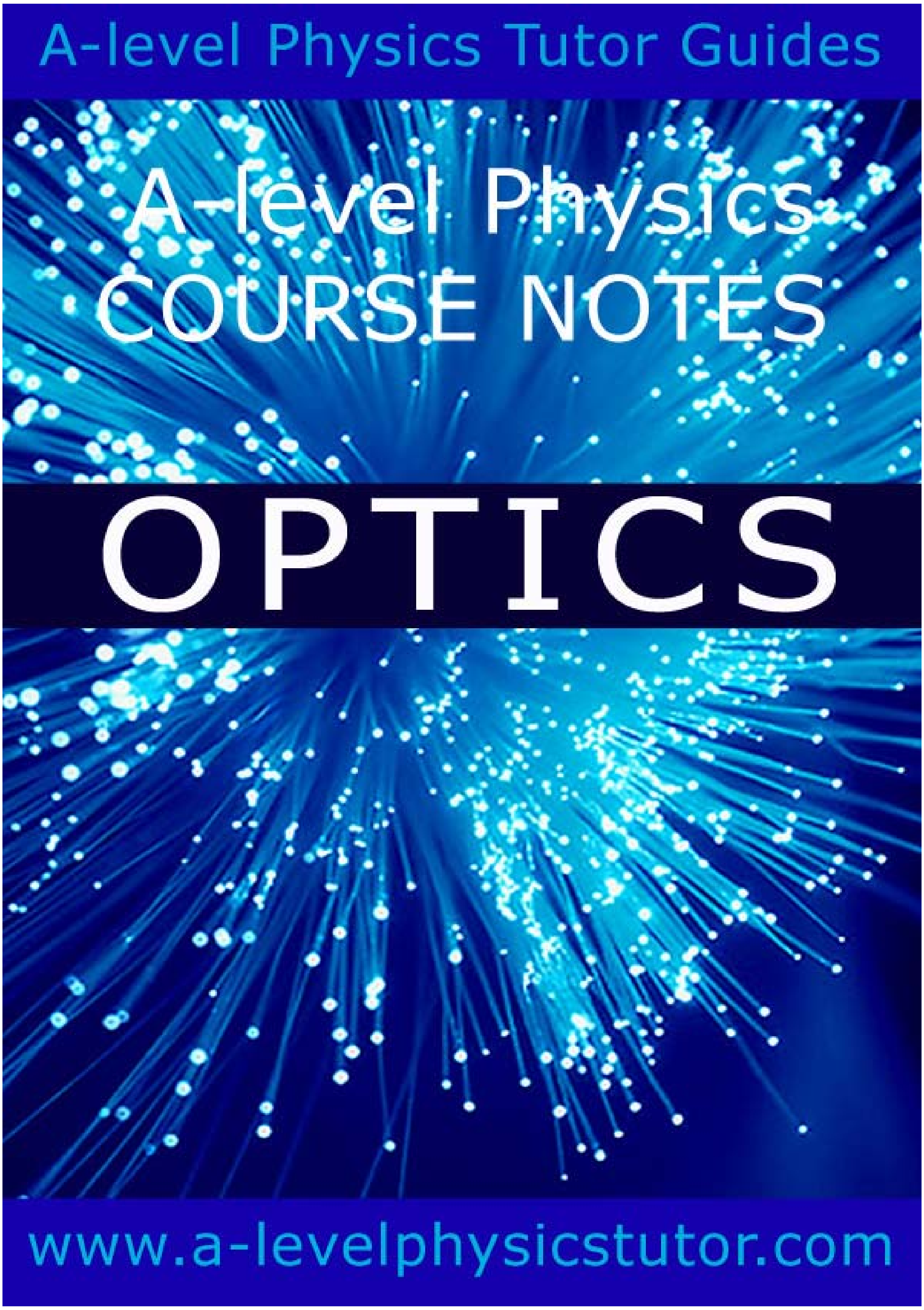

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CONTENTS

Refraction

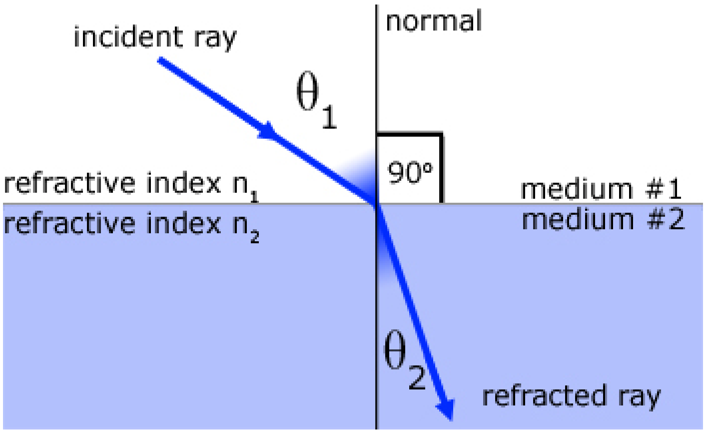
The Laws of Refraction

Consider a single light ray travelling through a low density material (eg air) and being refracted at the surface of a transparent material with higher density (eg glass).

The **normal** is a line drawn at right angles to the material's surface at the ray's point of entry.

The **angle of incidence** is the angle the light ray makes with the normal.

The **angle of refraction** is the angle the refracted light ray makes with the normal inside the material.



1) The incident ray, the refracted ray and the normal at the point of entry are all in the same plane.

2) The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a particular wavelength (Snell's Law).
The ratio constant is called the <u>relative refractive index</u> 'n' .
(older texts - the Greek letter 'μ' mu)

The relative refractive index between two media where a light ray travels through one medium (#1) and is refracted through the other medium (#2) is given by:

$$\frac{\sin\theta_1}{\sin\theta_2} = {}_1n_2$$

Refractive Index

The refractive index of a single medium can be defined as the ratio of the speed of light in a vacuum to the speed of light in the medium.
Here n_m is defined as the **absolute refractive index**

$$n_m = \frac{c_o}{c_m}$$

where,

c_o is the velocity of light in a vacuum
c_m is the velocity of light in the medium

let us consider our two materials(#1 & #2 from above). Their absolute refractive indices are given by:

$$n_1 = \frac{c_o}{c_1} \qquad n_2 = \frac{c_o}{c_2}$$

dividing the second equation by the first,

$$\frac{n_2}{n_1} = \left(\frac{c_o}{c_2}\right)\left(\frac{c_1}{c_o}\right)$$

that is,

$$\frac{n_2}{n_1} = \left(\frac{c_1}{c_2}\right)$$

$$\frac{n_2}{n_1} = {}_1n_2$$

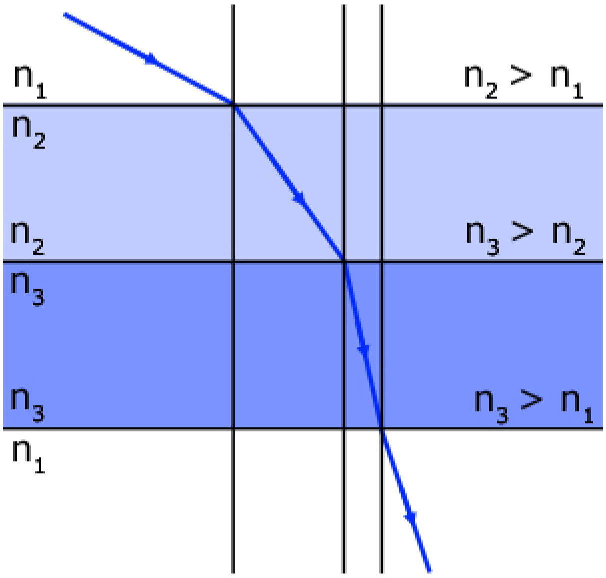
Snell's Law equation can now be rewritten as:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

or

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

note: when a light ray travels from a less dense medium to a denser medium, it bends towards the normal(and vice versa).



Common refractive indices

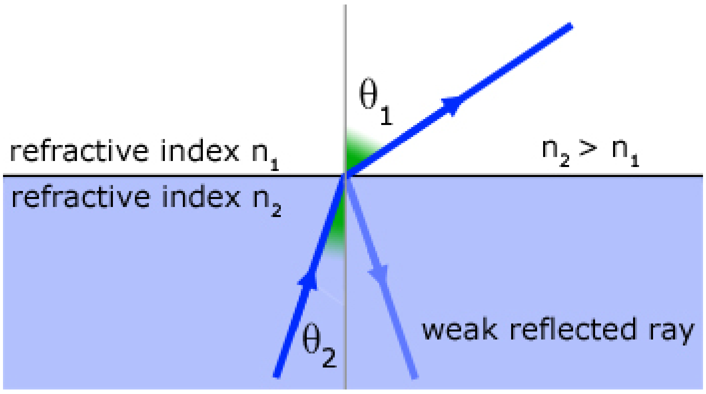
Material	***n*** (λ = 589.29nm)
Water	1.3330
Diamond	2.419
Amber	1.55
Fused silica	1.458
Sodium chloride	1.50
Liquid Helium	1.025
Water ice	1.31
Acrylic glass	1.490 - 1.492
Polycarbonate	1.584 - 1.586
Crown glass	1.50 - 1.54
Flint glass	1.60 - 1.62
Crown glass	1.485 - 1.755
Flint glass	1.523 - 1.925
Pyrex	1.470
Cryolite	1.338
Rock salt	1.516
Sapphire	1.762–1.778
Cubic zirconia	2.15 - 2.18
Moissanite	2.65 - 2.69

Total Internal Reflection

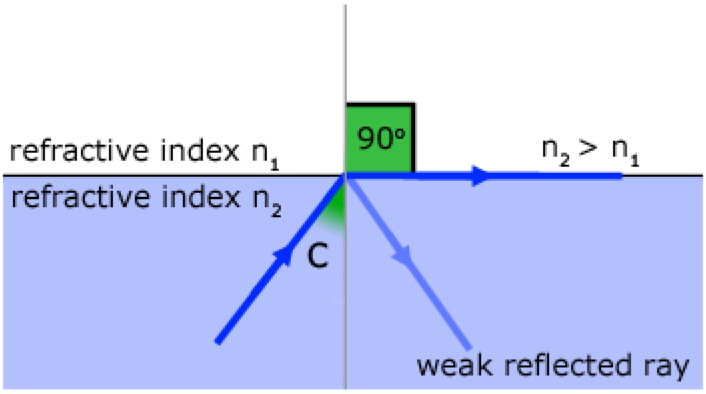
Critical Angle

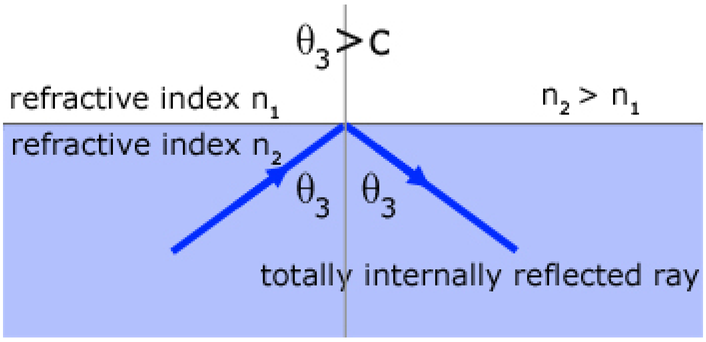
The **Critical Angle** (c^o)is the angle of incidence in a dense medium, such that the angle of refraction in the less dense medium is 90^o .

