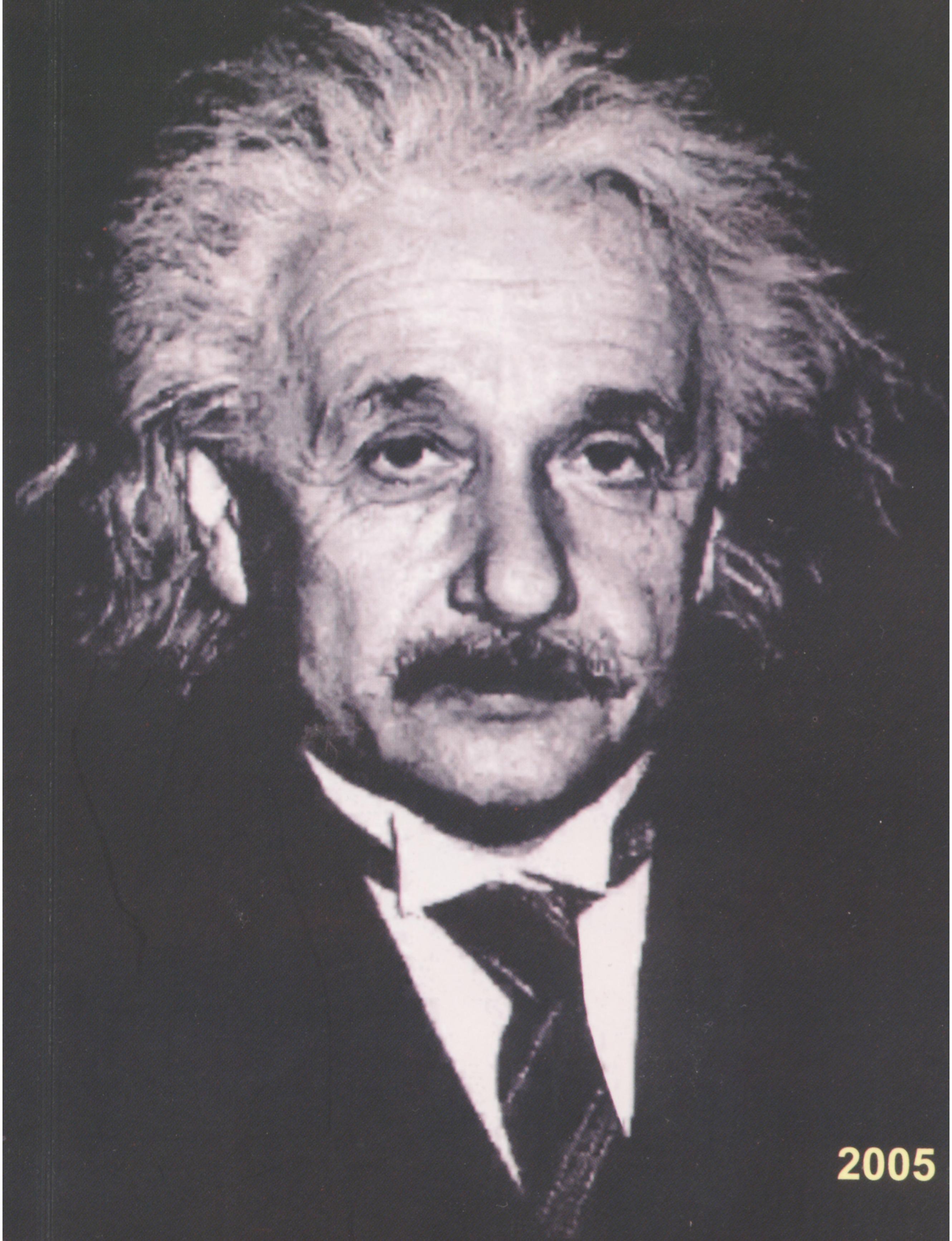
A Revolution in Physics :
Einstein's Discoveries of 1905 Made Simple



2005

A Revolution in Physics:
Einstein's Papers of 1905 Made Simple

2005

Contents

Preface

In entire human history, there have perhaps been five well documented revolutions in our physical picture of the world around us. The earliest of them was made by Euclid (320-275 BC), who codified all prevalent geometrical knowledge into a single logical system and gave human beings their first appreciation of the role of mathematics in their attempts to understand Nature. The second revolution came in the seventeenth century, when Newton laid down the laws of motion and gravity. The third revolution occurred in the nineteenth century, when Faraday and Maxwell laid down the laws of electromagnetism. The last two revolutions occurred in the twentieth century, when one man — Albert Einstein changed both our concepts of space and time *and* our understanding of matter and radiation. The first of these twentieth century revolutions is today known as *relativity theory* and the second as *quantum theory*.

What is amazing is that both these revolutions were brought about by one man and that too in a single year. In that year, sometimes called *Einstein's miraculous year*, he published the five papers listed below.

1. *On a new determination of molecular dimensions*, Ph.D. Dissertation, University of Zurich, Bern, 1905.
2. *On a heuristic point of view concerning production and transformation of light*, Annalen der Physik **17** (1905) 132–148. (Photoelectric effect).
3. *On the motion of small particles suspended in liquids at rest required by the molecular-kinetic theory of heat*, Annalen der Physik **17** (1905) 549–560. (Brownian Motion).
4. *On the electrodynamics of moving bodies*, Annalen der Physik **17** (1905) 891–921. (Special Relativity).
5. *Does the inertia of a body depend on its energy content?*, Annalen der Physik **18** (1905) 639–641. (Special Relativity).

In the second paper, he used the revolutionary *quantum hypothesis* to explain the photoelectric effect. In the third paper, he essentially laid to rest the debate on the nature of matter, by conclusively proving its atomic and molecular nature. In papers 4 and 5, he introduced what is today known as the *Special Theory of Relativity*, the most famous of his results. The world had to wait till 1916 for his *General Theory of Relativity*, which incorporated gravity into his theory of relativity.

We note here that even though Einstein was instrumental in bringing about the two revolutions in our conception of Nature, he did not achieve this on his own. His ideas on quantum theory were based on Max Planck's results of 1900, while his special relativity theory was built on ideas of Lorentz and Poincaré. Neither did these papers present the full theory. Others after him, like Niels Bohr, Heisenberg, Schrödinger, Dirac and Feynman developed quantum theory. The notion of four-dimensional space-time in the relativity theory was provided by the mathematician Hermann Minkowski. This idea was crucial to the formulation of the general relativity theory. Many scientists have since then worked on relativity theory and taken it far beyond these beginnings.

However, these papers of Einstein still presented radically different ideas, which have since then tremendously influenced both science and technology and hence our everyday life. From quantum optics and quantum field theory in physics, to the atom bomb, lasers and masers, to name only a few developments, we owe Einstein a lot today.

To commemorate the passing of a century after the publication of these landmark papers, the year 2005 is being celebrated as the International Year of Physics all over the world. As part of the celebrations, we bring to you simplified versions of four of the above papers. Paper 2, which laid the foundations of the quantum theory, and also contains the explanation of the photoelectric effect for which Einstein received his Nobel Prize is described in the article *Photoelectric Effect* by Professor Vijay A. Singh. Paper 3 which finally laid to rest the debate regarding the atomic nature of matter is described in the article *Brownian Motion* by Professor R.S. Bhalerao. Papers 4 and 5 are dealt with in the article *Special Relativity* by Professor T.P. Singh.

We thank the authors for putting in the tremendous effort required to explain the complex collection of ideas contained in these papers, in a language understandable even by high-school students.

We hope that you will enjoy reading these articles as much as we enjoyed preparing them for you.

Science Popularisation and Public Outreach Committee
Tata Institute of Fundamental Research
June 2005

Albert Einstein (1879–1955)

Albert Einstein was born in Ulm, Germany on 14 March 1879. When he was seven years old, he started going to a school in Munich and two years later entered Luitpold Gymnasium. Around 1889, he started studying higher mathematics on his own. In 1894, his family moved to Milan, Italy, while he remained in Munich to complete his schooling. However, he failed in the diploma examination which would have allowed him to join Eidgenössische Technische Hochschule (ETH) in Zurich for a career in electrical engineering which his family wanted him to do. He then joined his parents in Italy. In 1895, he went once again to Zurich to take the entrance examination which would allow him to join the ETH. Again, he failed the exam. He then joined a school in Aarau in order to prepare for the entrance exam once more. It was during this period that he gave up his German citizenship. He was to remain "stateless" for the next four years.

He passed the examination at ETH in 1896 and joined a four year course which would qualify him to be a teacher in mathematics and physics. His teachers here included Weber and Hermann Minkowski, the mathematician who would later be so important in his formulation of the general relativity theory. In 1900, he graduated from ETH and expected to be appointed to the physics department of ETH. However, even though his other classmates were given appointments, Einstein was not offered one. He obtained a temporary job in the Swiss Federal Observatory and became a Swiss citizen in 1901. In 1900, he published his first paper entitled *Deductions from the Phenomena of Capillarity*, in the Annalen der Physik. However, his applications for jobs with the German physical chemist Wilhelm Ostwald and Kammerlingh Onnes, the Dutch physicist at the University of Leiden (where he later became a honored visiting professor) were not even answered. In 1902, he accepted the post of Technical Expert (Third Class) in the Swiss Patent Office in Bern at the salary of 3500 francs a year. The post was temporary.

In 1904, the post was made permanent and in 1906 he was promoted to technical expert second class. In 1905, Einstein earned a doctorate from the University of Zurich with his paper *On a new determination of* *molecular dimensions*. In the same year he published four more papers (see Preface) which would revolutionize our conceptions of the physical world. In the first one, he laid the foundations of the quantum theory. In the second, he explained Brownian motion. In the third and fourth he introduced what is known now as the special theory of relativity. Recognition came slowly and in 1908, he became a lecturer at the University of Bern. In 1909 he became professor of physics at the University of Zurich and resigned from his lectureship at Bern and from his job in the patent office in Bern. In 1911 he moved to Prague as a full professor at the Karl-Ferdinand University. In 1912 he joined ETH at Zurich and in 1914 returned to Germany, to a research position in the Prussian Academy of Sciences, with a chair (but with no teaching duties) at the University of Berlin and the directorship of the Kaiser Wilhelm Institute of Physics in Berlin which was about to be established.

