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RESEARCH ARTICLE

Mixed convective elastico-viscous fluid flow past a porous plate in presence of induced magnetic field and energy dissipations

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ABSTRACT:

A mixed convective hydro-magnetic flow of visco-elastic fluid past an infinite vertical porous surface with constant suction has been investigated in presence of heat and mass transfer. Visco-elastic fluid flow is characterized by Walters liquid (Model B') for short relaxation memories. A magnetic field of strength B_0 is applied along the transverse direction to the surface. To study the governing fluid motion, we have considered the effects of induced magnetic field and magnetic dissipations of energy. Mechanism of heat transfer arises from mixed convection along with dissipation of energy due to viscosity. Let T_w and C_w be respectively the temperature and the molar species concentration of the fluid at the surface. To solve the governing equations of motion, we have used multi-parameter perturbation scheme. Two non-dimensional numbers taken as perturbation parameters are Eckert number and visco-elastic parameter. The results are discussed graphically for the various values of elastico-viscous parameter along with other values of flow parameters involved in the solution.

KEY WORDS: Elastico-viscous, induced magnetic field, perturbation scheme, current density, Eckert number.
2000 AMS Mathematics subject classification:, 76A05, 76A10

1. INTRODUCTION:

The analysis of visco-elastic phenomenon has attracted various researchers because of its application in various engineering and blood flow problems. The complex stress-strain relationships of visco-elastic fluid flow mechanisms are used in geophysics, chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil-physics, bio-physics, paper and pulp technology.

The concept behind the convection problem is the density difference caused by simultaneous heat and mass transfer characteristics. Researchers have extensively used the mechanisms of convection problems due to their applications in atmospheric field, geophysics and various engineering fields etc. An analysis of thermal convection in magneto-hydrodynamics problem has been studied by Singh and Cowling (1). Raptis and Kafousis (2) have analysed the characteristics of magneto-hydrodynamics free convective flow and mass transfer from a vertical plate in presence of heat flux. The combined heat and mass transfer effects in a porous medium has been investigated by Bejan and Khair (3). Trevisian and Bejan (4) have extended the heat and mass transfer problem by imposing the buoyancy effect in governing fluid flow mechanisms.

The phenomenon of magneto-hydrodynamics is widely used in many industrial purposes such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, textile industry etc. MHD is also used to study the principles of various engineering applications, in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and also in medical science. Acharya *et al.* (5) have studied the magnetic field effects on free convection and mass transfer flow through porous medium with constant suction and heat flux. The behaviour of steady MHD couette flow has been examined by Attia (6) by considering various temperature dependent physical properties. Simultaneous heat and mass transfer in a free convection flow past a flat plate under the influence of transverse magnetic field has been discussed by Singh *et al.* (7).

The effect of induced magnetic field has been neglected in above mentioned works. The assumption behind the neglecting of induced magnetic field is low conductivity of electrically conducting fluid. But for higher conductivity, the induced magnetic field cannot be omitted. Singh and Singh (8) have studied the MHD effects on heat and mass transfer by considering the influence of induced magnetic field. Ahmed (9) has analysed the mixed convection heat and mass transfer MHD flow from an infinite vertical porus plate in presence of induced magnetic field. Hydromagnetic free convective flow with induced magnetic field effects is examined by Ghosh *et al.* (10). Zueco and Ahmed (11) have investigated simultaneous heat and mass transfer in a mixed convective MHD flow in presence of heat source and induced magnetic field. The induced magnetic field effect with viscous/ magnetic dissipation bounded by a porous vertical plate in presence of radiation has been surveyed by Ahmed (12).

Walters liquid (Model B$^{/}$) is a type of visco-elastic fluid which resists shear flow and strains linearly with time under the application of an applied stress but when the stress is removed it quickly returns to its original position. The viscosity of the visco-elastic fluid enables the physics of the energy dissipated during the flow and its elasticity analyses the energy stored during the flow. As Walters liquid (Model B$^{/}$) exhibits elastic properties besides having fluid properties of Newtonian fluid.