Looking at the diagrams(below), as the angle of incidence in the dense medium is increased, the angle of refraction increases towards 90^o. During this time a weak reflected ray is also observed.



Only when the angle of incidence in the medium exceeds the Critical Angle does all the light become reflected internally.





We can formulate an equation for the critical angle using Snell's Law for two media of refractive index $\boldsymbol{n}_1$ & $\boldsymbol{n}_2$.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

When $\boldsymbol{\theta}_1 = 90^o$ and $\boldsymbol{\theta}_2 = c^o$,

$$n_1 \sin(90^o) = n_2 \sin(c)$$

but sin(90°) = 1, therefore:

$$n_1 = n_2 \sin(c)$$

$$\sin(c) = \frac{n_1}{n_2}$$

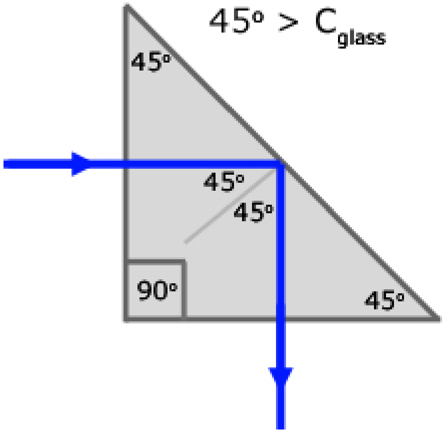
from work on relative refractive index,

$$\frac{n_2}{n_1} = {}_1n_2 \qquad \frac{n_1}{n_2} = \frac{1}{{}_1n_2}$$

$$\underline{\therefore \sin(c) = \frac{1}{{}_1n_2}}$$

90°deviation with a 90° prism

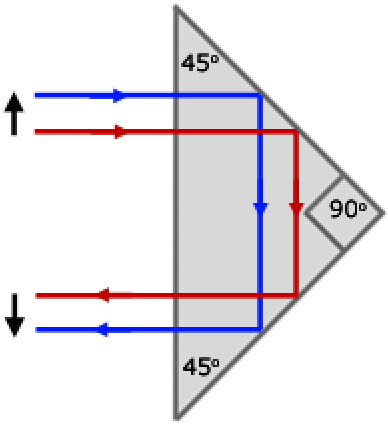
Total internal reflection in glass depends on the fact that its critical angle is approximately 42° . A light ray with an angle of incidence greater than this will be totally internally reflected. So a light ray with an angle of incidence of 45° will be reflected, and its angle of reflection will also be 45°. Hence the light ray is deviated through an angle of 90° .



For 90° deviation, a 90° isosceles glass prism is used. In this way the incident ray and the reflected rays are normal to the surfaces they enter and leave(and therefore unhindered). Total internal reflection occurs on the hypotenuse.

180°deviation with a 90° prism

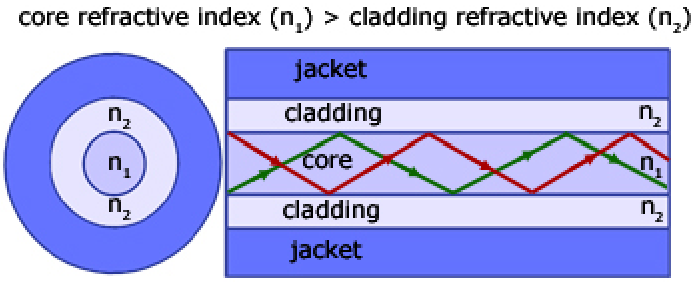
Besides a 90° deviation, an isosceles prism is also used to produce a 180° deviation, but this time reflection occurs on the equal, adjacent sides of the prism and not on the hypotenuse.



By tracing two light rays though the prism, it can be seen that the image produced is inverted. As a result of the image being reflected twice, lateral inversion is cancelled.

Optical fibres

Optical fibre typically consists of a **core** of high refractive index surrounded by **cladding** with a lower refractive index. Light is totally internally reflected down the fibre at the boundary of the two media.



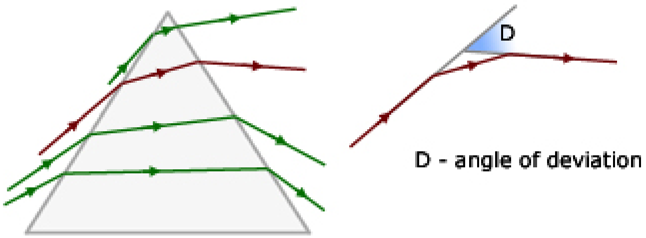
Optical fibre has a number of advantages over copper wire:

1. less attenuation
2. cheaper metre for metre
3. can carry more information
4. immune to electrical interference
5. safer - no fire risk as with electric currents
6. wire-tapping more difficult

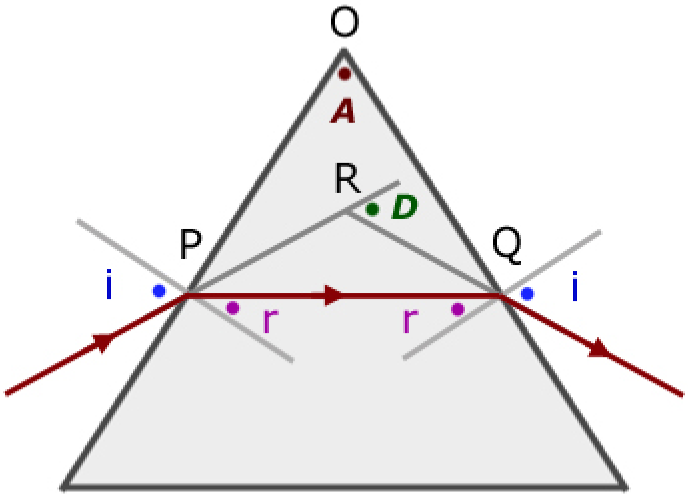
Prisms

Deviation

Deviation, measured in degrees, is the angle an incident ray is turned through after passing through a prism(or other optical component).
This deviation is a minimum for a prism when the path of a light ray is symmetrical about its axis of symmetry.



Derivation of Minimum Deviation ***D***



In $\triangle PQR$

$$D = \angle RPQ + \angle PQR$$ (i

(***D*** external angle of $\triangle PQR$)

around points **P** and **Q** respectively,

$$i = \angle RPQ + r \qquad i = \angle PQR + r$$

$$\angle RPQ = i - r \qquad \angle PQR = i - r$$

substituting into equation (i

$$\therefore \ \underline{D = 2(i - r)}$$

(ii

In $\triangle OPQ$

$$A + \angle OPQ + \angle PQO = 180^\circ$$
$$A + \left(90^\circ - r\right) + \left(90^\circ - r\right) = 180^\circ$$

$$\underline{A = 2r}$$

(iii

adding together equations (ii & (iii,

$$\begin{aligned} A + D &= 2r + 2(i - r) \\ &= 2r + 2i - 2r \\ &= 2i \end{aligned}$$

$$\underline{i = \frac{A + D}{2}}$$

(iv

rearranging equation (iii,

$$\underline{r = \frac{A}{2}}$$

(v

Using Snell's Law equation,

$$n_1 \sin(i) = n_2 \sin(r)$$

and substituting for **i** and **r** from equations (iv & (v above,

$$n_1 \sin\left(\frac{A+D}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

n_1 is the refractive index of air which approximates to 1. Hence the equation becomes:

$$\sin\left(\frac{A+D}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

<u>Example</u>

So for a 60^o equilateral prism made of glass(n = 1.5 approx.), the minimum deviation angle D is given by:

$$\begin{aligned}\sin\left(30^\circ + \frac{D}{2}\right) &= 1.5 \times \sin(30^\circ) \\ &= 1.5 \times 0.5 \\ &= 0.75\end{aligned}$$

It follows that:

$$30^\circ + \frac{D}{2} = \sin^{-1}(0.75)$$

remembering that $\sin^{-1}(x)$ means the angle whose sine is 'x',

$$\begin{aligned}30^\circ + \frac{D}{2} &= 48.59^\circ \\ \frac{D}{2} &= 48.59^\circ - 30^\circ \\ &= 18.59^\circ \\ D &= 2 \times 18.59^\circ \\ \underline{D} &\underline{= 37.18^\circ}\end{aligned}$$

Chromatic dispersion

The term **Chromatic dispersion** describes how refractive index changes with wavelength for a particular medium.