In 1916, he published the *General Theory of Relativity*, and when the British expeditions of 1919 confirmed his prediction that the gravity of the sun would bend light rays passing close by, he was idolized by the press. In 1921 he was awarded the Nobel prize in physics for his explanation of the Photoelectric effect (the first paper mentioned above). He was also awarded the Copley Medal of the Royal Society in 1925 and the Gold Medal of the Royal Astronomical Society in 1926.

With the Nazis coming to power in Germany, Einstein left Germany to join the newly formed Institute for Advanced Study, Princeton, U.S.A., never again to return to Germany. In 1940 he became a US citizen. In 1952, the Israeli government offered him the post of President of Israel, which he declined.

Apart from his tremendous contributions to physics, he made numerous contributions to peace during his lifetime.

Photoelectric Effect

Vijay A. Singh
Homi Bhabha Centre for Science Education
Tata Institute of Fundamental Research

1 Introduction

Calendars often mark their origin by a key historical event. The Gregorian calendar which is in common use starts with the birth of Christ. The starting point for the Islamic calendar is the Hegira, the period marking the emigration of the Prophet Mohammed from Mecca to Medina. If a physicist were to design a calendar, she would probably choose the year 1905 as the first year or origin year. An event would be either post 1905 or pre 1905. Thus we are currently hundred years from 1905 and this is year 100. India obtained her independence in the forty second year (1947). The publication of Newton's Principia (1687), which to a physicist would be another contender for the defining year, would be the two hundred and eighteenth year prior to 1905. Or mathematically speaking, year -218.

What is so special about 1905? The year is labelled "Annus Mirabilis" which in Latin means "miracle year". In that year Albert Einstein published immortal papers in three diverse topics, each in itself revolutionary and deserving of the highest accolade. Einstein was twenty-six years old at that time. His first paper was entitled, *On a heuristic point of view concerning the production and transformation of light*[1]. A heuristic argument is one which is plausible and enlightening but not rigorously justified. The paper endows light with *particle-like* properties and explains the then recently discovered photoelectric effect which is the subject matter of our present article.

---

[1] A. Einstein, Annalen der Physik, **17** (1905) 132–148.

2 The Photoelectric Effect

The interaction of light with matter gives rise to strange and wonderful effects. The blue sky and the redness of the setting sun are due to the interaction of sunlight with particulate matter. The photoelectric effect is yet another example. Consider a small spherical shiny piece of metal suspended from the roof by a cotton thread. Illuminate the piece with blue light. This is depicted in Figure 1. After a while we notice that the metallic sphere has acquired a positive charge. This could be verified by bringing a charged object, such as a glass rod rubbed with silk cloth, close to the sphere. A glass rod rubbed with silk cloth gets positively charged. So bringing it close to the illuminated sphere will cause the latter to deflect away. Our present-day knowledge permits us to solve this mystery of the positively charged sphere. Electrons which are negatively charged particles are present in copious amounts in the metal. Some of them have been energized enough by the blue illumination prompting them to leave the metal. You can say that they have been "coaxed out", "kicked out" or have "evaporated out" of the metal. It turns out that the phrase kicked out would be more appropriate than evaporated out but that would get us ahead of our story. An electron "jumping out" of the illuminated metallic sphere is depicted in Figure 1. Figure 1 is the bare, stripped down version of the photoelectric effect. We shall now elaborate on it.

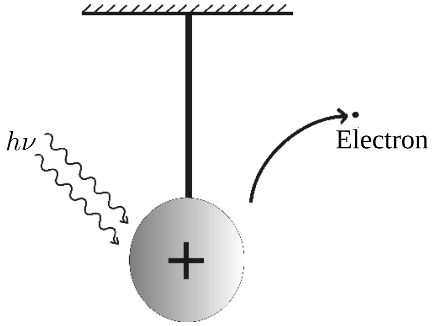


Figure 1: *The photoelectric effect. Electron emission occurs when light falls on the polished metallic sphere.*

The Phenomenon	**Corpuscular**	**Wave**
Reflection	Y	Y
Refraction	Y	Y
Interference	N	Y
Diffraction	N	Y
Polarization	N	Y
Photoelectric Effect	N	N

Table 1: *The table lists light-related phenomena and light-related theses. ‘Y” means that the thesis can account for the phenomenon. “N” means it cannot.*

3 Qualitative Explanation of the Photoelectric Effect

The photoelectric effect (PE) was discovered accidentally by the German physicist Heinrich Hertz in 1887. Hertz is credited with carrying out a masterful and wide-ranging study to establish the wave nature of light. So it is rather ironic that this accidental discovery would lead to a revaluation of this basic premise about light in the twentieth century. The commonly seen appearance of rainbow patterns in oil films can be explained by assuming that light waves *interfere*. The twin headlights of an approaching car cannot be distinguished from far. They appear as one single blur. As the car comes closer one gradually begins to distinguish them. This can be explained by assuming that light waves *diffract*. We do not go into explanations of these phenomena in this article. Suffice it to say that by the end of the nineteenth century there existed a solid body of evidence to support the wave thesis of light. Table 1 lists this evidence. For good measure we also remind ourselves that there existed an earlier thesis that light consists of *corpuscles* or particles that traveled in straight lines. One could account for the laws of reflection and refraction based on this thesis.

A number of scientists, notably the German Philip Lenard, conducted careful and exhaustive experiments to explore PE. Figure 2 depicts a sketch of the basic apparatus employed by them. A transparent evacuated tube contains two metal electrodes connected to an external circuit. The cathode is illuminated by light and a fraction of the emitted electrons is collected at the anode. The current can be measured by an ammeter (not shown) in the circuit. The experimental conditions that can be varied are the frequency of the incident light, the voltage drop, the polarity of the battery, thus transforming the anode to the cathode and vice versa, and at times the electrode material. As data began to accumulate, so did the bewilderment of the scientists concerned.

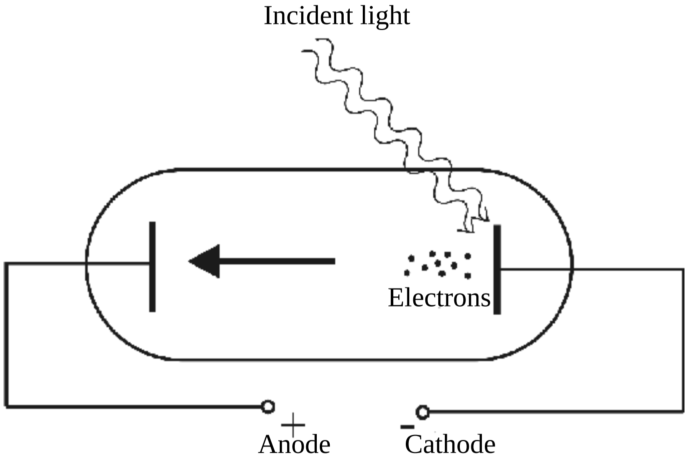


Figure 2: *Sketch of the basic apparatus for studying the photoelectric effect.*

A metal consists of a large number of electrons which are freely moving. If you heat a metallic filament, electrons are emitted. You might say they are “boiled off”. This phenomenon was known. Similarly, one may argue that electrons are emitted after the metal soaks in light energy. You might say that there should be no surprise or bewilderment.

However, one of the puzzling features of PE is that it is *instantaneous*. It should take some time for the light energy to be absorbed by the metal and the electrons to be emitted. If the light is of low intensity then it should take longer for the electrons to emerge. But this is not the case. Electrons are emitted almost instantaneously albeit in small amounts even at low-intensity illumination of the metal. Let us consider the analogy with boiling. Water in a pan would evaporate slower when put on slow heat. It takes longer for water molecules to leave the pan. Einstein suggested that a possible way out of the puzzle is to consider light to consist of particles. We shall call them photons, a word coined not by Einstein, but by the chemist Gilbert Lewis in 1928. These photons act *one-on-one* to eject the electrons. Maybe another analogy will help us understand this better. Consider the mango fruit hanging from a slender twig. A wave of wind is unable to sever it from its parent branch. But a well aimed pebble can. At low intensity there are fewer photons and hence a smaller number of electrons are ejected.

Another puzzling aspect was the dependence of the PE on the *frequency* of light. PE was observed in metals provided high frequency ultraviolet and blue light was used. Orange or red light, howsoever intense, did not cause the emission of *photoelectrons*. Most of us have observed the flow and ebb of tides on the seashore. A weak wave is unable to dislodge a stone. But a strong, large amplitude wave can roll a rock towards the shore. This is irrespective of the wavelength of the wave. And frequency is (inversely) related to wavelength and not related to the amplitude. If light were a wave then a high-intensity source of red light should ensure a yield of photoelectrons. Once again Einstein argued that light consists of the above mentioned photons whose energy is proportional to frequency ν. The higher the frequency, the higher the energy and hence the possibility of electron emission. The intensity is a monitor of the number of photons, not of its energy. This is contrary to the tenets of wave theory where the intensity is a measure of energy.

Figure 3 displays two curves of the photoelectric current. The lower one is when you illuminate the metal cathode with low-intensity light. As we increase the anode potential, the photoelectric current increases and then saturates, i.e., it attains a constant value. Note that the potential only serves to shepherd the emitted electrons towards the anode. Increasing the potential ensures maximal collection. The higher the intensity, the higher the saturation current. If we now reverse the polarity of the electrodes, the anode of Figure 2 becomes the cathode and vice versa. For sufficiently negative potential $(-V)$ no electrons reach this cathode. This negative potential is called the *stopping potential*. Interestingly, the

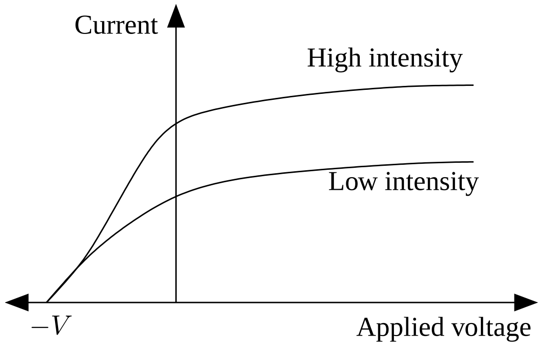


Figure 3: *The dependence of the photoelectric current on the electrode potential. The saturation of the current indicates that the potential serves merely to efficiently herd the emitted electrons and is not the cause of emission. The higher intensity light source ensures higher photocurrent. Intensity is also not the cause of emission as discussed in the text.*

stopping potential is the same irrespective of the intensity of light. Thus the potential is *not the cause* of emission.