The constitutive equation for Walters liquid (Model B$^{/}$) is

$$\sigma_{ik} = -pg_{ik} + \sigma'_{ik},\ \sigma'^{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik} \qquad (1.1)$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v_i is the velocity vector, the contravarient form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk}, \qquad (1.2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \qquad (1.3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau)d\tau \qquad (1.4)$$

N(τ) being the relaxation spectrum . This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)d\tau,\ n \geq 2 \qquad (1.5)$$

have been neglected.

Walters (13) reported that the mixture of polymethyl methacrylate and pyridine at 25^0 C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml fits very nearly to this model. For this mixture, the relaxation spectrum as given by Walters is

$$N(\lambda) = \sigma\eta_0\delta(\lambda) - \frac{1-\sigma}{\beta}\eta_0 \qquad (0 \leq \lambda \leq \beta)$$

$$= 0 \qquad \lambda > \beta$$

where $\sigma = 0.13, \eta_0 = 7.9$ poises, $\beta = 0.18$ sec and $\delta(\lambda)$ is the Dirac's delta function (Kapur et al. [14]).

Polymers are used in the manufacture of space crafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastic, engineering equipments, contact lens etc. Walters liquid (Model B') forms the basis for the manufacture of many such important and useful products.

2. MATHEMATICAL FORMULATION

The steady two-dimensional mixed convective flow of an electrically conducting Walters liquid (Model B') past an infinite vertical plate has been investigated. A magnetic field of uniform strength B_0 is applied in the transverse direction. The effect of induced magnetic field is also taken into account. In Magneto-hydrodynamics, the combined effect of current and magnetic field generates Lorentz force, which disturbs the fluid motion. The fluid velocity also changes the magnetic field by generating an induced magnetic field which disturbs the original field. Let x' axis be taken along the plate in the vertical upward direction and y' axis be taken normal to it. Velocity components along x' axis and y' axis are taken as u' and v' respectively. Let b_x' and b_y' be the magnetic induction vectors along x' axis and y' axis respectively. Let T_w and C_w be respectively the temperature and the molar species concentration of the fluid at the plate and T_∞ and C_∞ be respectively the equilibrium temperature and equilibrium molar species concentration of the fluid. The physical configuration of the problem is given in figure 1.

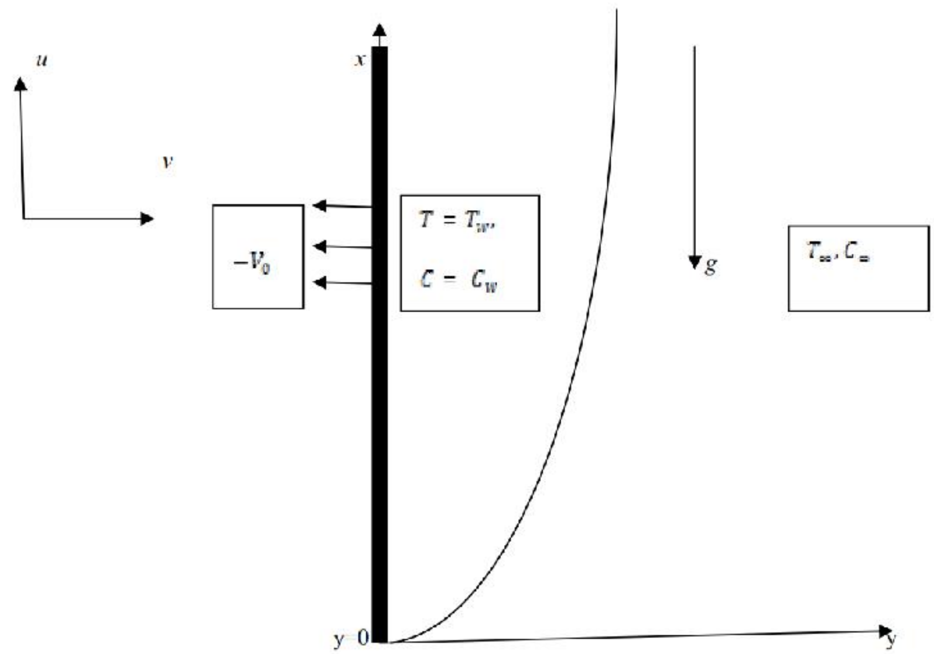

Figure 1: Physical configuration of the problem.

