

On The Stability of Immune-Tumor Model with Two Term Delay in Tumor

Anuradha Devi, Aditya Ghosh

Abstract-- The model presented here is a two variable model which include immune cell (I) and tumor cell (T). Two term delay T_d has been introduced in the tumor growth. The intrinsic behavior of the tumor cell by the introduction of two delay terms have been analyzed under the equilibrium points. A relation of immune and tumor cell under the effect of delay at equilibrium point have been established in this paper. The stability condition of the model have been analyzed numerically.

Index Terms--Mathematical model, Tumor cell, Immune cell, Eigenvalue, Stability, Delay, Equation

1 INTRODUCTION

Mathematical modeling and analysis of growth of tumor for cancer is a subject of great research interest. The complex behaviour of growth of tumor cell and it's effect on normal cell and immune cell already have been researched in [1],[9],[12]; between tumor and normal cells in [3],[6],[8], and under control using chemotherapy in [2],[3],[6],[10],[11],[14],[15],[16],[17]. As the growth of tumor cell shows delay, the modeling of tumor cells needs to be modified and analysed under delay. The behaviour of growth of tumor cell under one term delay has been analysed in [5]. The complexity of the two variable model of cancer with Immune (I) and Tumor (T) cell given in [4] has been modified with the introduction of one term delay in Tumor in [5]. Here, the same two variable model of [4] has been modified by introduction of delay in two terms in tumor.

2 MODEL DESCRIPTION

A Immune-Tumor Model described in [5] has been considered and modified by introducing delay in two terms for tumor growth. A mathematical model of Immune-Tumor with delay in one term has been already introduced in [5].

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2.1 Model Equation

2.1.1 Immune-Tumor Model with delay in Tumor growth

Let $I(t)$ denote the number of immune cell at time t that kill tumor cells, $T(t)$ denote the tumor cells at time t . Immune and tumor cells together behaves as a predator-pray model of interacting species [19]. Immune cells in human body have a constant source and also get stimulated and recruited by the presence of tumor cells. Immune cells shows natural death at a rate d_1 . Hence the growth of immune cells can be modelled as

$$\frac{dI}{dt} = s + \frac{rI(t)T(t)}{\sigma + T(t)} - d_1I(t) \quad (2.1)$$

where

s = The constant immune cells present in the body.

σ = stepness coefficients.

r = recruitment rate of immune cells stimulated by tumor.

Furthermore, the interaction of immune cells and tumor cells can result in either the death of tumor cells or the deactivation of immune cells, resulting in the two competition terms

$$\frac{dI}{dt} = -c_1I(t)T(t) \quad (2.2)$$

and

$$\frac{dT}{dt} = -c_2I(t)T(t) \quad (2.3)$$

The tumor cells follows a logistic growth $aT(t)(1 - bT(t))$ undergo a delay. So the growth of tumor is modified by

$$aT(t - \tau)(1 - bT(t - \tau)) \quad (2.4)$$

Hence, the model equation for immune-tumor cell growth with one term delay may be represented as

$$\frac{dI}{dt} = s + \frac{rI(t)T(t)}{\sigma + T(t)} - c_1I(t)T(t) - d_1I(t) \quad (2.5)$$

$$\frac{dT}{dt} = aT(t - \tau)(1 - bT(t - \tau)) - c_2I(t)T(t) \quad (2.6)$$

Here,(2.6) represents equation of tumor growth with two term delay.

where

c_1 =Tumour deactivation rate of effectors.

d_1 =Natural death rate of immune cells.

a =intrinsic tumor growth rate

$\frac{1}{b}$ =tumour population carrying capacity

c_2 =death rate of tumor cell

τ =delay term

3 STABILITY ANALYSIS

3.1 Immune-Tumor Model with two term delay

3.1.1 Equilibrium points

Immune-Tumor delay model is a model which has $T(t)$ as well as $T(t - \tau)$. Considering the following equation

$$aT(t - \tau)(1 - bT(t - \tau)) - c_2I(t)T(t) = 0 \quad (3.1)$$

Leads to the equilibrium points

$$T_1 = \frac{M}{c_2I_1} \quad (3.2)$$

$$T_2 = \frac{M}{c_2I_2} \quad (3.3)$$

$$M = -aT(t - \tau) + abT(t - \tau)^2 \quad (3.4)$$

where, I_1 and I_2 are roots of the equation

$$(-d_1c_2\sigma)I^2 + I(sc_2\sigma + rM - c_1M\sigma - d_1M) + \left(sM - \frac{c_1}{c_2}M^2\right) = 0$$

Now, for the value $T(t) = 0$ we get $I(t) = \frac{s}{d_1}$. Hence, there exists a tumor free equilibrium point $(\frac{s}{d_1}, 0)$.