4 Einstein's Photoelectric Equation

In his 1905 paper (see footnote 1) Einstein went further and suggested a quantitative test for his light quanta (photon) hypothesis. The equation he wrote down is now famous as Einstein's photoelectric equation. It is an energy balance equation for the photoelectrons. Let us develop this equation with the help of an analogy.

In Figure 4 we show a bucket of water being drawn from a well of height H. The bucket can hold water of mass m. To draw the water from the well we need to provide energy mgH which is the energy required to overcome the gravitational pull. Here g is the acceleration due to gravity and has a value of approximately 9.8 m/s^2 near the earth's surface. In order to irrigate a nearby patch of land we need to throw the water with some speed v. This requires an additional energy, called the kinetic energy, of $\frac{1}{2}mv^2$. Thus if we designate E as the total energy required of us

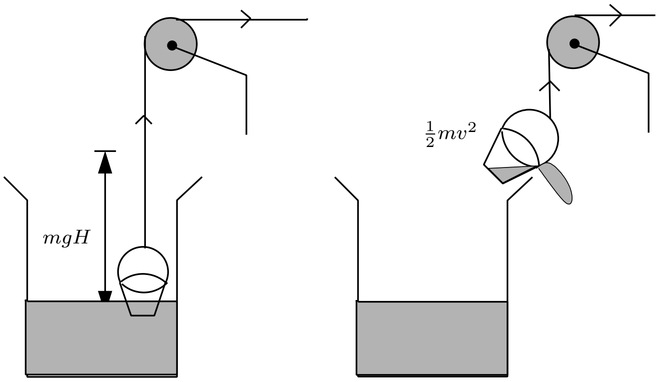


Figure 4: *The energy required to draw a bucketful of water from a well and throw it with some speed* v*. The figure illustrates equation* (1).

in this endeavour, our equation for the bucketful of water would read as

$$E = mgH + \frac{mv^2}{2}. \tag{1}$$

We can now develop Einstein's equation by analogy. We have at our disposal the energy of the photon. This, as argued above, is proportional to the frequency of light ν. The constant of proportionality is Planck's constant h. It was introduced only a few years earlier by the famous theoretical physicist Max Planck in connection with black body radiation. Einstein was influenced by Planck's seminal study on black body radiation but that would constitute another story and we shall not dwell on it here. So the photon of energy $h\nu$ is related to the bucketful of energy E of equation (1).

$$E \longrightarrow h\nu.$$

Next some energy is needed to overcome the attractive pull of the metal on the electron. If this energy barrier was not present, electrons would leave the metal at random. This energy is called the *work function* of the metal, in other words the work required to draw the electron from the metal. Just as different wells may have different depths H, different metals have different work functions. We designate the work function by

ϕ. Our analogy is thus

$$mgH \longrightarrow \phi.$$

Thus the photon must have a minimum energy ϕ for photoemission to take place. This is the reason why red light cannot cause photoemission. There is simply not enough energy in the "red" photon. However a "blue" photon or an "ultraviolet" photon may have energy in excess of ϕ. This will not only provide the electron enough energy to leave the parent metal but will also endow it with some speed. We designate this as v_{max}, the maximum speed of the photoelectron. Some photoelectrons could have less speed because of energy loss due to collisions and other processes which occur on the way out.

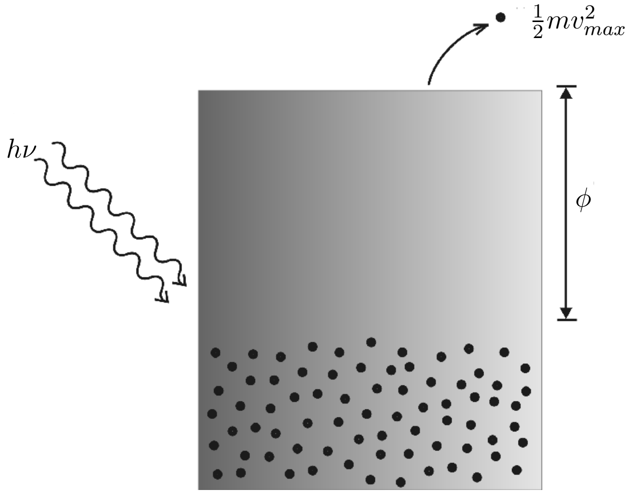


Figure 5: *The energy required to eject an electron from a metal and endow it with a speed v_{max}. The figure illustrates Einstein's photoelectric equation* (2).

The above mentioned process is illustrated in Figure 5. The Einstein equation for the photoelectric effect is thus,

$$h\nu = \phi + \frac{mv_{max}^2}{2}. \tag{2}$$

This is a simple, testable equation. It implies that the relationship between the maximum kinetic energy of the electron (last term on the right) and the frequency ν should be a straight line as indicated in Figure 6. The intercept indicating threshold frequency will depend on the work function of the metal. And as a bonus, the slope should yield Planck's constant!

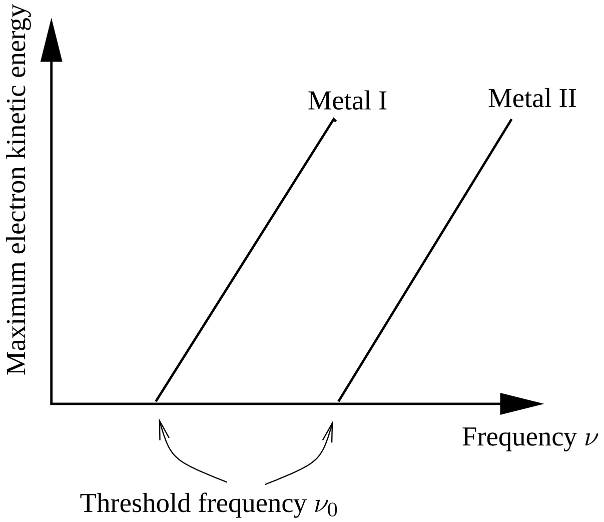


Figure 6: *Sketch of dependence of the maximum kinetic energy of the photoelectrons on the frequency of the incident light as implied by equation* (2).

In 1905, many would have shaken their heads in disbelief at equation (2). The concept that light, when it interacts with matter, acts like a particle with a well defined energy seemed fanciful. Even the title of Einstein's article indicated hesitance: note the word "heuristic" in the title (see page 1).

In the decade following the publication of the 1905 paper, careful experiments were conducted to verify equation (2). The American physicist Robert Millikan who measured the charge on the electron was in the forefront of these investigations. The equation stood vindicated and is now found in all elementary textbooks of physics.

5 Conclusion

In his 1905 paper Einstein wrote of light quanta. As mentioned earlier he did not use the term "photon" for it. The word "photon" was coined much later in 1928 by the chemist Lewis. Although Einstein never used this word, it has come to stay.

Some experts consider that calling the 1905 paper (see footnote 1) Einstein's photoelectric paper is a misnomer[2]. The paper is largely concerned with radiation in an enclosure. The photoelectric equation follows as a "by-product" of his treatment of radiation in thermodynamic terms. In explaining the photoelectric effect Einstein placed his faith on the solidity of thermodynamics and statistical mechanics. Consider a sample of the type of arguments he employed. There are billions and billions of air molecules in the room you are sitting in right now. The probability that all of them would be spontaneously gathered to one small corner of the room leaving the rest of the room empty is very low. One can make this mathematical. An elementary law of statistical mechanics is that a gas of N independent atoms in an enclosure of volume V_0 has a probability $(V/V_0)^N$ of occupying a partial volume V of the same enclosure. Einstein used a similar argument for photons. If the energy of light of frequency ν, in an enclosure, is W, then Einstein showed that the probability that the photons in a cavity of volume V_0 would occupy a partial volume V would be $(V/V_0)^N$ where N is given by $W/h\nu$. It is this derivation which suggests that the light in the enclosure consists of N photons each with energy $h\nu$.

Since the photon has a well defined energy, one may argue that it should also have a momentum. Such is indeed the case and the American physicist Arthur Compton is credited with studying the change in momentum during the collision of a photon with an electron. Photons need not transfer energy only to electrons. They could also transfer energy to the atoms in a molecule enhancing their vibrational or rotational energy. Very careful experiments concerning the interaction of photons with vibrating molecules were conducted by the Indian physicist C. V. Raman. He demonstrated that the photons picked up energy from the molecule or lost energy to them in discrete amounts. In photochemistry we have

---

[2] Einstein's paper (see footnote 1 above) consists of nine sections. Only one section, namely the eighth section, is devoted to the photoelectric effect.

the Stark-Einstein law: a single photon can cause photochemical reaction only in a single absorbing molecule and not in multiple molecules. A photon on emission from its parent body can actually “kick” it back! This is similar to the recoil experienced by a soldier firing his rifle. A variant of this effect is employed in solids and is called the Mossbauer effect. A fundamental theory of the interaction of light with matter was proposed by Richard Feynman, Julian Schwinger and Sini-toro Tomonaga. It is called quantum electrodynamics and it fetched its protagonists the Nobel Prize in 1965. You could literally let your imagination run riot once you get used to the fact that light when it interacts with matter behaves as a particle. Table 2 lists some of the spin-offs from the photon concept.

The Spin-off	**Person(s)**	**Remarks**
Photon Momentum	A. Compton	Nobel Prize 1927
Raman Effect	C. V. Raman	Nobel Prize 1930
Stark-Einstein Law	J. Stark and A. Einstein	Photochemical Reaction
Recoilless Emission	R. Mossbauer	Nobel Prize 1961
Masers and Lasers	T. Maiman*, C. Townes, N. Basov and A. Prokhorov	Nobel Prize 1964
Bose-Einstein Statistics	S.N. Bose* and A. Einstein	

* Not recognized by a Nobel Prize. However, Bose-Einstein condensation, which is predicted on the basis of Bose-Einstein statistics was experimentally discovered in 1995 and fetched its discoverers the Nobel Prize in 2001.