With these physical considerations and using Boussinesq approximation, the equations governing the steady motion of an incompressible electrically conducting Walters liquid (Model B') in presence of magnetic field are:

$$\frac{\partial v'}{\partial y'} = 0 \qquad (2.1)$$

$$\frac{\partial b_y'}{\partial y'} = 0 \qquad (2.2)$$

$$-V_0\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + \frac{k_0}{\rho}V_0\frac{\partial^3 u'}{\partial y'^3} + \frac{\sigma B_0^2}{\rho}(U - u') + \frac{V_0\sigma B_0}{\rho}(b_0 - b_x') + g\beta(T' - T_\infty) + g\beta^*(C' - C_\infty) \qquad (2.3)$$

$$-V_0\frac{dT'}{dy'} = \frac{K}{\rho C_p}\frac{d^2T'}{dy'^2} + \frac{\nu}{C_p}\left(\frac{du'}{dy'}\right)^2 + \frac{k_0}{\rho C_p}V_0\frac{du'}{dy'}\frac{d^2u'}{dy'^2} + \frac{\sigma}{\rho C_p}\{B_0(U - u') + V_0(b_0 - b_x')\}^2 \qquad (2.4)$$

$$-V_0\frac{dC'}{dy'} = D\frac{d^2C'}{dy'^2} \qquad (2.5)$$

$$0 = \eta \frac{d^2 b_x^{'}}{dy^{'2}} + B_0 \frac{du'}{dy'} + v_0 \frac{db_x^{'}}{dy'} \qquad (2.6)$$

The corresponding boundary conditions of the problem are

$$y^{'} = 0 : u^{'} = 0, T^{'} = T_w, C^{'} = C_w, b_x^{'} = 0 \ \& \ y^{'} \to \infty : u^{'} = U, T^{'} = T_\infty, C^{'} = C_\infty, b_x^{'} = b_0 \qquad (2.7)$$

Integrating equation (2.1) and (2.2), subject to the boundary conditions, we get

$v^{'} = -V_0$ & $b_y^{'} = B_0$

Introducing the following non-dimensional quantities

$$u = \frac{u^{'}}{U}, y = \frac{y^{'}V_0}{\nu}, \theta = \frac{T^{'} - T_\infty}{T_w - T_\infty}, \phi = \frac{C^{'} - C_\infty}{C_w - C_\infty}, Gr = \frac{\nu g \beta (T_w - T_\infty)}{U^{'} V_0^2}, Gm = \frac{\nu g \beta^* (T_w - T_\infty)}{U^{'} V_0^2}$$

$$E = \frac{U^{'2}}{C_p (T_w - T_\infty)}, Pr = \frac{\eta_0 C_p}{K}, Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2}{\rho V_0^2}, b_x = \frac{b_x^{'}}{b_0}, H = \frac{b_0}{B_0}, Pm = \frac{\nu}{\eta}, \lambda = \frac{V_0}{U}, k = \frac{k_0 V_0^2}{\rho \nu^2}$$

where u is dimensionless velocity, y is dimensionless displacement variable, θ is dimensionless temperature, ϕ is dimensionless concentration, Gr is the Grashoff number for heat transfer, Gm is the Grashoff number for mass transfer, E is Eckert number, Pr is Prandtl number, Sc is Schmidt number, M is Hartmann number, b_x is dimensionless magnetic induction along x-axis, H is the ratio of induced magnetic field to the applied magnetic field, Pm is magnetic Prandtl number, λ is the ratio of suction velocity to the free stream velocity and k is non-dimensional visco-elastic parameter.

