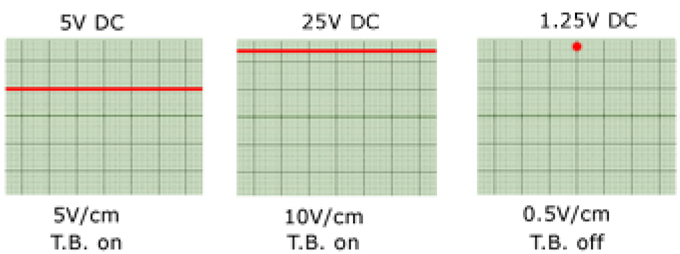
The electron beam is focussed by passage through several annular anodes. These '**collimate**' the beam into a narrower, faster stream of electrons, producing a smaller, sharper dot on the screen.

time-base

This controls how fast the dot moves across the screen. With the time-base switched off, the dot appears static in the centre. With higher settings the dot appears as a horizontal line.
The control has units of '**time/cm**' or '**time/division**', with settings in the range 100 ms - 1 μs per cm/division.

sensitivity/gain

This controls the vertical deflection of the dot. It has units of '**volts/cm**' or '**volts/division**'.



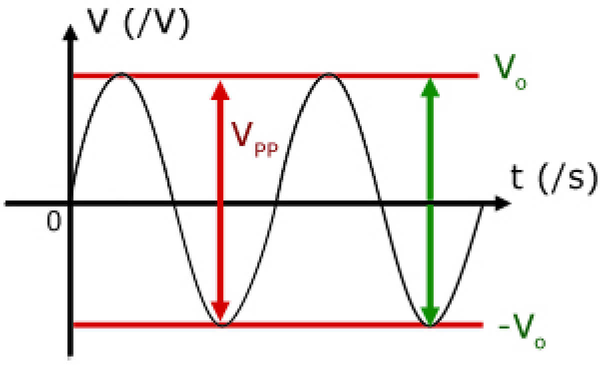
Measurement of alternating voltage

It is important to know the names of vertical measurements on a waveform.

The **peak voltage** (V_0) is the maximum vertical displacement measured from the time-line.

The **peak-to-peak voltage** (V_{PP}) is the vertical displacement between the minimum and maximum values of voltage.

$$V_{PP} = 2 V_0$$



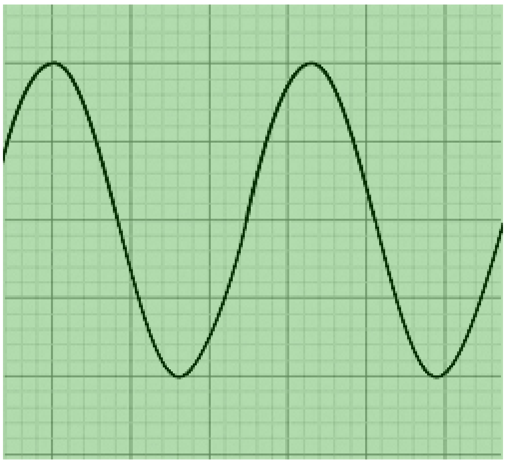
Measurement of frequency

Frequency is measured by finding the 'wavelength' of the waveform along the time-line. In other words to find the**period** (**T**) of the wave (the time interval for one complete oscillation).

T = (horizontal dist. on screen in cm) x (time-base setting)

Once **T** has been measured, the frequency **f** can then be found. The period **T** is inversely proportional to frequency **f** :

$$T = \frac{1}{f}$$



Example: In the example above, measuring from the first crest to the second horizontally, the distance is 3.3 cm.

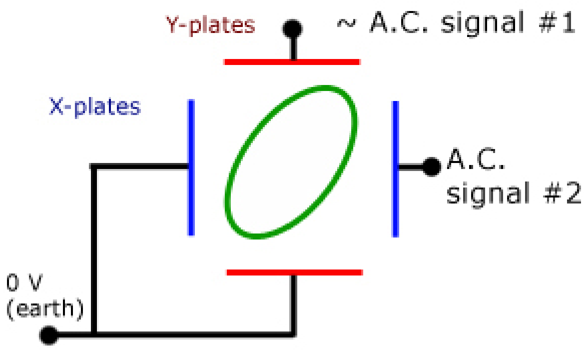
If the time-base is 2ms/cm (2 x 10^{-3} sec./cm), then the period (T) is:

$$T = 3.3 \times 2 \times 10^{-3} = 6.6 \times 10^{-3} \text{ secs.}$$

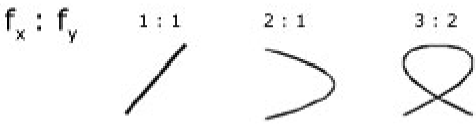
Hence the frequency (f) is:

$$f = (6.6 \times 10^{-3})^{-1} = \underline{151.5 \text{ Hz}} \text{ (1 d.p.)}$$

Another way of measuring frequency is by **Lissajous figures**. These images are obtained by switching off the time-base and inputting an AC signal into it. We then have the case where we have two AC signals, one across the X-plates and the other across the Y-plates.



Different ratios of frequency give different images. In this way an unknown frequency can be identified, provided it is in a simple ratio with the first frequency.



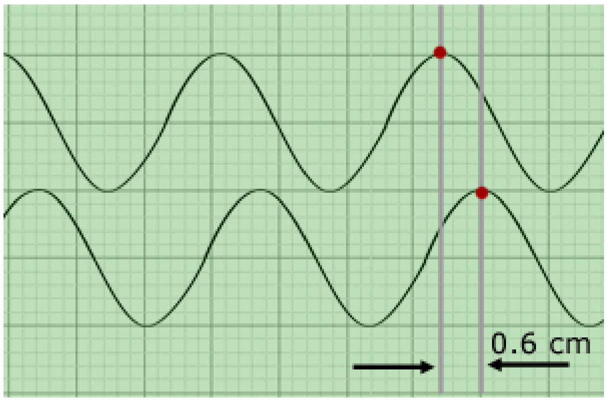
Measurement of phase difference

On a double beam oscilloscope the phase difference in cm is be measured along the time-line between similar points on each wave.

To measure the phase difference in radians, the horizontal length of one wave (in cm) must also be known (the period).

Then the phase difference is given by:

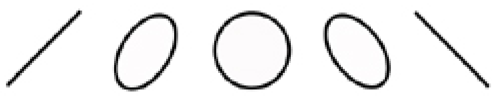
$$\text{phase diff. (rads.)} = \frac{\text{phase diff. (cm)}}{\text{wavelength (cm)}} \times 2\pi$$



Example: The phase difference is 0.6 cm and the wavelength is 3.1 cm. therefore :

phase difference (φ phi) =(2 π x 0.6)/3.1 = 1.2 rads.

Besides frequency, Lissajous figures can also be used to identify phase differences.



0   π/4   π/2   3π/4   π