Table 2: *The table shows some of the spin-offs from the photon concept.*

Of the three seminal pieces of work that Einstein published in 1905, he considered his work on the photoelectric effect to be the most revolutionary. He was awarded the Physics Nobel Prize in 1921 for explaining the photoelectric effect and for his services to theoretical physics. His work constituted one of the cornerstones of early quantum theory. He saw the quantum "movement" blossom before his eyes. But the deep thinker and radical critic that he was, he never could reconcile himself to the wave-particle duality and the uncertainty espoused by the quantum theory. He noted that the quantum theory of light which treats it as a particle, explicitly involves a wave concept, namely the frequency. As a new generation of physicists rushed headlong into exploring the consequences of quantum theory, his dissenting voice became a cry in the wilderness. His feelings on this matter are reflected in a poignant letter he wrote to his friend Michelle Angelo Besso in 1951: "All these fifty years of conscious brooding have brought me no closer to the question, 'What are light quanta ?' Of course today every knave thinks he knows the answer, but he is deluding himself."

Acknowledgment

The work is supported by the National Initiative on Undergraduate Science (NIUS) undertaken by the the Homi Bhabha Centre for Science Education (TIFR), Mumbai, India.

Brownian Motion

R.S. Bhalerao
Department of Theoretical Physics
Tata Institute of Fundamental Research

1 Introduction

Let us do a “thought experiment”. What is a thought experiment? It is an experiment carried out in thought only. It may or may not be feasible in practice, but by imagining it one hopes to learn something useful. There is a German word for it which is commonly used: *Gedankenexperiment*. It was perhaps A. Einstein who popularized this word by his many gedankenexperiments in the theory of relativity and in quantum mechanics.

Coming back to our thought experiment: Imagine a dark, cloudy, moonless night, and suppose there is a power outage in the entire city. You are sitting in your fourth floor apartment thinking and worrying about your physics test tomorrow. Suddenly you hear a commotion downstairs. You somehow manage to find your torch and rush to the window. Now suppose your torch acts funny: it turns on only for a moment, every 15 seconds. Initially, i.e., at time $t = 0$ seconds, you see a man standing in the large open space in front of your building. Before you make out what is happening, your torch is off. Next time it lights up, i.e., at $t = 15$ sec, you see him at a slightly different location. At $t = 30$ sec, he is somewhere else and has changed his direction too. At $t = 45$ sec, he has again changed his location and direction. You have no idea what is going on. But, you continue observing him for some more time. When the lights come back, you mark his positions on a piece of paper (see Figure 1). At $t = 0$, he is at point A, at $t = 15$, he is at B, at $t = 30$, he is at C, and so on. Connect point A to B, B to C, C to D, and so on, by straight lines. (Go ahead, grab a pencil and do it.) What do you see? A zigzag path.

Random walk in one dimension:

Here is an experiment you can do yourselves. You will need a plain paper, a ruler, a pencil, a one-rupee coin, a small stone and a lot of patience. Draw a number line with markings at $-10, -9, ..., \ 0, ..., \ 9, 10$. Place the stone at 0. Toss the coin. The rule is, if it is heads (H), the stone is moved one place to the right and if it is tails (T), it is moved one place to the left. For example, if you get H, H, T, ..., the stone moves from 0 to 1, 1 to 2, 2 to 1, ... Toss the coin 10 times. Note the final position of the stone. Call it x_1. Obviously, $-10 \leq x_1 \leq 10$.

Replace the stone at 0 and repeat the experiment. Again note the final position of the stone. Call it x_2. Obviously, x_2 may or may not be equal to x_1.

If you were to repeat this experiment a very large number of times, say 1000 times, and then take the average ($\bar{x}$) of $x_1, \ x_2, \ x_3, \ ..., \ x_{1000}$, what result do you think you will get? Since each toss of the coin is equally likely to result in an H or a T, $x_1, \ x_2, \ x_3, \ ..., \ x_{1000}$ will be distributed symmetrically around the origin 0. Hence $\bar{x}$ is most likely to be zero.

Interestingly, however, the average ($\overline{x^2}$) of $x_1^2, \ x_2^2, \ x_3^2, \ ..., \ x_{1000}^2$, will not be zero, since these are all non-negative numbers. In fact, $\overline{x^2}$ turns out to be equal to the number of times you toss the coin in each experiment, which is also equal to the number of steps (N) in the random walk. (This is 10 in our experiment.) Thus $\overline{x^2} = N$ or $\left(\overline{x^2}\right)^{1/2} = N^{1/2}$. Since the left-hand-side is the square root of the mean (= average) of the squares, it is called the *rms* displacement and is denoted by $x_{\rm rms}$. Thus $x_{\rm rms} = N^{1/2}$.

What is the meaning of the statement $\bar{x} = 0$, but $x_{\rm rms} = N^{1/2}$? It means, in a random walk, the object is as likely to be found on one side of the starting point as on the other, making $\bar{x}$ vanish. But at the same time, as the number of steps increases, the object is likely to be found farther and farther from the starting point.

Equivalently, imagine 1000 drunkards standing at the origins of 1000 parallel lines, and then starting simultaneously their random walks along these lines. If you observe them after a while, there will be nearly as many of them to the right of the centres as there are to the left. Moreover, the longer you observe them, the farther they are likely to drift from the centre.

Conclusions: (a) $\bar{x} = 0$. (b) $x_{\rm rms} = N^{1/2}$ if each step is of unit length. (c) $x_{\rm rms} = N^{1/2} l$ if each step is of length l.

What do you think was going on? Did you say “a drunken man wandering around aimlessly”? Right. That was easy. One does not need an Einstein’s IQ to figure that out. In physicists’ language, the above is an example of a *random walk in two dimensions*: **two** dimensions because the open area in front of your building has a length and a breadth. (Strictly speaking, a walk can be said to be random if the direction of each step is completely independent of the preceding step. For simplicity, the steps may be taken to be of equal length.) Before you read further, close your eyes and imagine a random walk in 1 dimension and then a random walk in 3 dimensions.

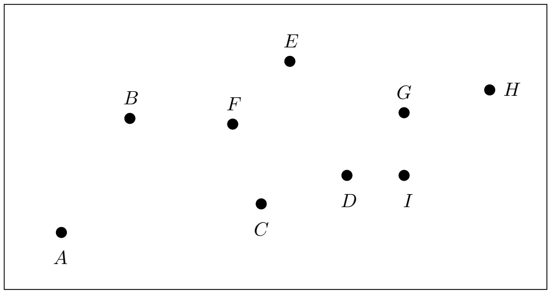


Figure 1

Let us perform another thought experiment. Suppose you are sitting in a big stadium, watching a game of football or hockey, being played between two equally good teams. As in the previous thought experiment, you mark on a piece of paper the position of the ball every 15 seconds, and then connect these positions in sequence. What do you see? Again a zigzag path. The ball is moving almost like the drunken man. Would you say the ball is drunk? Of course, not. The ball is moving that way because it is being hit repeatedly by the players in two competing teams. This is another example of an (almost) random motion in two dimensions.

Want to impress someone? Remember this: Random processes are also called stochastic processes. Chance or probability plays an essential role in these processes.

What you learnt above is the ABC of the branch of physics, called *Statistical Mechanics*.

2 History

He is happiest who hath power to gather wisdom from a flower — Mary Howitt (1799 - 1888).

Now I want to describe a real (not a gedanken) experiment. Robert Brown was a British botanist. In 1827, he observed through a microscope pollen grains of some flowering plants. To his surprise, he noticed that tiny particles suspended within the fluid of pollen grains were moving in a haphazard fashion.[1] If you were Robert Brown, how would you understand this observation? (Remember, science in 1827 was not as advanced as it is today. Many things written in your science textbook were not known then.) Would you suspect that the pollen grain is alive? Or would you get excited at the thought that you may have discovered the very essence of life or a latent life force within every pollen? Or perhaps this is just another property of organic matter? What other experiments would you perform to test your suspicions?

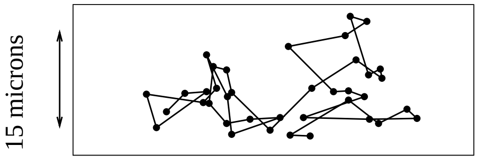


Figure 2

Brown repeated his experiment with other fine particles including the dust of igneous rocks, which is as inorganic as could be. He found that any fine particle suspended in water executes a similar random motion. This phenomenon is now called *Brownian Motion*. Figure 2 shows the result of an actual experiment: the positions of the particle were recorded at intervals of 30 seconds. (From J. Perrin, *Atoms*, D. Van Nostrand Co.,

---

[1] Sometimes it is wrongly stated that Brown observed irregular motion of the pollen grains themselves. Secondly, he was not the first to notice this phenomenon. But he was the first to stress its ubiquitousness and to rule out its explanations based on the so-called life force. As a result of his work, this subject was removed from the realm of biology into the realm of physics.

Inc., 1923.) Similar observations were made for tiny particles suspended in *gases*.

Scientists in the 19th century were puzzled by this mysterious phenomenon. They attempted to understand it with the help of ideas such as convection currents, evaporation, interaction with incident light, electrical forces, etc. But they had no satisfactory explanation for it. With your knowledge of modern science, can you provide a rudimentary explanation? Obviously, the suspended particle is not moving on its own unlike the drunkard in our first gedankenexperiment. Why then is it moving? And why in an erratic way (see Figure 2)? Think, before you read further.

Want a hint? Recall our second gedankenexperiment.

3 Basic Understanding

If you have not already guessed, here is the rational explanation for the mysterious jerky movement of tiny particles suspended in fluids, which made Mr. Brown famous:

- The size — the radius or diameter — of the suspended particle is roughly of the order of a few microns (1 micron = 10^{-6} m). The size of an atom is of the order of 10^{-10} m. The size of a water molecule (H_2O) is somewhat larger. Thus the suspended particle is a monster, about 10000 times bigger compared to a water molecule. Also note that a spoonful of water contains about 10^{23} water molecules. (The atomic or molecular theory of matter which says that matter consists of atoms and molecules, is well-established today. It was not so in 1827!)

- You also know that molecules of water (or molecules in any sample of a liquid or gas) are not at rest. They are perpetually moving in different directions, some faster than others. As they move, they keep colliding with each other, which can possibly change their speeds and directions of motion.