Medium	Violet 400 nm	Red 650 nm
Crown glass	10.00	10.37
Acrylic	10.46	10.87
Fused quartz	11.30	11.58

data courtesy of Serway & Jewett

Using refractive index theory we can formulate equations in terms of the wavelength of the light involved:

Remembering that: n_m is defined as the **absolute refractive index**

$$n_m = \frac{c_o}{c_m}$$

where,

c_o is the velocity of white light in a vacuum
c_m is the velocity of white light in the medium

It follows that: n_v is defined as the **absolute refractive index for violet light**

$$n_v = \frac{c_o}{c_{m(v)}} \qquad \text{(i}$$

where, $c_{m(v)}$ is the velocity of violet light in the medium

Similarly, n_r is defined as the **absolute refractive index for red light**

$$n_r = \frac{c_o}{c_{m(r)}} \qquad \text{(ii}$$

where, $c_{m(r)}$ is the velocity of red light in the medium

Since frequency f is constant throughout,

$$c_o = f\lambda_o$$

and

$$c_{m(v)} = f\lambda_{m(v)} \qquad c_{m(r)} = f\lambda_{m(r)}$$

where,

λ_o is the wavelength of <u>white</u> light in a vacuum
$\lambda_{m(v)}$ is the wavelength of <u>violet</u> light in the medium
$\lambda_{m(r)}$ is the wavelength of <u>red</u> light in the medium

Substituting for $c_{m(v)}$, $c_{m(r)}$ and c_o into equations (i and (ii ,

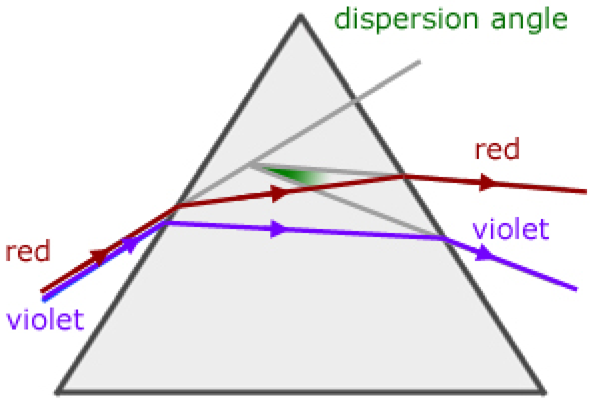
$$n_v = \frac{f\lambda_o}{f\lambda_{m(v)}} \qquad n_r = \frac{f\lambda_o}{f\lambda_{m(r)}}$$

cancelling ***f*** in each equation,

$$n_v = \frac{\lambda_o}{\lambda_{m(v)}} \qquad n_r = \frac{\lambda_o}{\lambda_{m(r)}}$$

<u>Minimum angle of dispersion</u>

This is measure of the angle of 'spread' of a spectrum when it leaves a prism. For minimum angular dispersion, the angle is derived from the difference in deviation between red and violet rays of light.



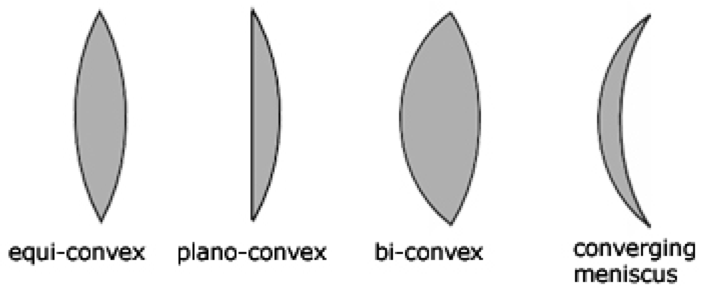
Replacing $\boldsymbol{n_2}$ with $\boldsymbol{n_r}$ for red, and $\boldsymbol{n_v}$ for violet light in the minimum deviation equation below,

$$\sin\left(\frac{A+D}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

we can calculate values of deviation for each colour. Subtracting the angles gives the minimum dispersion angle for white light.

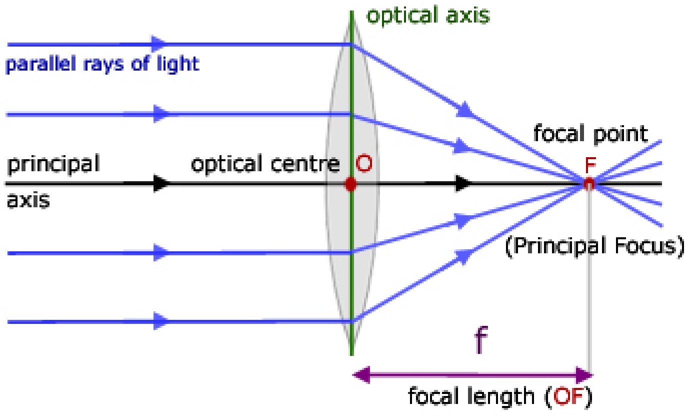
Convex Lenses

Types of lens



All four types of convex lens are converging lenses. That is, they bring parallel rays of light to a focus, producing a real image.

Basic ray diagram



The basic ray diagram for a convex lens introduces a number of important terms:

principal axis - the line passing through the centres of curvature of the lens

principal focus - a point on the principal axis where rays of light parallel to the principal axis converge

focal length - the horizontal distance between the principal focus and the optical centre of the lens

optical centre - an imaginary point inside a lens through which a light ray is able to travel without being deviated

centre of curvature - the centre of the sphere of which the lens surface is part

Power of a lens

$$P = \frac{1}{f}$$

The power P of a lens is the inverse of its focal length f. Since f is measured in metres 'm' the units of lens power are m^{-1}.

The power also depends on the type of lens. **Convex** lenses have **positive** powers, while **concave** lenses all have **negative** powers.

For example, a 10 cm focal length convex lens has a power of +10 m^{-1}; while a 20 cm focal length concave lens has a power of -5 m^{-1}.

Ray diagrams

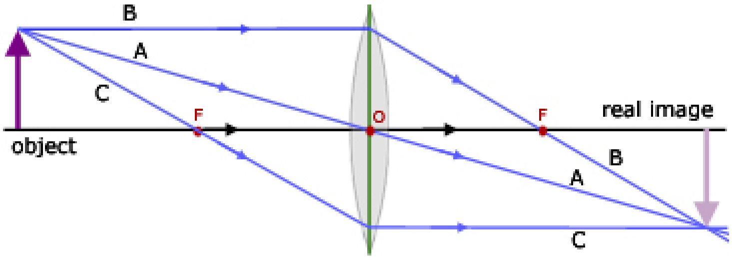
To understand ray diagrams it is important to know something about images. Images come in two categories :

real images - are produced from actual rays of light coming to a focus (eg a film projected onto a screen)

virtual images - are produced from where rays of light appear to be coming from (eg a magnifying glass image)

Ray diagrams are constructed by taking the path of three distinct rays from a point on the object:

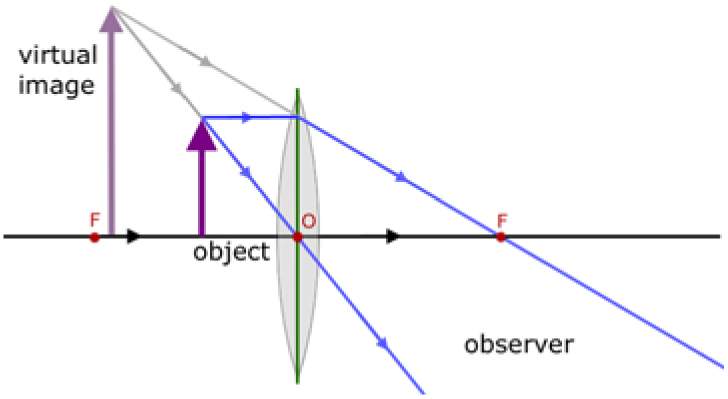
note - the lens is considered to be so thin as to be represented by the axis of the lens(green)



- **A**) a ray passing through the optical centre of the lens
- **B**) a ray parallel to the principal axis, which refracts through the lens, passing through the principal focus
- **C**) a ray passing through the principal focus(on the same side as the object) and being refracted through the lens, emerging parallel to the principal axis

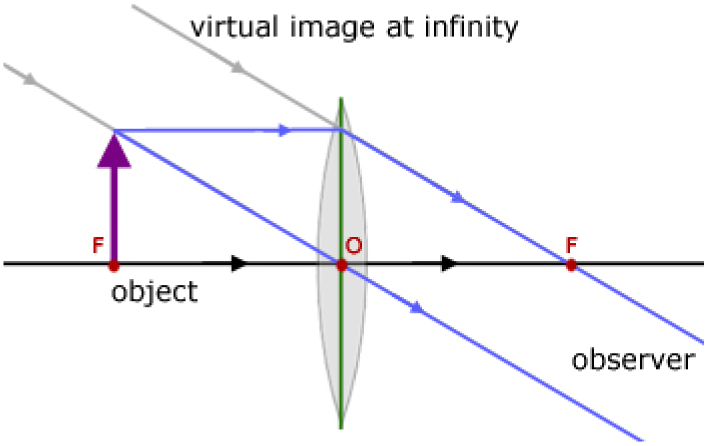
The diagrams represent the formation of an image from an object positioned between *f* and the lens, at *f*, between f and 2f and at 2*f* from the lens

object between *f* and the lens (magnifying glass)



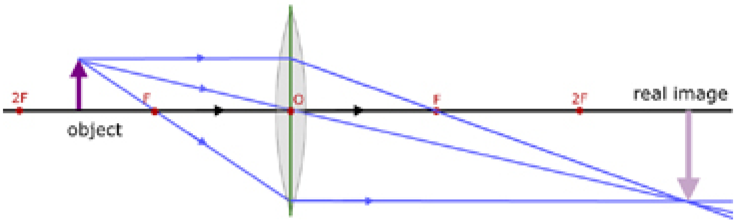
The image is the same side of the lens as the object and is upright, virtual and magnified.

object at *f*



The image is formed at infinity from parallel rays that do not converge. Therefore no image is formed.

object between f and 2f (projector)



The image is on the opposite side of the lens than the object. It is real, inverted and magnified.

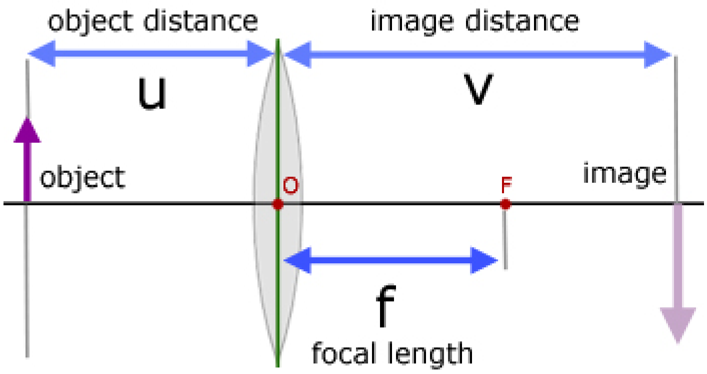
object at 2*f*

The diagram is very similar to the one showing the three construction rays.
The image is the same distance behind the lens as the object is infront.
The image is inverted, real and the same size as the object.

object at infinity

The image is formed at the focal point of the lens. It is real, inverted and diminished in size.