The co-existing equilibrium point (I_1, T_1) is determined from (3.2), (3.3), (3.4).

3.1.2 Stability Analysis for (I_1, T_1)

Linearizing around (I_1, T_1) gives a linear system of equation

$$\begin{pmatrix} \dot{I} \\ \dot{T} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} I - I_1 \\ T - T_1 \end{pmatrix} \quad (3.5)$$

Where

$$A = \frac{rT(t)}{\sigma + T(t)} - c_1T(t) - d_1 \quad (3.6)$$

$$B = \frac{rI(t)(\sigma + T(t)) + rI(t)T(t)}{(\sigma + T(t))^2} \quad (3.7)$$

$$C = -c_2T(t) \quad (3.8)$$

$$D = -c_2I(t) \quad (3.9)$$

With eigenvalues

$$\lambda_1 = \frac{(A + D) + \sqrt{(A - D)^2 + 4BC}}{2A} \quad (3.10)$$

$$\lambda_2 = \frac{(A + D) - \sqrt{(A - D)^2 + 4BC}}{2A} \quad (3.11)$$

The model will be stable if

$$\lambda_1 < 0 \quad (3.12)$$

And

$$\lambda_2 < 0 \quad (3.13)$$

Here the eigen values λ_1 and λ_2 depend on all of the system parameters. This means that if $\lambda_1 < 0, \lambda_2 < 0$, then the system will be stable, means when λ_1 and λ_2 both non-negative, then the system will be unstable and immune cells will not be able to completely irradiate the tumor and some external control needs to be introduced in the system.

4 ANALYSIS AND CONCLUSION

The delay term given in (2.4) indicates the delay in the growth of tumor cell and an effect of it a quadratic delay in the carrying capacity. The condition for stability for the mathematical model given by the equations (3.12) and 3.13 together with (3.10),(3.11). The condition for stability depends upon the parameter involved in the model. Numerical analysis shows that, when we consider parameter for which λ_1 and λ_2 both positive, the growth of tumor shows exponential growth and which implies instability of the system[Fig. 1]. When λ_1 and λ_2 both negative for the parameter chosen as such shown stability in both immune and tumor cells [Fig.2, Fig.3]

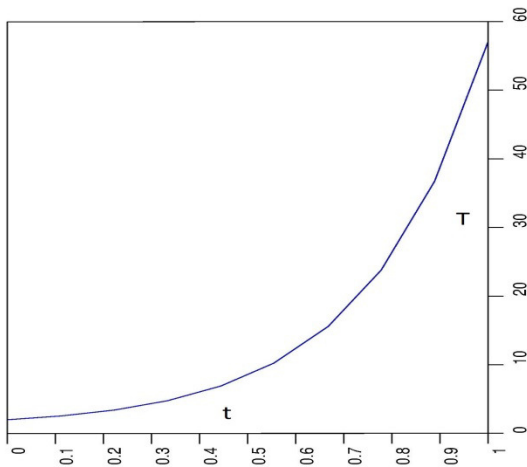


Fig. 1. I-T-delay model tumor instability

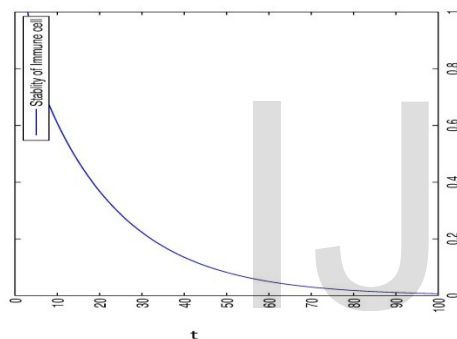


Fig2. . I-T-delay model stability condition for Immune

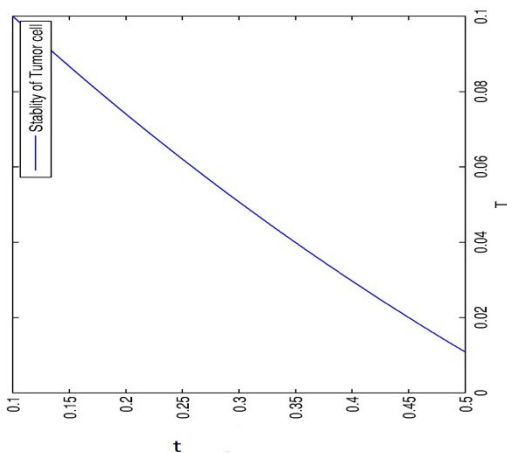


Fig3. . I-T-delay model stability condition for Tumor

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