- Now you can very well imagine the fate of the particle unfortunate enough to be placed in the mad crowd of water molecules. The

> poor fellow is getting hit, at any instant, from all sides, by millions of water molecules. The precise number of water molecules which hit the particle at any instant, the exact points where they hit it, their speeds and directions — all keep changing from time to time. (It is practically impossible and also *unnecessary* to have this information.) This results in a net force which keeps fluctuating in time, i.e., its magnitude and direction keep changing from one instant to another. The particle keeps getting kicks in the direction of the instantaneous net force. The end result is that its position keeps changing randomly as in Figure 2.

4 A Quiz

Ready for a quiz? Here are a few easy questions:

(1) Imagine the game of football played by invisible players. (Nothing except the ball is visible.)

(2) See Figure 2. If the positions of the particle were recorded every 60 seconds, instead of every 30 seconds, how will the pattern look like?

(3) How will the Brownian Motion be affected if (a) water is cooled, or (b) instead of water a more viscous liquid is taken, or (c) the experiment is done with a bigger particle?

5 Einstein's Contribution

You now have a *qualitative* understanding of the Brownian Motion. But that is usually not enough. Scientists like to develop a *quantitative* understanding of a phenomenon. This allows them to make precise numerical predictions which can be tested in the laboratory.[2] For example, one would like to know how far (on an average) will the particle drift from its initial position in say 10 minutes? How will its motion be affected if the water is cooled by say 5 C, or if the viscosity of the liquid is increased by 10%, or if the particle size is exactly doubled?

---

[2] However good a theory may appear, if its predictions do not agree with experimental data, it is discarded.

In 1905, Einstein published a detailed mathematical theory of the Brownian Motion, which allowed one to answer these and many other interesting questions. How did he do it? I will only give you a flavour of what is involved.

What is an ensemble?

Recall that chance or probability plays an important role in random processes. Hence, in the Introduction, when we discussed the random walk (see the box), you were asked to do the experiment 1000 times and then average the results. If you do it only a few times, $\bar{x}$ may not vanish and $\overline{x^2}$ may not equal 10. If you are too lazy to do the experiment 1000 times, there is a way out: Get hold of 1000 friends of yours, ask each of them to prepare a similar experimental set-up, and let each of them do the experiment only once. If you then take the average of the results obtained by them, you will find $\bar{x} \approx 0$ and $\overline{x^2} \approx 10$.

Similarly, if you observe the Brownian Motion of a particle only a few times, based on these observations, you would not be able to make quantitative statements about its average behaviour. You need to repeat the experiment a large number of times and take the average of all the results. Alternately, you could prepare a large assembly of identical particles, observe each of them once under identical experimental conditions, and then take the average.

Physicists use the word *ensemble* to describe such an imaginary assembly of a very large number of similarly prepared physical systems. The average taken over all the members of the ensemble is called an ensemble average. In the following, when we talk about an average behaviour of a Brownian particle, we mean an ensemble average.

Let us ask ourselves a few simple questions about the average behaviour of a tiny particle suspended in a liquid. Taking the initial location of the particle as the origin, imagine drawing x, y and z axes in the liquid. Let (x, y, z) denote the coordinates of the particle.

$\boxed{\bar{x}}$: Where will the particle be after some time? In other words, what will be the values of $\bar{x}$, $\bar{y}$ and $\bar{z}$ after some time? (Remember the overhead lines denote ensemble averages.) Since this is a case of a random walk[3] in 3 dimensions, $\bar{x} = \bar{y} = \bar{z} = 0$.

---

[3] Here we assumed that successive time intervals are very small compared with the

$\boxed{\bar{v}_x}$: What will be the average velocity of the particle parallel to the x axis? When we talk of the velocity of a particle, we have *two* things in our mind: its speed (fast or slow) and its direction of motion. Since the particle is as likely to move in the positive-x direction as in the negative-x direction, v_x is as likely to be positive as negative. Hence $\bar{v}_x = 0$. Similarly, $\bar{v}_y = \bar{v}_z = 0$.

$\boxed{\overline{v_x^2}}$: What will be the value of $\overline{v_x^2}$? This ensemble average will not be zero since v_x^2 is either positive or zero — never negative. I already said that molecules of water are not at rest. They are perpetually moving in different directions, some faster than others. Now, heat is a form of energy. When we heat water, we give energy to it. As a result, the water molecules start moving faster. Their average kinetic energy rises. On the other hand, when we heat water, its temperature also rises. Thus temperature is a measure of the average kinetic energy of the water molecules. It turns out that when the suspended particle is in thermal equilibrium with the water, its average kinetic energy is proportional to the temperature:

$$\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}kT,$$

where m is the mass of the suspended particle, k is a constant and T is the absolute temperature of the water. Hence $\overline{v_x^2} = kT/m$. This implies that heavier particles will have smaller $\overline{v_x^2}$. Similar statements can be made about the motion in y and z directions.[4]

$\boxed{\overline{x^2}}$: How far from the origin will the particle be after some time? Equivalently, what will be the value of $\overline{x^2}$? Before I answer this question, note the important complication in the present problem. Now, not only the direction but also the size of each step is a variable and is completely independent of the preceding step. (Why? Remember the fluctuating force mentioned in Section 3.)

Using the ideas from Statistical Mechanics, Einstein derived the fol-

---

observation time but still large enough that the motion of the suspended particle in any time interval can be considered to be completely independent of its motion in the preceding time interval.

[4] Einstein showed that it is practically impossible to measure $\overline{v_x^2}$ and suggested that experimentalists should rather measure $\overline{x^2}$.

lowing result:

$$\overline{x^2} = \frac{kT}{3\pi\eta a} t,$$

where η is the viscosity of the liquid, a is the radius of the suspended particle (assumed to be spherical) and t is the elapsed time. Thus the mean square displacement $\overline{x^2}$ increases *linearly* with time (i.e., the power of t in the above equation is unity).

Looking at the last equation, can you now answer the question no. (3) in the Quiz above? Please find out yourselves, before you read the answers given here:

(a) The Brownian Motion is less vigorous in cold water than in hot water. (b) The Brownian Motion will be damped if water is replaced by a more viscous liquid. (c) A bigger particle will drift less than a smaller particle — we do not notice the Brownian Motion of fish, people or boats. Do we?

Using Einstein's result, one can also answer quantitatively the more specific questions listed at the beginning of Section 5.

6 Importance

- In 1908, the French physicist Jean Baptiste Perrin verified Einstein's result experimentally. He measured $\overline{x^2}$ as a function of time t. Knowing the temperature T and viscosity η of the water, and radius a of the particle, he could obtain the value of the constant k. Using this, he obtained a reasonably good value for Avogadro's number (no. of molecules in a mole of a substance).

- Einstein's explanation of the Brownian Motion and its subsequent experimental verification by Perrin[5] were historically important because they provided a convincing evidence for the molecular theory of matter. In other words, they showed that atoms and molecules are real physical objects. Skeptics who doubted their existence were silenced.

- Fluctuating force on a Brownian particle is but one example of a fluctuating physical quantity even when the system is in equilibrium. Another example is a fluctuating current in some electric

---

[5] Perrin was honoured with the Nobel Prize for Physics in 1926, for this work.

> circuits. Einstein's work on the Brownian Motion laid the foundations of the study of fluctuation phenomena as a branch of statistical mechanics.

We have reached the end of our story of the Brownian Motion. You must have realized how a lowly pollen grain can tell us so much about the constitution of matter. Note that nothing of this would have been possible without the inquisitive mind of the scientist. The following quotation comes to my mind:

There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact — Mark Twain (1835 - 1910).

Special Relativity

T. P. Singh
Department of Astronomy and Astrophysics
Tata Institute of Fundamental Research

Many years ago, there used to live in Colaba two smart young sisters, Amrita and Rujuta. Amrita was at that time studying in the tenth standard, and Rujuta was in eighth. Both were very interested in science, and their teachers and parents used to tell them wonderful things about the world around them. One day Amrita came home from school, all excited. She said to her sister, “You know Ruju, today my teacher told us about Einstein’s special theory of relativity, and she did such a good job of explaining it, that I think I understand it quite well. You want to know?” Of course I do, said Rujuta. Tell me right now. OK then, lets grab some chips, and get down to work! Its going to take some time, but it will be worth the wait. In the end you will have learnt something fascinating about space and time, and about light, and about how things move, when they move very fast.

1 Motion is Relative

When you look at the world around you, you see lots of things. Some may be moving, and some may be at rest. You might see a car moving by at high speed, and you might see a table which is not moving. You are also aware that all these things exist in space. And you are also aware of time — time that seems to be passing by. There is yesterday, today and tomorrow. Time that is measured by clocks and watches. The special theory of relativity, which was discovered by Einstein in the year 1905, tells us new things about space and time — things that we simply cannot imagine by looking at our day to day world.

Lets start by talking about motion of objects. First, I want to make sure you understand the concept of speed. Speed means how fast something is moving. If the speed of an object does not change with time, we say it is moving at a *uniform* speed. This speed can be calculated by dividing the distance travelled by the time taken to cover that much distance. For example, if one day you decide to walk to your school, which is 3 kms away, and if you take one hour to do this, your speed will be three kilometers per hour. Please remember one thing. We will only be talking about uniform speeds here. It would be too much to keep on saying uniform speed, uniform speed all the time. But that's the kind of speed we will always be talking about.

Now Rujuta, there is a very important thing you must understand. All speeds are *relative*! What on earth does that mean? It is quite simple actually. Suppose we are both sitting side by side in a bus, and the bus is moving at a uniform speed of sixty kilometers per hour. Then what is your speed? Its sixty kilometers per hour. What is my speed? Also sixty kilometers per hour. Both of us are moving, but are we moving away from each other? No, not at all. We are all the time sitting side by side. So you are not moving with respect to me. As seen by me, you are in fact at rest! So we say that our relative speed is zero. We are both moving, but with respect to the ground outside, not with respect to each other. Our speed relative to the ground is sixty kilometers per hour, but our speed relative to each other is zero.