Into the equations (2.3) to (2.6), we get

$$k\frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} + \frac{du}{dy} - Mu = -Gr\theta - Gm\phi - M + M\lambda H(b_x - 1) \qquad (2.8)$$

$$\frac{d^2\theta}{dy^2} + Pr\frac{d\theta}{dy} = -EPr\left(\frac{du}{dy}\right)^2 - kEPr\frac{du}{dy}\frac{d^2u}{dy^2} - MEPr\{(1-u) + \lambda H(1-b_x)\}^2 \qquad (2.9)$$

$$\frac{d^2\phi}{dy^2} + Sc\frac{d\phi}{dy} = 0 \qquad (2.10)$$

$$\lambda H b_x^{''} + Pmu^{'} + Pm\lambda H b_x^{'} = 0 \qquad (2.11)$$

The dimensionless boundary conditions are

$$y = 0 : u = 0, \theta = 1, \phi = 1, b_x = 0 \ \& \ y \to \infty : u = 1, \theta = 0, \phi = 0, b_x = 1 \qquad (2.12)$$

3. METHOD OF SOLUTION:

Solving the equation (2.10) by using the boundary conditions (2.12), we get

$$\varphi = e^{-Scy} \qquad (3.1)$$

Since the Eckert number is small for all incompressible fluids, so the velocity u, temperature θ and induced magnetic field b_x in the neighbourhood of the plate are assumed to be of the form

$$u = u_0 + Eu_1 + o(E^2), \theta = \theta_0 + E\theta_1 + o(E^2), b_x = b_{x_0} + Eb_{x_1} + o(E^2) \qquad (3.2)$$

Substituting equations (3.2) into the equations (2.8), (2.9) and (2.11) and equating the like powers of the perturbation parameter E, we get

Zeroth-order equations:

$$ku_0^{'''} + u_0^{''} + u_0^{'} - Mu_0 = -Gr\theta_0 - Gme^{-Scy} - M + M\lambda H(b_{x_0} - 1) \qquad (3.3)$$

$$\theta_0^{''} + Pr\theta_0^{'} = 0 \qquad (3.4)$$

$$\lambda H b_{x0}^{''} + Pm\lambda H b_{x0}^{'} + Pmu_0^{'} = 0 \qquad (3.5)$$

First-order equations:

$$ku_1^{'''} + u_1^{''} + u_1^{'} - Mu_1 = -Gr\theta_1 + M\lambda H b_{x_1} \qquad (3.6)$$

$$\theta_1^{''} + Pr\theta_1^{'} = -Pru_0^{'2} - kPru_0^{'}u_0^{''} - MPr\{(1-u_0) + \lambda H(1-b_{x_0})\}^2 \qquad (3.7)$$

$$\lambda H b_{x1}^{''} + Pm\lambda H b_{x1}^{'} + Pmu_1^{'} = 0 \qquad (3.8)$$

The modified boundary conditions are

$$y = 0 : u_0 = u_1 = 0, \theta_0 = 1, \theta_1 = 0, b_{x_0} = b_{x_1} = 0$$

$y \to \infty: u_0 = 1, u_1 = 0, \theta_0 = \theta_1 = 0, b_{x_0} = 1, b_{x_1} = 0$ (3.9)

Solving the equation (3.4) subject to the boundary condition (3.9), we get

$\theta_0 = e^{-Pry}$ (3.10)

Combining (3.3) and (3.5), we get

$ku_0^v + (1 + kPm)u_0^{iv} + (1 + Pm)u_0^{'''} + (Pm - M)u_0^{''} = -Gr(\theta_0^{'} + Pm\theta_0^{'}) + GmSc(Pm - Sc)e^{-Scy}$ (3.11)

Combining (3.6) and (3.8), we get

$ku_1^v + (1 + kPm)u_1^{iv} + (1 + Pm)u_1^{'''} + (Pm - M)u_1^{''} = -Gr(\theta_1^{''} + Pm\theta_1^{'})$ (3.12)

To solve the equations (3.11), (3.12), we use another perturbation scheme by assuming α as a perturbation parameter.