The Lens Formula



$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

When using this equation a sign convention must be obeyed:

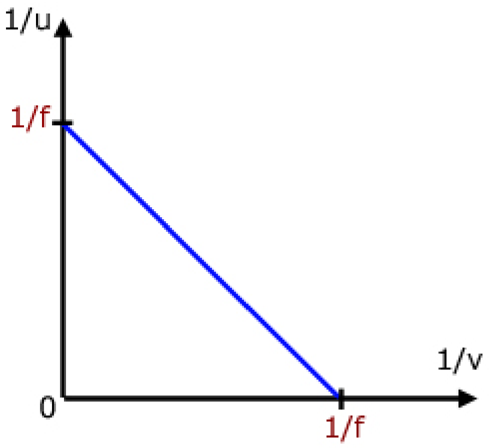
Distances from lenses to real objects & real images are positive

Distances from lenses to imaginary objects and imaginary images are negative.

Focal lengths of convex lenses are positive.

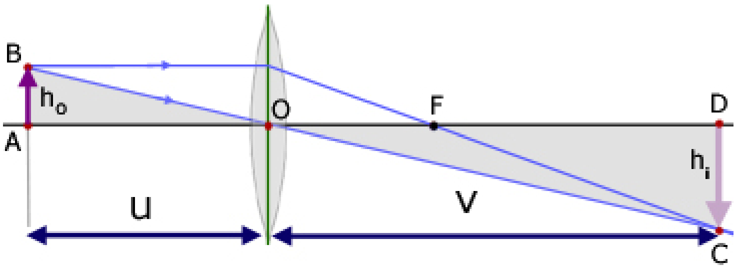
Focal lengths of concave lenses are negative.

The focal length *f* of a lens can be found quite accurately by moving an illuminated object infront of a lens so that an image is cast on a screen. By taking readings of image and object distances for different positions, a graph can be drawn.



Plotting 1/u against 1/v gives a straight line with a negative gradient. The focal length can be found from the intercept, which is 1/f on both the x and y axes.

Magnification



Magnification (***m***) is simply the image height divided by the object height.

$$m = \frac{h_i}{h_o}$$

To obtain a relation involving object distance (***u***) and image distance (***v***), consider the image formed by two rays from a point on the object.

Triangles **AOB** and **COD** are similar.

Therefore,

$$\frac{CD}{AB} = \frac{DO}{AO}$$

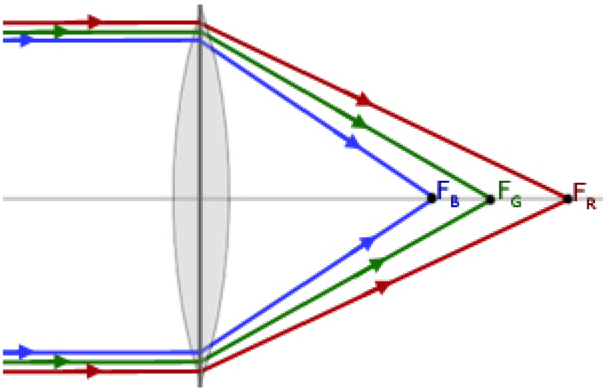
$$\frac{h_i}{h_o} = \frac{v}{u}$$

$$\underline{m = \frac{v}{u}}$$

Chromatic Aberration

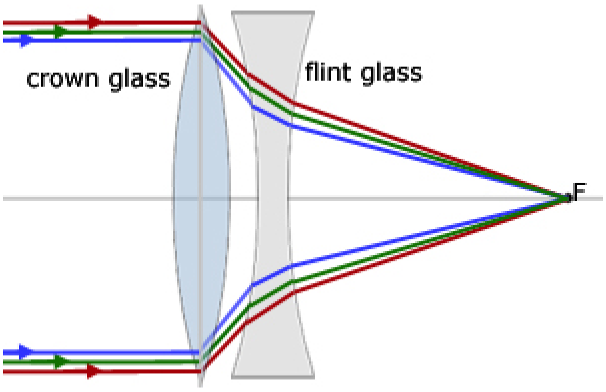
Chromatic aberration is the dispersion of white light by a convex lens. The different coloured components of white light are brought to different foci according to their wavelength. Since 'red rays refract least', red light produces the longest wavelength and violet the shortest.

In the diagram, $\mathbf{F_B}$, $\mathbf{F_G}$, $\mathbf{F_R}$ are the principal foci for blue, green and red light.



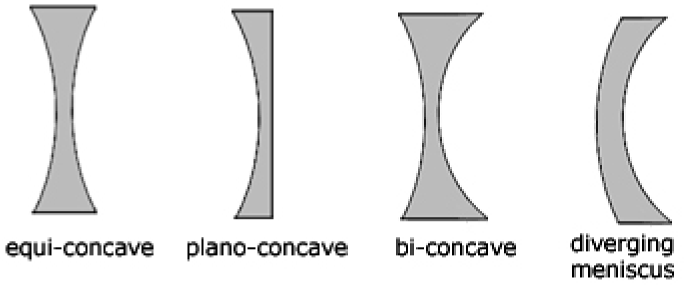
Chromatic aberration affects the image by making it appear blurred with fringes of colour around it. This is a result of only one colour being in focus at a time.

Chromatic aberration can be corrected using a **chromatic doublet**. This is a combination of two lenses, one convex and the other concave. The lenses are of different types of glass (crown glass & flint glass) . The pair are cemented together (not shown) using Canada Balsam glue. This has a refractive index mid-way between the two glass types. In any event, the glue layer is extremely thin and has little effect.



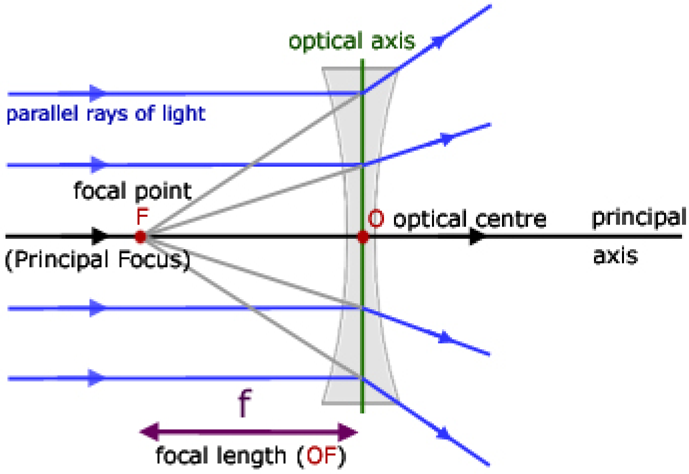
Concave Lenses

Types of lens



All four types of concave lens are diverging lenses. That is, they diverge parallel rays of light from a focus, producing a virtual image.

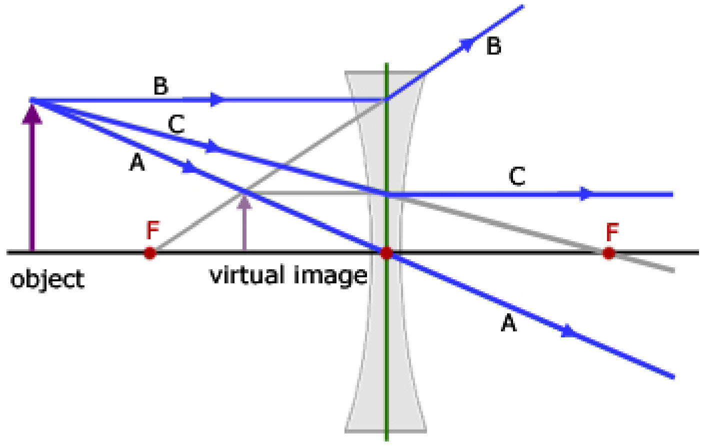
Basic ray diagram



Ray diagrams

Ray diagrams are constructed by taking the path of three distinct rays from a point on the object:

note - the lens is considered to be so thin as to be represented by the axis of the lens(green)



A) a ray passing through the optical centre of the lens

B) a ray parallel to the principal axis, which refracts through the lens and appears to have come from the principal focus

C) a ray heading towards the principal focus(on the opposite side of the lens) and being refracted through the lens, emerging parallel to the principal axis

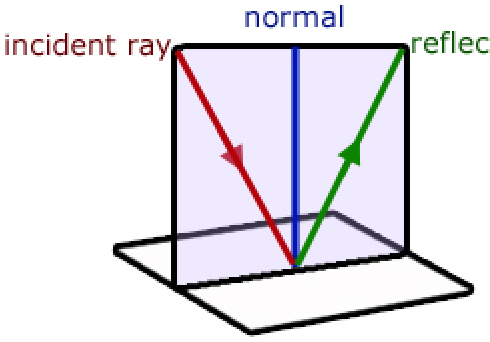
For all the object positions listed below,

object between *f* and the lens
object at *f*
object between f and 2f
object at 2*f*
object at infinity

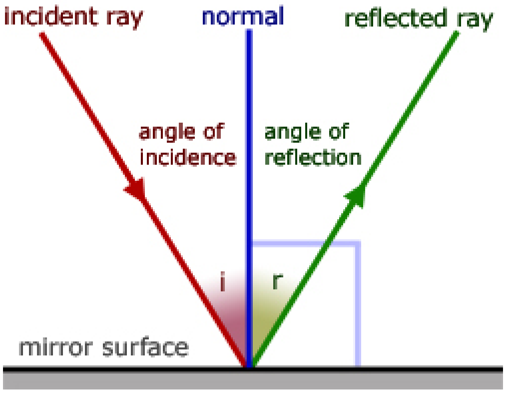
the ray diagrams are virtually the same as in the diagram above. Hence the result is the same. The image produced is virtual, upright, diminished and on the same side of the lens as the object.

Plane Mirrors

Laws of Reflection

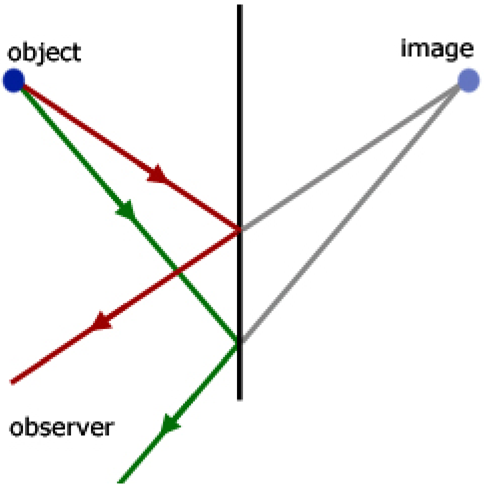


1. The incident ray, the reflected ray and the normal, at the point of incidence, all lie in the same plane.



2. The angle of reflection equals the angle of incidence

Hence an image can be located by taking two light rays from a point object and retracing them after reflection.