To make this clearer, let us imagine that our dear friend Kaustubh is standing on the road and watching the bus go by. We say that our speed relative to Kaustubh is sixty kilometers per hour. We might even imagine that the bus was not moving at all, but Kaustubh, and the ground, and all the other things outside, were rushing past us in the opposite direction, at a speed of sixty kilometers per hour!

So my dear little sister, remember, all speeds are relative. In fact, all *motion is relative*. The bus is moving relative to the ground. We, in the bus, are moving relative to the ground. We are *not* moving relative to each other. Kaustubh is not moving relative to the ground. But he is moving, relative to us.

How can we calculate the relative speed of moving objects? Suppose I am in a car moving at fifty kilometers per hour, and Kaustubh is in another car just by the side of my car, also moving in the same direction at fifty kilometers per hour. Then its obvious that our cars are going to remain side by side all the time, and that our relative speed is zero. We got this result by subtracting one speed from the other. This answer is expected, because we are at rest with respect to each other.

Now think of a situation where Kaustubh's car is moving faster than my car, say at seventy kilometers per hour. What is our relative speed now? Is it zero? No, it cannot be. Since he is moving faster, he will overtake me and appear to move away from me. So our relative speed cannot be zero. At what speed will he appear to go away? Seventy kilometers per hour? No. That is his speed with respect to the ground, not with respect to me. His speed relative to me is the difference of our speeds, seventy kilometers per hour minus fifty kilometers per hour, that is, twenty kilometers per hour. To me, he will appear to be moving at twenty kilometers per hour. To him, I will appear to be moving at twenty kilometers per hour, in the opposite direction.

What if Kaustubh's car was actually coming towards my car from the opposite direction, at seventy kilometers per hour? Since we are both moving towards each other, it should be obvious that our relative speed will be higher than seventy, and higher than fifty. It is in fact the sum of our speed, seventy plus fifty, equal to one hundred twenty kilometers per hour.

So you have probably now understood the rule for finding relative speeds of two objects moving in the same direction or in the opposite direction. In the first case, you subtract their speeds, and in the second case you add them, in order to find their relative speeds.

Consider another example. This time you, Rujuta, are sitting in a train which is going past a station platform on which I am standing. Suppose again that the train is moving at a speed of fifty kilometers per hour. And lo and behold, on the train's roof, a crazy man is running towards the engine (like they sometimes do in the movies)! If he is managing to run towards the engine, then clearly he is not at rest relative to the train. (In fact, Ruju you are at rest relative to the train, because you are sitting lazily on your seat). So he has some speed relative to the train. Let us suppose his speed relative to the train is ten kilometers per hour. Are you clear what that means? It means that if Rujuta, you could see through the roof of the train, you would find him moving forward at ten kilometers per hour.

The question is, what is the the man's speed relative to me? Remember, I am standing on the ground. Is the man's speed relative to the ground ten kilometers per hour, or fifty kiolometers per hour? Neither of these, in fact. Look at it this way. You, Ruju, are moving, say to my left, at fifty kilometers per hour. This would also have been the man's speed, had he not been crazy enough to run on the roof. But in addition to having this speed of fifty kilometers per hour, he also has an extra speed, ten kilometers per hour, relative to the train, and in the same direction as the train. So Ruju, as should be obvious to you, the man is moving to my left faster than you are! His speed relative to me (and relative to the ground) is actually fifty kilometers per hour, *plus* ten kilometers per hour, which makes it sixty kilometers per hour.

Let us summarize this important result. The man's speed relative to the train is ten kilometers per hour. His speed relative to the ground is a *different* number, sixty kilometers per hour, obtained by adding the man's speed relative to the train, and the train's speed relative to the ground.

Rujuta was quite excited with all this, but felt like she needed a break. It was play-time, so the sisters decided to call it off for a while, and went down to play with their friends. At the back of Rujuta's mind, though, there was this man running on the train — hey man, be careful, don't fall off . . . . But she wondered what special relativity was all about. Was it about people running on trains, and about people going past each other in fast moving cars? As she would soon find out, there was more to it, much more. Things that she could never have even dreamt of.

2 The Speed of Light

After dinner, the sisters were at it again. And the next day was Saturday, so Mummy was not going to tell them to sleep right away. Huddled in their bedroom, Amrita started once again.

Ruju, I now want to tell you about light. Of course, you know what light is. Like every evening we say, put on the lights. Or, sunlight, it is light from the sun. Or, you switch on a torch, and you see a beam of light coming out from the torch. Next time round, when you have a torch in your hand, go out into the dark night, and point the torch to the sky before switching it on. And when you switch it on, a beam of light will shoot out of the torch, and go out far, far and even farther. It will keep going.

Going? Yes, going. And that means that light travels, and anything that travels must have a speed! So you see, we are back again to talking about speed. What then is the speed of light? It is not an easy thing, to measure the speed of light. But many many years ago, scientists did succeed in finding out how fast light moves. And it moves very, very fast indeed. The speed of light is three lakh kilometers per second. Wow! That's huge. It takes light about one second to reach from the earth to the moon. And while an aeroplane (which I thought moves quite fast!) would take about an hour or two to go from Mumbai to Delhi, a beam of light would cover that distance in much less than one second.

Now, Rujuta, will you please get back on to the train? Thanks. I want to talk about relative speeds again. The relative speed of light. Remember, you were on the train, moving past the station platform, at fifty kilometers per hour. And the man on top the roof, moving at ten kilometers per hour relative to you, and at sixty kilometers per hour relative to the ground.

Now we are going to replace the man by a beam of light. Suppose you are sitting in the train facing the engine. Now switch on a torch, and let the beam of light travel straight ahead. So, instead of the man running on the roof, we have this beam of light moving forward, towards the engine, very fast. What is the speed of light as measured by you, Rujuta? Let us accept that you do an experiment, and find the speed of light to be three lakh kilometers per second. So the speed of light relative to you is three lakh kilometers per second.

Now we come to the most important part of our dialogue today. The most important fact about special relativity, from which so many extraordinary consequences follow. I ask you, Rujuta, what is the speed of light relative to the ground, that is relative to me (Amrita) standing on the station platform. You would use the same reasoning as you did for the man running on the roof. Add the speed of light relative to the train, and the speed of the train relative to the ground. So you would say that the result is three lakh kilometers per second plus fifty kilometers per hour. The speed of light relative to the ground is three lakh kilometers per second plus fifty kilometers per hour.

Right? NO! WRONG! Completely wrong. Experiments show that the speed of light relative to the ground is the same as its speed relative to the train! Three lakh kilometers per second. The rule of finding relative speed by adding speeds does not apply to light. Light moves at the same speed relative to everyone.

How can that be, you say. How? Prove it, you say, prove it to me.

Take it easy dear. Scientists have learnt this by doing experiments. There is nothing to prove here. It is a fact established by experiment, and we have to simply accept it.

You see, we learn facts about the world around us by doing experiments. Then we make rules and theories which explain those facts in a nice and simple way. Such rules are generally referred to as laws. A few hundred years ago, Galileo and Newton discovered the laws of motion of bodies, on the basis of experiments on how objects move. These are known as Newton's laws of motion. You have probably learnt these laws at school.

Newton's laws of motion correctly explain the motion of objects that we see in our day to day life. Moving balls, moving vehicles and moving planets, for example. From these laws it follows that relative speeds are to be found by adding speeds (remember: adding the man's speed relative to the train and the train's speed relative to the ground gave his speed relative to the ground).

Some two hundred years after Newton, scientists succeeded in doing an experiment to find the relative speed of light. And they found that light moves at the same speed relative to everyone. This means that Newton's laws of motion do not apply to light.

So, Newtons's laws must be replaced by new laws of motion. In the year 1905, Einstein discovered these new laws of motion, according to which the speed of light is the same relative to everyone. These new laws came to be known as Einstein's *Special Theory of Relativity*.

Does this mean that Newton was wrong? No, not at all. Newton's laws of motion correctly describe the motion of bodies so long as the speed of the body is much smaller than the speed of light. If a body starts to move very fast, at a speed comparable to the speed of light, then Newton's laws do not correctly describe its motion. But Einstein's laws do. And, very importantly, Einstein's laws agree with those of Newton when the speed of the body is much smaller than the speed of light. We say that Newton's laws of motion are an approximation to Einstein's laws — an approximation that holds good to a great accuracy for bodies moving slowly, compared to light. The motion of light itself can be correctly described only by Einstein's theory, which tells us, in particular, that light moves at the same speed relative to everyone.

In Einstein's theory, the rule for finding relative speeds is different from Newton's. Relative speed is not found merely by adding speeds. The rule is such that the speed of light comes out the same for everyone. And moreover, for bodies moving slowly compared to light, the rule becomes the same as that of Newton: relative speeds of such bodies are to be found by adding speeds, just as Newton found. Isn't that nice?

In fact, if you are comfortable with a little bit of mathematics, I can write down for you the rule for adding speeds, as given by Einstein. Suppose v_b is the speed of a body on the train (this is the man on the roof, or the beam of light). Let us denote by v the speed of the train. Let v_r be the speed of the body relative to the ground. Einstein found that v_r is given by the rule

$$v_r = \frac{v_b + v}{1 + \frac{v_b v}{c^2}}$$

In this formula, c denotes the speed of light. You see, when the speed of the body and the speed of the train are much smaller than the speed of light, the term to the right of the 'one' in the denominator is very small and can be ignored, and this rule becomes the same as the addition rule of Galileo and Newton! It is in this sense that Newton's laws of motion are an approximation to those of Einstein.

On the other hand, suppose the body in question is a beam of light, so you must set $v_b = c$. You can easily check that then you get $v_r = c$. Isn't that great? The speed of light comes out the same relative to everyone, according to this rule.

Einstein also found that no body can move faster than light. Things can move only slower than light, or at best as fast as light. If something is moving at the speed of light, it must always move at the speed of light! It cannot slow down. And if something is moving slower than light, it can never move as fast as light.

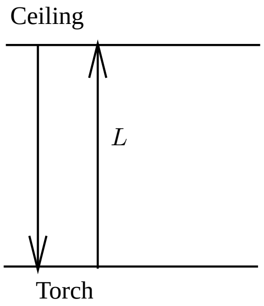
By now Rujuta was tired and sleepy. So, with their minds full of relativity and what not, Amu and Ruju fell asleep. And no wonder they dreamt of beams of light, of Einstein and Newton.