$u_0 = u_{00} + ku_{01} + o(k^2), \quad u_1 = u_{10} + ku_{11} + o(k^2)$ (3.13)

Since $\alpha \ll 1$ due to small shear rate. Substituting (3.13) into the equations (3.11), (3.12) and (3.7) and equating the like terms of elastic-viscous parameter α, we get

$u_{00}^{iv} + (1 + Pm)u_{00}^{'''} + (Pm - M)u_{00}^{''} = L_1 e^{-Pry} + L_2 e^{-Scy}$ (3.14)

$u_{01}^{iv} + (1 + Pm)u_{01}^{'''} + (Pm - M)u_{01}^{''} = -u_{00}^{v} - Pmu_{00}^{iv}$ (3.15)

$u_{10}^{iv} + (1 + Pm)u_{10}^{'''} + (Pm - M)u_{10}^{''} = -Gr(\theta_{10}^{''} + Pm\theta_{10}^{'})$ (3.16)

$u_{11}^{iv} + (1 + Pm)u_{11}^{'''} + (Pm - M)u_{11}^{''} = -u_{10}^{v} - Pmu_{10}^{iv} - Gr(\theta_{11}^{''} + Pm\theta_{11}^{'})$ (3.17)

where, $\theta_1 = \theta_{10} + \alpha\theta_{11}$

Relevant boundary conditions of the problem are

$y = 0: u_{00} = u_{01} = u_{10} = u_{11}$ & $y \to \infty: u_{00} = 1, u_{01} = u_{10} = u_{11} = 0$ (3.18)

The equations (3.14) to (3.17) are solved by using the boundary conditions (3.18) and the equation (3.7) is solved by using boundary condition (3.9). The solutions and constants are not presented here for the sake of brevity.

4. RESULTS AND DISCUSSION:

The velocity profile is given by

$u = (u_{01} + ku_{01}) + Ec(u_{10} + ku_{11})$ (4.1)

The shearing stress at the plate is given by

$$\sigma = \frac{\sigma_{xy}^{'}}{\rho V_0 U} = \frac{\partial u}{\partial y} + k\frac{\partial^2 u}{\partial y^2} \text{ at } y = 0 \quad (4.2)$$

The co-efficient of heat transfer in terms of Nusselt number at the plate is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0} \quad (4.3)$$

The co-efficient of mass transfer in terms of Sherwood number at the plate is given by

$$Sh = -\left(\frac{d\phi}{dy}\right)_{y=0} \quad (4.4)$$

The current density distribution in the non-dimensional form at the plate is given by

$$J = \frac{J^{'}}{\sigma U B_0} \quad (4.5)$$

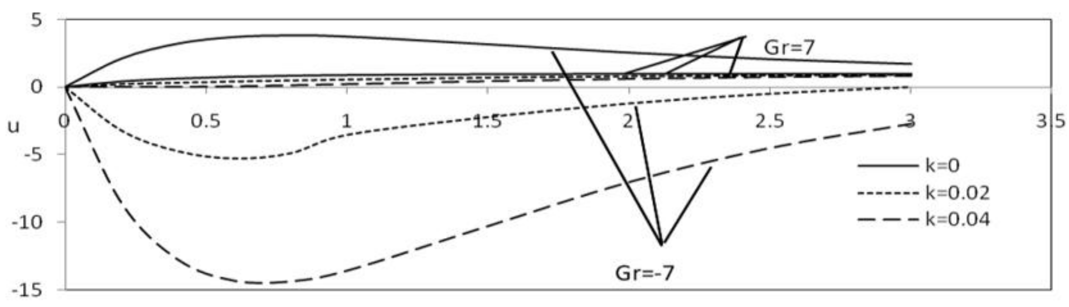

Figure 2: Velocity profile against y for Gm=3, Pr=10, Sc=6, Pm=2, M=0.2, H=0.5, λ=0.5, E=0.01.