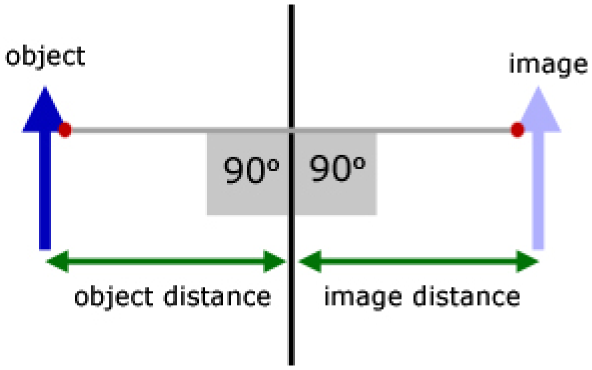


Plane mirror images

1. All images are virtual. That is, they cannot be projected on to a screen.

2. The image produced in a mirror is as far behind the mirror as the object is infront.

object distance = image distance



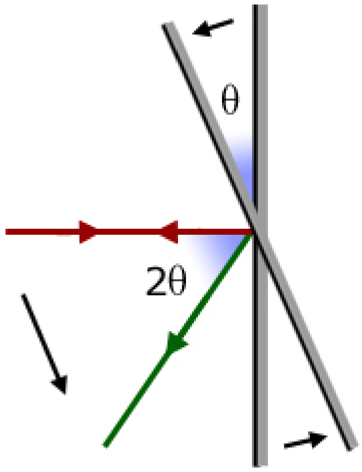
3. The image is the same size as the object.

4. A line joining a point on the image to a corresponding point on the object is perpendicular to the mirror.

5. The image is laterally inverted (sideways upside down).

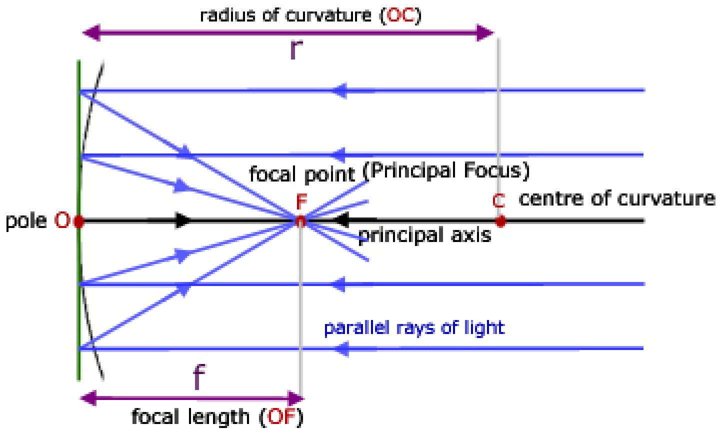
Mirror rotation

When a light ray is normal to a mirror it reflects back the way it came. However, if the mirror is tilted say, through θ °(theta) then the reflected light ray rotates through 2θ ° .



Concave Mirrors

Basic ray diagram



The basic ray diagram for a concave mirror introduces a number of important terms:

aperture - the diameter of the circular mirror

pole - where the principal axis meets the mirror surface

centre of curvature - the centre of the sphere that the mirror forms part

radius of curvature (*r*) - radius of the sphere

principal axis - the line through the centre of curvature and the pole of the mirror

focal length (*f*) - equal to half the radius of curvature $f = r / 2$

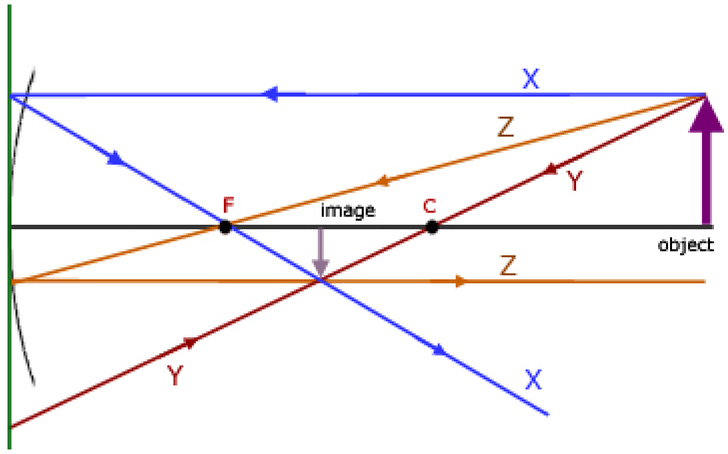
Ray diagrams

Ray diagrams are constructed by taking the path of three distinct rays from a point on the object:

X) a ray parallel to the principal axis reflected through **F** (the principal focus)

Y) a ray passing through **C** which is then reflected back along its original path

z) a ray passing through F, which is then reflected parallel to the principal axis



note - the concave mirror is considered to be so thin as to be represented by a vertical line

The image types and positions for a concave mirror are very similar to those of a convex lens.

object between F and the lens

The image is upright, virtual and magnified.

object at F

The image is formed at infinity from parallel rays that do not converge. Therefore no image is formed.

object between F and C

The image is real, inverted and magnified.

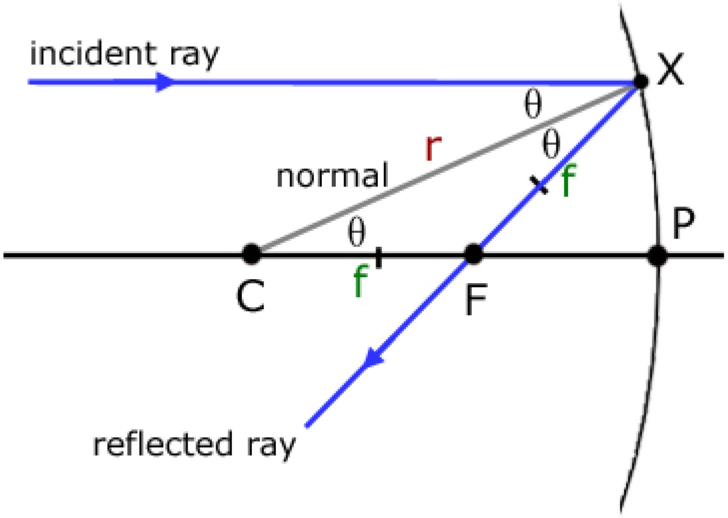
object at C

The image is formed at C.
The image is inverted, real and the same size as the object.

object at infinity

The image is formed at the focal point of the lens. It is real, inverted and diminished in size.

Proof of r = 2f



The angle of incidence equals the angle at C (corresponding angles).

From the diagram it is apparent that **XF** does not equal **PF**. However, when point **X** is closer to the principal axis, **XF** approaches the size of **PF**. So for rays close to the principal axis we can say that **XF = PF**.

In other words,

XC = PF + FC

r = f + f

r = 2f

The Mirror Formula

$$f = \frac{r}{2} \qquad \frac{1}{f} = \frac{2}{r}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \underline{\frac{1}{u} + \frac{1}{v} = \frac{2}{r}}$$

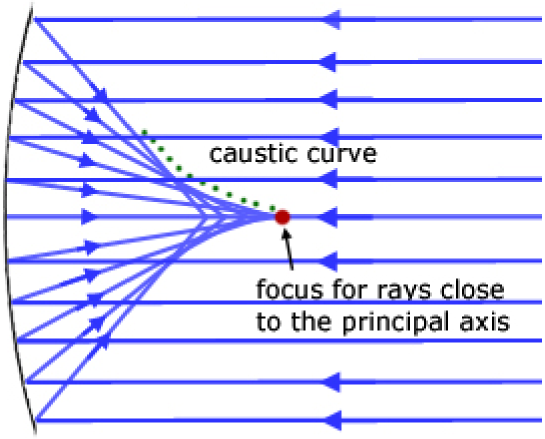
The sign convention, '**real is positive**' is used:

1) focal length (*f*) and radius of curvature (*r*) are both positive for concave mirrors

2) distances to real images and real objects are positive

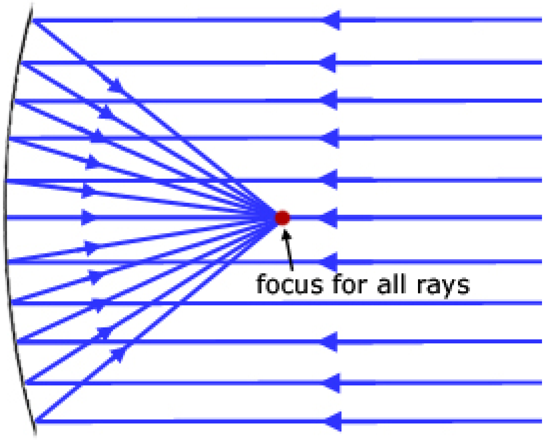
3) distances to virtual images and virtual objects are negative

Caustic Curves



The **caustic** is the name given to the region of a concave mirror where parallel rays of light come to different foci. This happens for rays away from the principal axis. The further away they are, the closer is the focus to the mirror.

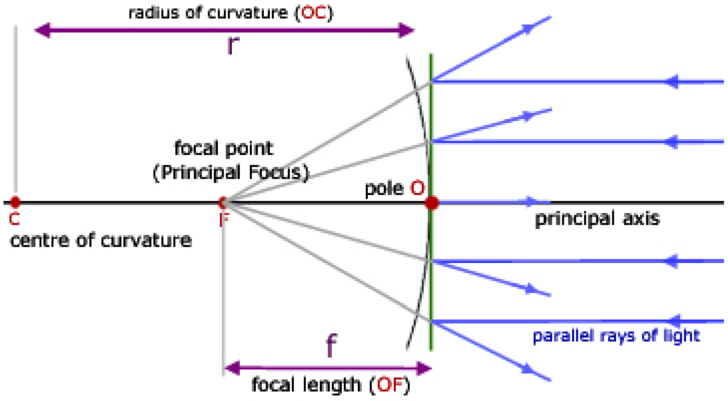
Parabolic mirrors



A parabolic mirror produces <u>one</u> focus for all rays parallel to the principal axis, irrespective of their distance from it. Parabolic mirrors have uses in telescopes, solar furnaces, and car headlights/torches/floodlights etc.