3 Space and Time

No sooner had they woken up on what turned out to be a nice and rainy morning, Amrita began talking again.

Rujuta, the fact that the speed of light is same for everyone leads to dramatic consequences regarding our understanding of space and time. I will tell you about these now. It's not difficult, what I am going to tell you now, but surely it's very surprising! And something that we are not used to in our day to day life.

Do me a favour, Ruju, will you, please? Get on to our good old train, which moves past me at the platform, at the speed of fifty kilometers per hour. And be ready with your torch. You are again to put on the torch so as to release a beam of light. Only this time, please point the torch upwards, towards the ceiling, where there happens to be a mirror, so that the beam will be reflected and come back exactly to where it started from. This is what you, Rujuta, will see as the path of light.

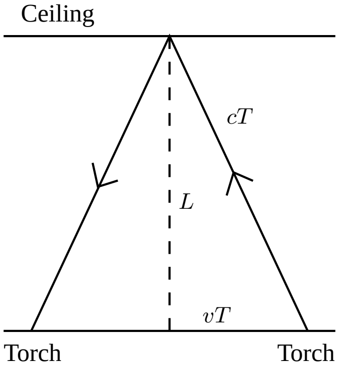


Path of light as seen by Rujuta:

Distance to ceiling $= L$
Time taken to reach ceiling $= t$
Hence $L = ct$

Figure 1

Of course for you the beam will simply go up and down, and the total distance travelled by the beam is twice the distance from the starting point to the ceiling. But what will I, standing on the ground, see as the path of the beam? By the time the beam comes back to the starting point, the train will have moved slightly to the left, so the path of the beam, as seen by me, will *not* be just up and down, but slanting, as shown in the figure below.



Path of light as
seen by Amrita:

Time taken by light
to reach ceiling $= T$

Distance travelled by
train while light reaches
ceiling $= vT$

Figure 2

Does the beam of light travel the same distance, as seen by either of us? No, of course not. I find that the beam travels more distance than claimed by you, Ruju. That's because for you the beam simply travels up and down, but for me it takes a slanting path. Clearly, for me the beam travels a longer distance.

Now the fun can begin. We agreed a while ago that the speed of light is the same for everyone. The time taken to go up and come down is simply the distance travelled divided by the speed. Speed of light is same for the person on the train, as well as the person on the ground. But distance travelled is different. So to come back to the starting point, light takes more time for me, than it does for you, Ruju!

Let me make that clearer with an example. Both of us are wearing a watch each. Suppose when you, Ruju, switch on the torch, both our watches show the time to be ten a.m. When the beam reaches back at the torch after the reflection, let your watch read five minutes past ten. According to our reasoning above, according to me, light will take more time to come back to the torch, so my watch will show a larger time, say seven minutes past ten!

With a little bit of maths, it is not hard to calculate the relation between the rates at which the the two clocks move. Look at the two figures above. Suppose according to you light takes a time $2t$ to come back to the torch. This means it will take a time t to reach the ceiling. If L is the distance to the ceiling, we have the relation $L = ct$. According to me, let the light beam take a time $2T$ to come back to the torch. So it will take a time T to reach the ceiling, and hence cover the distance cT along the slanting direction. Also, in this much time the train travels a distance D given by $D = vT$, where v is the speed of the train. By the application of the Pythagoras theorem to the triangle in Figure 2 we have the relation

$$(cT)^2 = (vT)^2 + (ct)^2$$

from which we immediately get, upon simplification, the relation between t and T:

$$T = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Again you see that if the speed of the train is much less than the speed of light, the term after ‘one’ under the square-root can be ignored, as it is very small, and we get $T = t$, which is the assumption made by Galileo and Newton. We once again see that Newton’s mechanics is an approximation to Einstein’s mechanics, valid when the speed of the moving bodies is much smaller than the speed of light.

What is this telling us? If two persons are moving relative to each other, their clocks do not run at the same rate. I, Amrita, standing on the ground, concludes that Ruju’s clock is going slower. This is simply a consequence of light having the same speed for everyone. According to Newton’s laws of mechanics, speed of light would not have been same for both of us, and in fact, clocks run at the same rate for everyone. (Of course they do, in our daily lives we do not go about quarreling and disagreeing about how fast our clocks are running, even though we move about a lot everyday, relative to each other).

Wait a minute, says Ruju, I am a bit confused. You said the correct laws of motion are those of Einstein, Newton’s laws are only approximately true. So why do we not have a problem with our clocks. If I sat on my train, my clock should run slower, shouldn’t it?

Good point, said Amu, good point. Your clock will indeed run slower, but only by a very very tiny amount. The difference in the rate of your clock and my clock will be very slight, because the train is moving at a speed so much smaller than the speed of light. The difference in the rates of clocks would become easily noticeable if your train starts moving at a speed comparable to the speed of light.

Ah, said Ruju, I get that one. But there is something else bothering me. You had earlier told me that all motion is relative. So I, Ruju, could as well have imagined that Amrita, you are the one who is moving. So I should conclude that my (Ruju's) clock should be showing more time than Amu's, clock, unlike what you are claiming. Smart, Ruju, smart, said Amrita. You are warming up to the scene. (It turns out that both of us are right (funny!) but I am going to leave it to you to sort out that one . . .)

Something similar happens also for lengths of rods, and in general for the size of things. Suppose my dear sister, you are again travelling in the train (poor you, so much train travel!). Suppose you have a straight rod with you, placed exactly along the direction in which the train is moving, and you measure the length of the rod, and find it to be some number, say one meter. And me, Amrita, standing on the ground, also measures the length of that rod you are carrying, by doing some experiment. I will find the rod to be less than one meter long! Crazy, but true. And again this is a consequence of the speed of light being the same for everyone.

If you, Rujuta measure the length of the rod, placed along the direction of motion of the train, to be L, then according to Einstein, I, Amrita will measure the length of the rod to be a number D given by the the relation

$$D = L\sqrt{1 - \frac{v^2}{c^2}}.$$

Once more, if the speed v of the train is much smaller than the speed of light, the last term under the square root is very small and can be ignored, and we get the expected result of Newtonian mechanics: $D = L$. In Newton's mechanics different persons moving relative to each other agree on the size of things, just as they agree on the rates at which clocks move.

Moving objects appear to be shorter than they actually are, according to special relativity, and moving clocks appear to run faster than the actual rate at which they are running. Hence, according to Einstein, there is nothing absolute about space, neither is there anything absolute about time. How much time will have passed, and how much size a body will have, depends on who is making the measurement. People in relative motion do not agree about the results of these measurements. This is completely different from what Newton said. Once again, you might think Newton was wrong. No, he wasn't wrong, he was approximately right. And right to a very good approximation. If the bodies on which we make measurements are moving much slower than light, we will agree that clocks are running at the same rate, and that sizes are same, as seen by people in relative motion.

But Amu, it is frustrating. According to Einstein, people in relative motion seem to disagree about everything! I don't like it. I like people to agree with each other, and be friendly. Of course, lil sis, of course. You forgot something! People in relative motion agree about the speed of light. They all agree that light moves at the same speed, three lakh kilometers per second. It is because they agree about speed of light that they are forced to disagree about length and time.

And yet, there is still one more wonderful fact. Something made out of both space and time is there, on which two people in relative motion agree. I will give you an example. Please get back on the train, Rujuta, and this time take Kaustubh with you. Let us imagine that Kaustubh is sitting on the seat opposite you, at a distance one meter away, as measured by *you*. Now throw a ball to him, and suppose it reaches him after one second, again this time is as measured by *you*.

I, Amrita, am standing on the ground, and watching this ball throwing business. Of course, I am not going to agree with you on how far Kaustubh is sitting from you, nor am I going to agree with you on how much time the ball took to reach Kaustubh. That's what we have learnt from Einstein.

But suppose you calculate the following number. Take the square of the time passed, which is one second in your case. Multiply it by the square of the speed of light. From the result subtract the square of the distance between you and Kaustubh, one meter as measured by you. Lets call the resulting number *Rujuta's Number*.

Now I, Amrita, will do a similar calculation. Having found the time the ball took to reach Kaustubh (as measured by *me* standing on the ground) I will square this time. I will multiply that by the square of the speed of light. From the result I will subtract the square of the distance between Rujuta and Kaustubh (distance as measured by me). Let us call this answer *Amrita's Number*. Here then is the surprise:

$$\text{Amrita's Number} = \text{Rujuta's Number}.$$

I have not proved it to you, but this is another consequence of the speed of light being same relative to everyone.

So, according to Einstein, space is not absolute, and time is not absolute, but something made out of space and time together is absolute. Hence we talk of space and time as being parts of a new concept, space-time. As if to say that space does not exist without time, and time does not exist without space, both exist in togetherness, as space-time.

What exactly does this number mean, that we made out of space and time, and on whose value people in relative motion agree? To understand this, we shall introduce the idea of an event. An event is specified by telling when and where something happened. So when you and Kaustubh were exchanging the ball, the first event happened just when you threw the ball. The position of this event was your (Ruju's) position, and the time of this event was the time on your clock when you threw the ball. This is a space-time event, whose location in space and in time is as I just mentioned.

The second space-time event happened when Kaustubh received the ball. The position of this event is where Kaustubh is sitting, and the time of this event is the time (according to your clock) when Kaustubh received the ball. The number that we called Rujuta's Number above, measures how 'far' the first space-time event is from the second space-time event, as seen by you, Rujuta.

Now on the ground, I (Amrita) also observe these two space-time events. I also measure the positions of Rujuta and Kaustubh, and the times of the two events, and the number that we called Amrita's number above measures how 'far' the two space-time events are from each other, as seen by me.

The fact that these two numbers are equal means that people in relative motion agree on the 'distance' between two space-time events, even though they do not agree on the amount of time between these two events, or on the length of space between these two events. Remember, this ‘distance’ between space-time events is not the ordinary distance in space; it is distance in space-time! We also call it the space-time interval between two events.

As you know Rujuta, space is three dimensional, because there are three directions in space. To this we add the one dimension of time. And we say that space-time is four dimensional. We live in a four dimensional space-time. Our Universe consists of matter living in a four dimensional space-time. Once and for all, the distinction between space and time has been blurred, and this had to happen because the speed of light is the same for everyone!