Convex Mirrors

Basic ray diagram



The basic ray diagram for a convex mirror introduces a number of important terms:

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centre of curvature - the centre of the sphere that the mirror forms part

radius of curvature (*r*) - radius of the sphere

principal axis - the line through the centre of curvature and the pole of the mirror

focal length (*f*) - equal to half the radius of curvature $\boldsymbol{f = r/2}$

Ray diagrams

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X) a ray parallel to the principal axis reflected through **F** (the principal focus)

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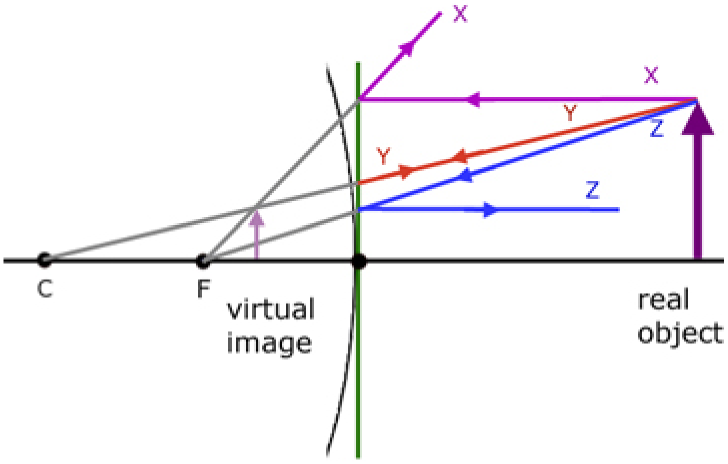
Z) a ray passing through F, which is then reflected parallel to the principal axis

note - the convex mirror is considered to be so thin as to be represented by a vertical line

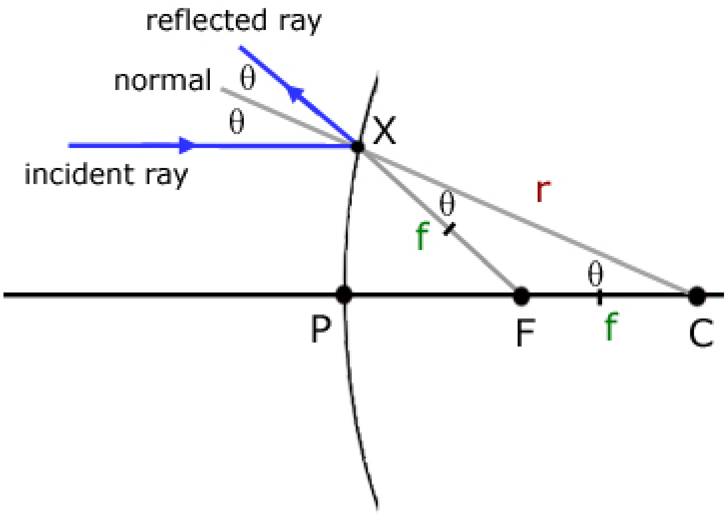


For all the object positions listed below,

object between *f* and the mirror
object at *f*
object between f and 2f
object at 2*f*
object at infinity

the ray diagrams are virtually the same as in the diagram above. Hence the result is the same. The image produced is virtual, upright and diminished.

proof of r = 2f



The angles in the triangle are vertically opposite and corresponding angles from the angles of reflection and incidence, respectively.

From the diagram it is apparent that **XF** does not equal **PF**. However, when point **X** is closer to the principal axis, **XF** approaches the size of **PF**. So for rays close to the principal axis we can say that **XF = PF**.

In other words,

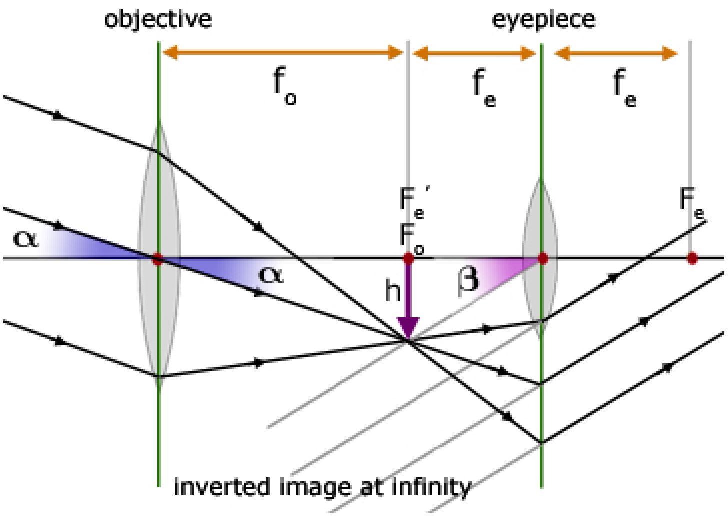
$$XC = PF + FC$$

$$r = f + f$$

$$r = 2f$$

Telescopes

Astronomical refracting telescope



The angle subtended by the object ***α*** (alpha), is given by:

$$\alpha = \frac{h}{f_o}$$

The angle subtended by the image ***β*** (beta), is given by:

$$\beta = \frac{h}{f_e}$$

Angular magnification is by definition:

$$M = \frac{\beta}{\alpha}$$

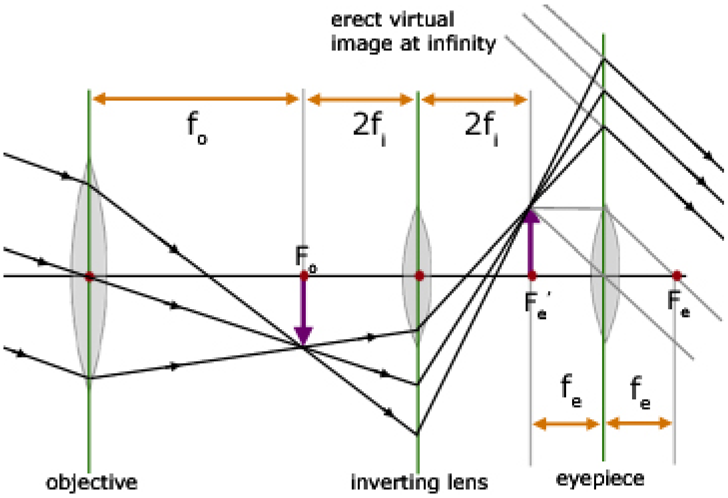
Substituting for ***α*** (alpha) and ***β*** (beta) from above,

$$M = \left(\frac{h}{f_e}\right)\left(\frac{f_o}{h}\right)$$

$$\underline{M = \frac{f_o}{f_e}}$$

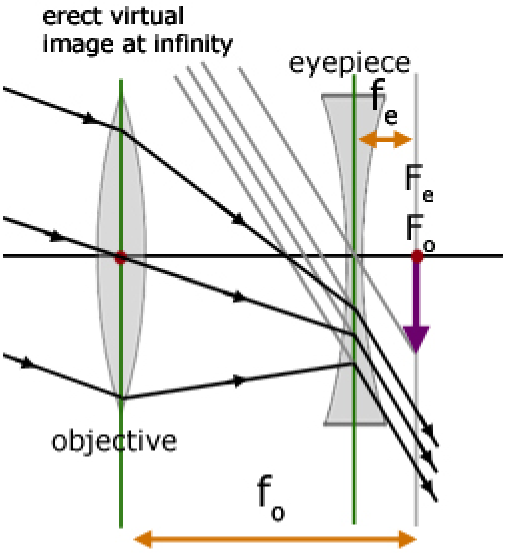
So for maximum magnification, a telescope is requires a long objective focal length and a short eyepiece focal length.

Terrestrial telescope



The terrestrial telescope has one more convex lens than an astronomical telescope. The lens is used to invert the intermediary image, while not affecting the image size. This is done by placing the image from the objective at 2f from the lens. Another image is then produced at 2f on the other side of the lens. This image in turn is magnified by the eyepiece.

Galilean telescope



The Galilean telescope produces an erect image from a convex objective len and a concave lens eyepiece.

The telescope is much shorter than both the astronomical and terrestrial telescopes. It's simple design makes it ideal as 'opera' glasses. However, for important astronomical and field work it is severely limited by having a very small 'field of view'.

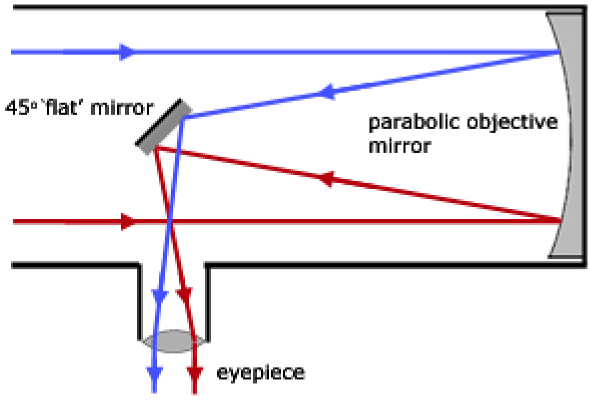
Reflecting telescopes

Reflecting telescopes all use a concave mirror to produce the primary image.

It is true that refracting telescopes score points by their compactness. However, that aside, reflectors do have a number of advantages:

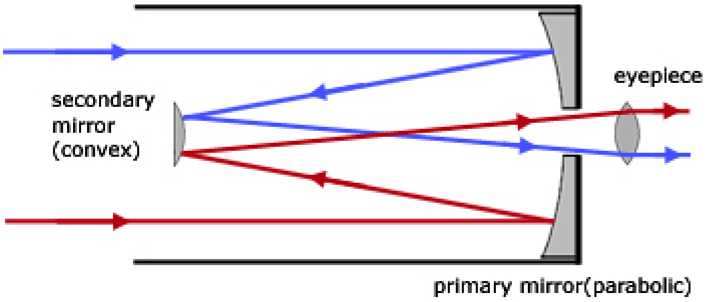
1) Mirrors can be made metres in diameter. Lenses are limited by their weight, which must be supported around the edges. Approximately one metre in diameter is the practical limit for a lens.
2) Mirrors do not produce chromatic abberation.
3) Mirror glass need not be of as high a purity as a lens glass.
4) Mirrors produce less spherical aberration than lenses.

Newtonian reflector



The primary image from the parabolic mirror is directed at 45°from the principal axis by a plane (flat) mirror into an eyepiece located at the side of the telescope.

Cassegrain reflector

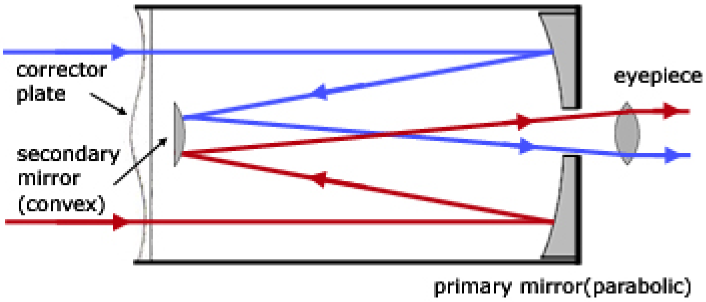


In the Cassegrain reflector light from the primary mirror is reflected backwards from a secondary, convex mirror and directed through the middle of the mirror. here the image is further magnified by an eyepiece.

The Cassegrain has an advantage over a Newtonian by having a large f-number (ratio of focal length to primary mirror diameter). This allows much greater magnification to be attained.

One other important advantage is the length of the telescope. Cassegrains are much shorter, typically less than half the length.

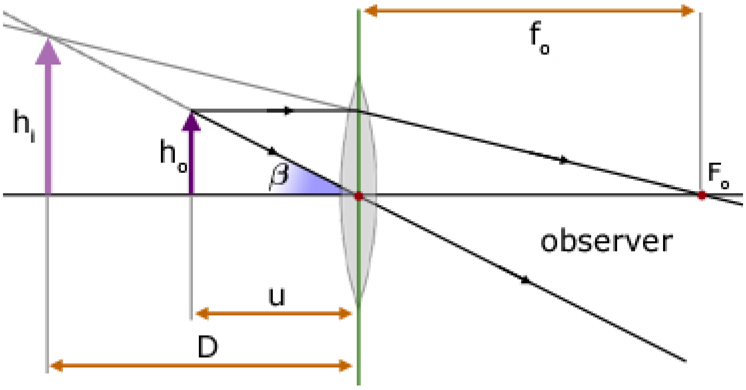
Schmidt-Cassegrain telescope (SCT)



Along with the other advantages of a Cassegrain, a SCT reduces spherical aberration to a minimum. It does this using a 'corrector plate'. This is a specially designed lens, having properties of both convex and concave lenses.

Microscopes

Magnifying Glass/Simple Microscope (image at Near Point)



D is the **Near Point** of the eye. This is the closest an object can be to the eye and remain in focus.

By definition, magnification **M** is the height of the image h_i divided by the height of the object h_o :

$$M = \frac{h_i}{h_o}$$

From the diagram, the angle β (beta) is given by:

$$\beta = \frac{h_o}{u} = \frac{h_i}{D}$$

rearranging,

$$\frac{h_i}{h_o} = \frac{D}{u}$$

$$\underline{M = \frac{D}{u}}$$

If we now use the lens equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

In this case, **v = - D** . The image is virtual. So the sign is negative.

Hence,

$$\frac{1}{u} - \frac{1}{D} = \frac{1}{f}$$

Multipling both sides by **D**, and taking the second term over to the right,

$$\frac{D}{u} - 1 = \frac{D}{f}$$

$$\frac{D}{u} = \frac{D}{f} + 1$$

From our derivation of magnification M (above),

$$M = \frac{D}{u}$$

therefore our equation becomes,

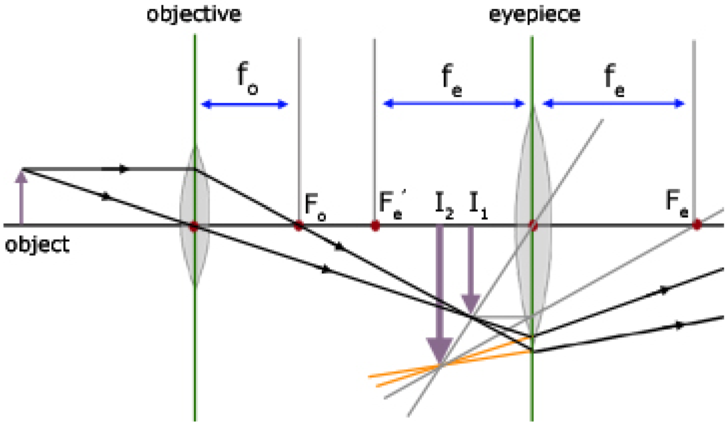
$$\underline{M = \frac{D}{f} + 1}$$

With the image at the near point, the magnification of an object by a magnifying glass can be simplified as:

$$M = \frac{25}{f} - 1$$

where ***f*** is measured in centimetres(cm)

Compound microscope



A microscope is very similar in arrangement to a telescope, the difference being in the focal length of the objective lens.

Microscope lens focal lengths are measured in mm, while telescope focal lengths can be measured in metres.

Essentially a real image is formed by the objective and this in turn is magnified by the eyepiece to form a virtual, erect image.

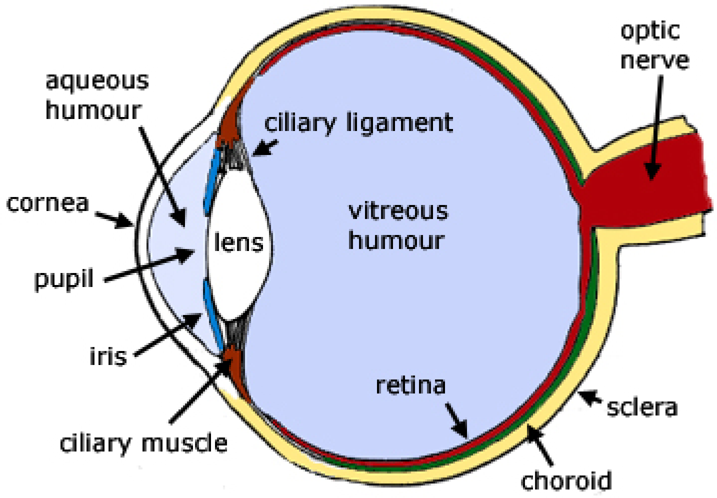
The first image (I_1) is positioned infront of the eyepiece, between f and the lens. The eyepiece produces the virtual image (I_2) behind the first image.

For the second image to be in focus, the distance between it and the eye must be at least 25 cm (D).

The **magnifying power** of a microscope is the product of the eyepiece and objective lens magnification.

The Human Eye

Eye biology



retina - a light sensitive region on the rear of the inner surface of the eyeball.

accommodation - the ability of the eye to produce a focussed image on the retina. This is done by altering the shape of the eye-lens by muscle in the shape of a ring, called the **ciliary muscle**. The **ciliary ligament** transfers force between the muscle and the lens.

iris - a ring of muscle controlling the amount of light entering the eye.

lens - made of clear cartilage. In old age the lens can become opague. In a simple procedure it can be replaced with a plastic lens.

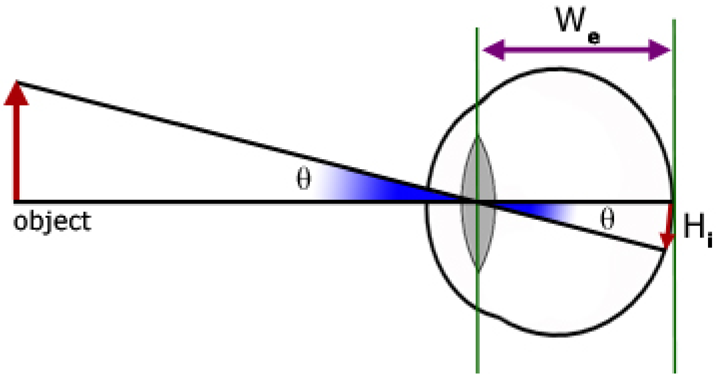
cornea - front part of the eye. Most of the deviation of light coming from an object occurs at the air/cornea boundary. Old age/disease can cause the cornea to become fogged, eventually causing blindness. Transplants from cadavers can remedy this.

pupil - the space inside the iris. This appears black because it leads into the eyeball. Inside the eyeball is caverness and dimly lit from light entering.

humours - the aqueous and vitreous humours are clear liquids within the eye with similar refractive indices.

aqueous humour (1.33)
vitreous humour (1.34)
eye lens (1.41)
cornea (1.38)

Visual Angle



The size of objects perceived by the eye depends on the size of their image on the retina.

Suppose an object subtends an angle ***θ*** at the eye, producing an image of height $\mathbf{H_i}$ on the retina.

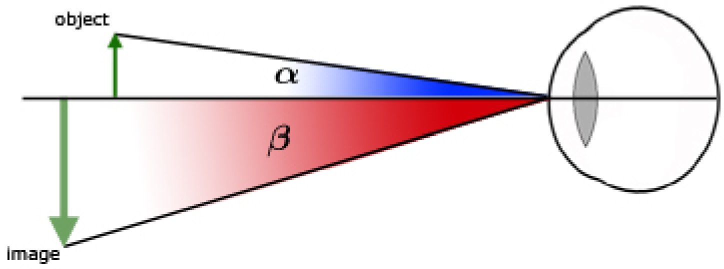
If $\mathbf{W_e}$ is the width of the eyeball between lens and retina, then for small angles, to a good approximation:

$$H_i = W_e\theta$$

Angular Magnification (magnifying power)

This is used in optical instruments - microscopes, telescopes.

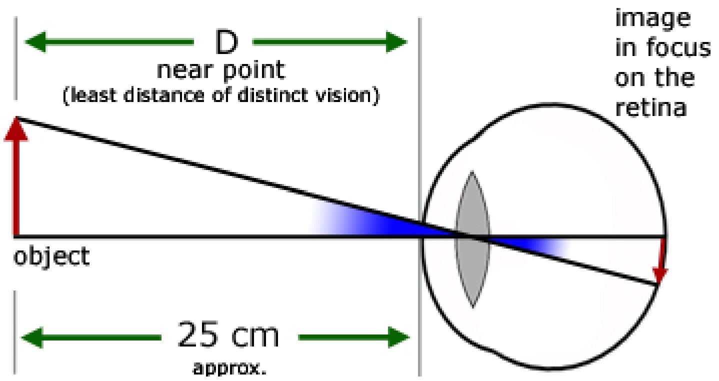
Angular magnification is the ratio of the angle subtended at the eye by the image (***β***°), to the angle subtended by the object (***α***°).



$$M = \frac{\beta}{\alpha}$$

Near Point

The **near point** (**D**) is the closest distance infront of the eye where a positioned object is in focus. It is sometimes referred to as '**the least distance of distinct vision**'.