Once again, the kids needed a break. Also, it was lunchtime. And they still had to take their baths. And do their home-work. And go for a birthday party in the evening. So it was decided that the story of special relativity was to be continued on Sunday morning.

4 Mass and Energy

Newton gave us the laws of motion. The first law says, as I am sure you have studied in school, that *unless it is acted on by an external force, a body at rest continues to be at rest, and a body in uniform motion continues to be in uniform motion*. This law stays as such, unchanged, in Einstein’s special theory of relativity.

Newton’s second law says that *the state of uniform motion of a body can only be changed by applying a force to the body*. In fact you may have also learnt that the force applied to the body is equal to the rate of change of its momentum. Now what is momentum? Well let me give you a simple-minded picture: momentum is what you get by multiplying the mass of a body with its speed. Momentum is an important concept because momentum is what changes when a force is applied to a body. It is a measure of the mass of a body as well as of its speed.

In special relativity too, the second law of motion is true, except that here the definition of momentum changes. The momentum still depends on mass and speed, but it depends on speed in a way different from that in Newton’s theory. In fact if a body starts moving very, very fast and its speed approaches that of light, its momentum starts to become infinitely large.

The definition of momentum in special relativity is:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Here v is the speed of the body, c is the speed of light, and m is the mass of the body. You can see for yourself that as v approaches c, the momentum becomes infinite. This is one way of understanding why nothing can move faster than light. Also, you can see that for speeds small compared to the speed of light, you can ignore the term after one in the square root, and you recover Newton's definition for momentum.

In Newton's mechanics one also talks of the energy of motion of a body, which we call its kinetic energy. Ruju you do have an idea as to what we mean by the energy of a body, don't you? In Newton's mechanics, the kinetic energy of a body is calculated by multiplying the mass of the body by the square of its speed, and then dividing the result by two. You can easily check that the kinetic energy is also equal to the square of the momentum divided by two times the mass of the body. This is the relation between the energy and the momentum of the body in Newton's mechanics. If the momentum is zero, the energy is also zero.

You can also think of kinetic energy in another way. A force acting on a body does work on it, and this work done on the body is converted into its kinetic energy. In fact the mathematical expression for kinetic energy is what it is, precisely because of the second law, and because momentum is mass times speed.

If you apply force to a body, you change its speed, and hence you also change its kinetic energy. The same is true in special relativity, except that the definition of kinetic energy in terms of speed is again different from that in Newton's mechanics. The most interesting thing is that the relation between energy and momentum is now very different. If you square the energy of the body and subtract from it the square of the momentum multiplied by the square of the speed of light, the resulting number is always equal to the square of the mass multiplied by the fourth power of the speed of light! The speed of light never appears in the energy-momentum relation in Newton's mechanics, but in special relativity it does.

In special relativity the kinetic energy E of a body is defined as

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which becomes infinite when the speed of the body approaches the speed of light. Using this expression and the definition of momentum given above we easily see that

$$E^2 - p^2c^2 = m^2c^4.$$

This relation between the energy and momentum of a body is true from everyone's point of view. Thus if you, Ruju was in the moving train and measured the energy and momentum of a ball moving in the train, she would find the above relation. And if I, standing on the station platform, measured the energy and momentum of the body, I would get different answers for the values of energy and momentum, compared to you, but the relation between energy, momentum and mass would be the same as obtained by you.

The real interesting thing is that unlike in Newton's mechanics, the energy of the body does not go to zero even when its speed (and hence momentum) goes to zero. From the above relation, we instead get, for a body at rest, $E = mc^2$. This is one of the most famous, if not *the* most famous, equations of physics.

So we learnt for the first time, from Einstein, that there is really no difference between mass and energy! Mass is just a form of energy. If a body at rest has some mass m, and if you want to find out how much energy it has, just multiply its mass by the square of the speed of light. Today, physicists all the time use this relation between mass and energy, essentially with their eyes closed (!), but when Einstein first discovered this relation (again as a consequence of the speed of light being the same for everyone) it was considered revolutionary.

Indeed, this relation between mass and energy has been verified experimentally many, many times. It also plays a very important role in nature, and in the man-made world. The relation also tells us that mass and energy can be converted into each other. For example, did you know that the energy of sunlight comes because the sun is converting its mass to energy? Deep inside the sun, hydrogen is converted into helium, and because a helium nucleus has less mass from the four hydrogen nuclei from which it is formed, the lost mass actually shows up as the energy of sunlight!

And did you know that the atom bomb works on the same principle? Conversion of mass into enormous energy. Alas, during the second world war Einstein's ideas were put to misuse, and the atom bombs dropped on Hiroshima and Nagasaki in Japan lead to enormous destruction. But that was not Einstein's fault, you see. We as human beings have to learn to put scientific discoveries to good use. For example, the energy obtained from converting mass is a useful source of energy in our daily lives.

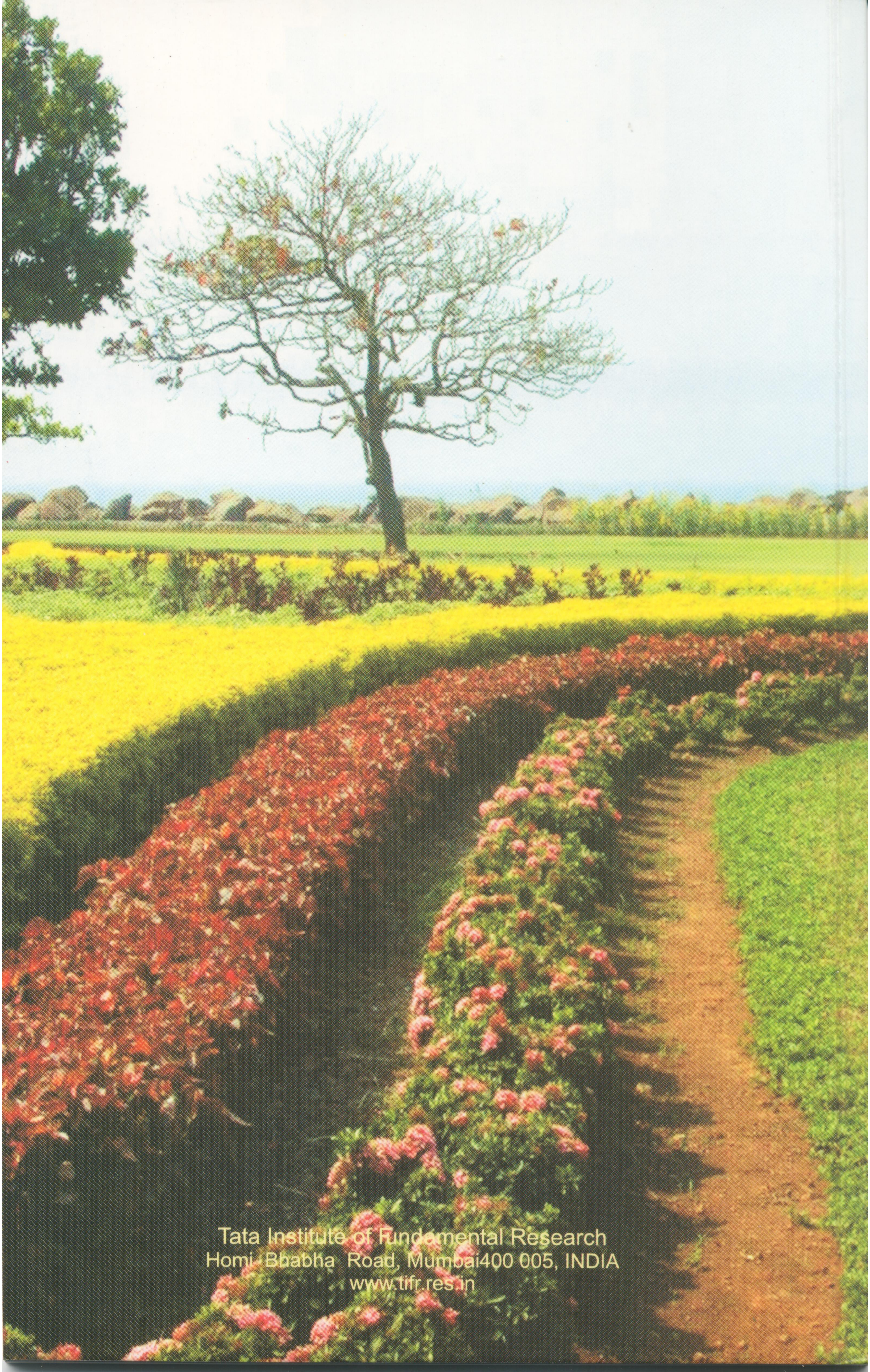
5 What Next?

With that, dear Ruju, I have come to the end of my story. I have told you most of what my teacher told me on this topic. This is the *special theory of relativity*, that Einstein discovered in 1905. By the way, I should add that the name 'Special Relativity' was not given to the theory in 1905. In fact, the famous research paper of Einstein, in which this discovery was reported, was titled *On the electrodynamics of moving bodies*. Scientists report their results by writing research papers and by publishing them in science journals.

The name 'Special Theory of Relativity' was given by Einstein some ten years later, when he discovered another theory, which he called the *General Theory Of Relativity*. In the General Theory, Einstein proposed a new law of gravitation, and modified Newton's law of gravitation. (You would think Einstein was really hell-bent on changing everything that Newton did! But Einstein had the highest respect for Newton, and regarded Newton as the greatest physicist of all times).

You know Ruju, many other great physicists also did important work on this topic, and on the speed of light, *before Einstein*. Some of them were Maxwell, Lorentz, Poincaré, Michelson and Morley, and Fitzgerald. We must not forget them. Their contribution was also important. Einstein happened to come on top because his findings were the clearest, and the most general.

In the end I want to tell you the most fascinating thing my teacher told me. She said many physicists today are not satisfied with Einstein's special theory of relativity! They think there are reasons why Einstein's theory should be replaced by another theory, just as there were reasons to replace Newton's mechanics by a new theory. And scientists are working on this problem with lot of excitement and enthusiasm. You know what, I think I am going to be a scientist when I grow up. What about you?



Tata Institute of Fundamental Research
Homi Bhabha Road, Mumbai400 005, INDIA
www.tifr.res.in