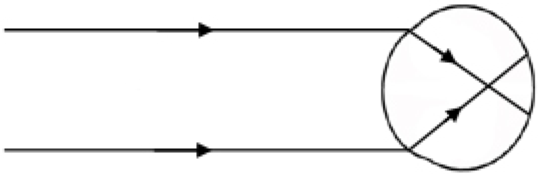
The near point varies from person to person, but a good average is 25 cm.

Short Sight (Myopia)

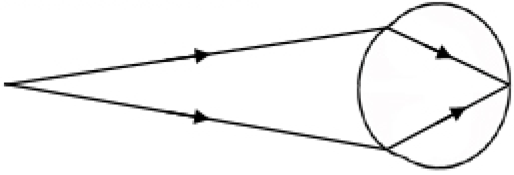
As the name implies, a person with 'short sight' can see objects close up, but not in the distance.

Short sight is a result of an abnormally long eyeball or the eye lens being too strong. The consequence is that the image of a distant object is formed infront of the retina. The resulting image actually reaching the retina is therefore out of focus.

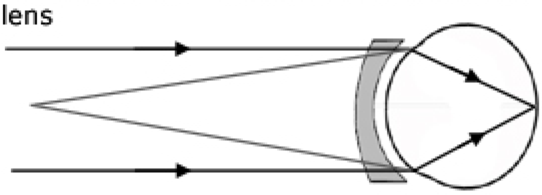
distant image out of focus on the retina



object closer than the near point,
in focus on the retina



correction of short sight using a concave lens



Correction of short sight is by concave lens. A concave lens is a diverging lens. So the effect is to push the image back towards the surface of the retina, where it will be perceived as in focus.

<u>Long Sight</u> (Hypermetropia)

As the name implies, a person with 'long sight' can see objects far away , but not close up.

Long sight is a result of an abnormally short eyeball or the eye lens being too weak. The consequence is that the image of a distant object is formed behind the retina. The resulting image perceived on the retina is therefore out of focus.

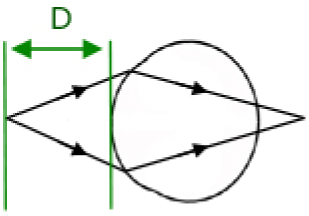
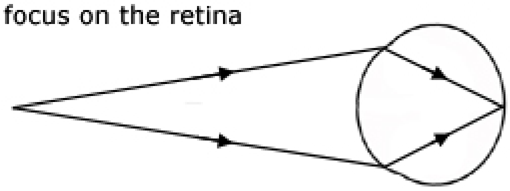
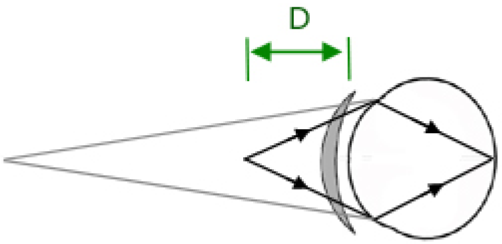


image formed behind the eyeball at the near point

object further away than the near point, in focus on the retina



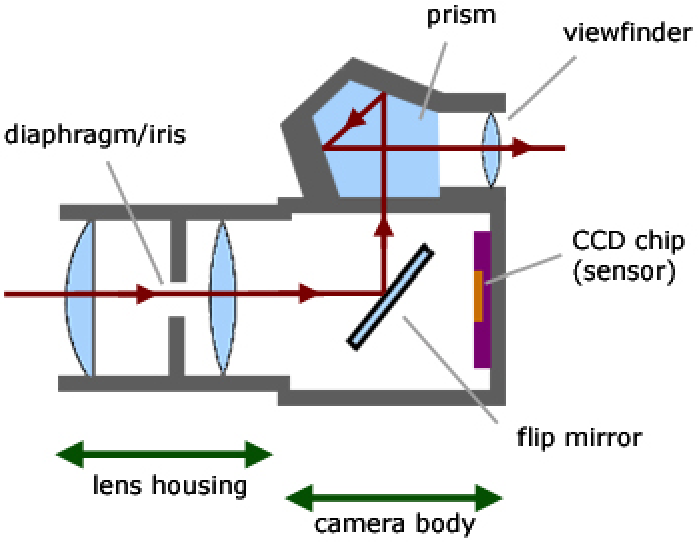


correction of long sight using a convex lens

Correction of long sight is by convex lens. A convex lens is a converging lens. So the effect is to pull the image back from behind the eyeball towards the surface of the retina, where it will be perceived as in focus.

The Camera

Introduction - the single lens reflex camera (SLR)



lens - lenses are compound in nature and of sophisticated design, with most offering the facility to zoom in on an image.

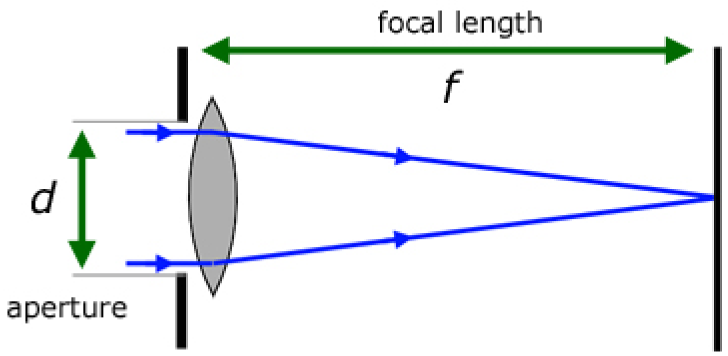
shutter - this controls the flip mirror, which rotates about its top edge into the camera body.

viewfinder - the image can be viewed directly through the lens system via the flip mirror and the penta-prism.

diaphragm / iris - is a variable aperture. By rotation, the light entering the lens increases/decreases. Hence the f-number of the lens can be altered.

c.c.d. (charge coupled device) - a silicon chip consisting of an array of capacitor-like elements that store charge when light falls on them. The amount of electric charge stored in each element is proportional to the light intensity at that point.

f-number



As might be expected, the total amount of light (***L***) falling on a sensor/film is proportional to the area of the aperture (***A***), which in turn is proportional to diameter squared (d^2).

$$L \propto A$$

The area ***A*** of a circular aperture of radius ***r*** is given by:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{d}{2}\right)^2 \\ A &\propto d^2 \\ L &\propto A \end{aligned}$$

hence,

$$L \propto d^2$$

It can be shown that the area of an image(A_i) is proportional to the square of the focal length (***f***).

$$A_i \propto f^2$$

The light per unit area of image is given by the the total amount of light in the image (***L***) divided by the area of the image (A_i) .

Hence,

$$\frac{L}{A_i} \propto \frac{d^2}{f^2}$$

So the amount of light in the image relates to both the aperture diameter and the focal length. For a bright image the aperture must be large and the focal length small. Note the telephoto lenses used by photographers at sports events. Object lenses are wide with a short tapering barrel.

Further, it can be shown that exposure time $\boldsymbol{E_T}$ is inversely proportional to the light per unit area of image($\boldsymbol{L}/\boldsymbol{A}_i$).

$$\frac{1}{E_T} \propto \frac{L}{A_i}$$

$$E_T \propto \frac{A_i}{L}$$

hence,

$$E_T \propto \frac{f^2}{d^2}$$

$$E_T \propto \left(\frac{f}{d}\right)^2$$

The ***f*-number** (relative aperture) is defined as the lens focal length (***f***) divided by the aperture (***d***).

$$f\text{-number} = \frac{f}{d}$$

therefore,

$$\text{exposure time} \propto (\text{f-number})^2$$

f-number settings (blue) on a camera have discrete values.

2	2.8	4	5.6	8	11	16	22
4	7.84	16	31.36	64	121	256	484

The square of each number (red) in the series is approx. double that of the square of the number preceding it.

Since exposure time ($\boldsymbol{E_T}$) is directly proportional to the square of the f-number, by decreasing the f-number by one setting the exposure time is halved.

depth of field (DOF)

This is the distance between objects in the foreground and the background that appear in focus.

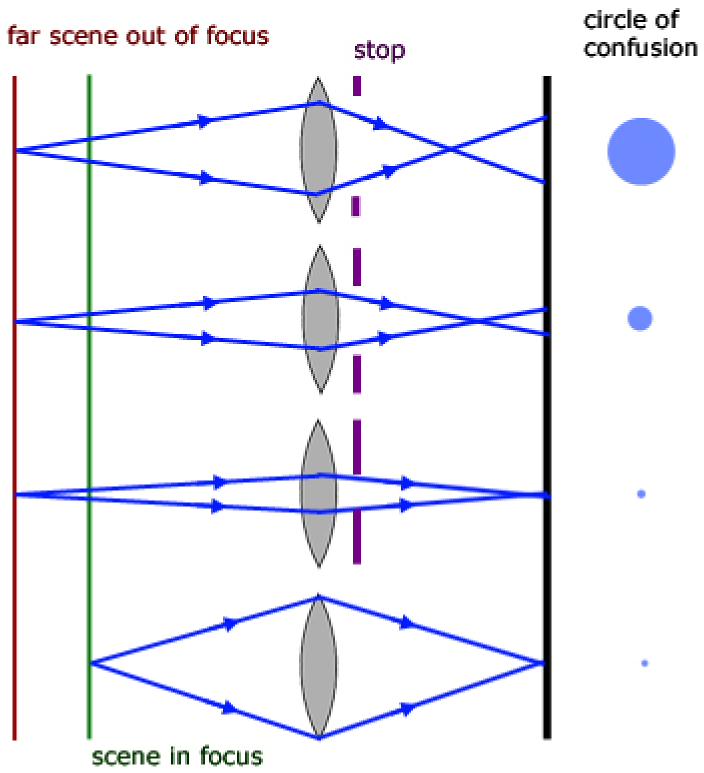
Depth of field depends on:

camera-to-subject distance : the further a subject is away, the sharper the image.

lens focal length : a short focal length lens gives a shallow DOF and vice versa.

***f*-number** : a high f-number gives a deeper DOF.

circle of confusion CoC : a point is imaged as a spot rather than a point as a result of light being brought to focus infront and behind the prime image.



The DOF can be defined in terms of the CoC as: the region where the CoC is less than the resolution of the human eye (or the medium being used).

In the top three diagrams you can see how a region (in red), further away from from the lens, is out of focus. This is because light rays do meet at a point on the screen.

As the lens is stopped (aperture lowered), the blurred area becomes smaller and smaller. As a result of the limitations of the eye/media, a point is reached when a blurred spot is indistinguishable from a focussed point. The scene then has regions infront and behind that appear in focus. A depth of field is perceived.