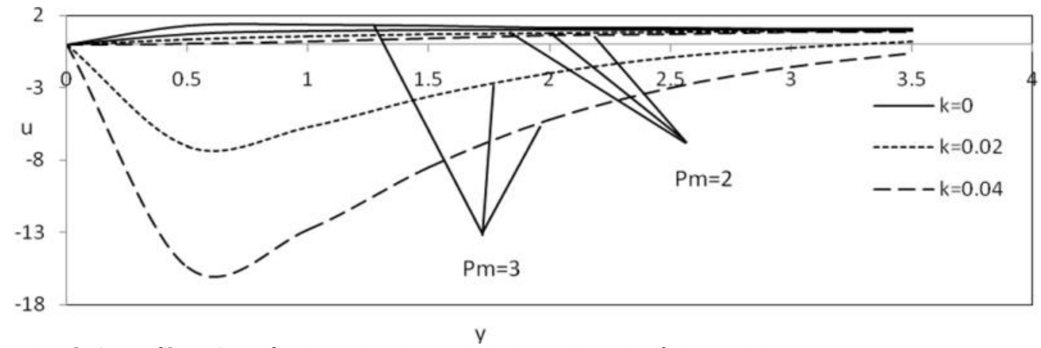

Figure 3: Velocity profile against y for Gr=7, Gm=3, Pr=10, Sc=6, M=0.2, H=0.5, λ=0.5, E=0.01.

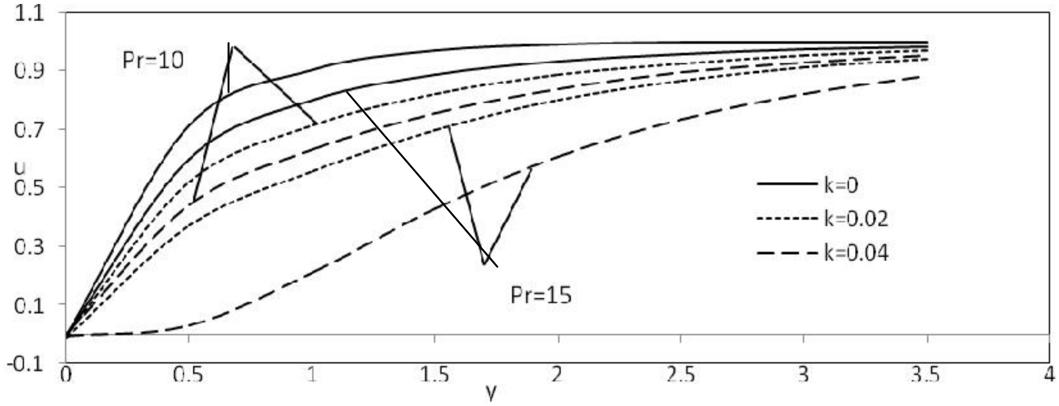

Figure 4: Velocity profile against y for Gr=7, Gm=3, Pm=2, Sc=6, M=0.2, H=0.5, λ=0.5, E=0.01.

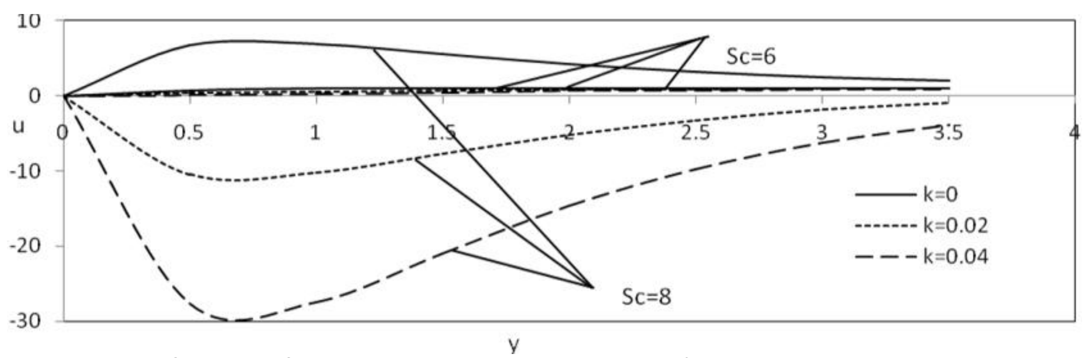

Figure 5: Velocity profile against y for Gr=7, Gm=3, Pm=2, Pr=10, M=0.2, H=0.5, λ=0.5, E=0.01.

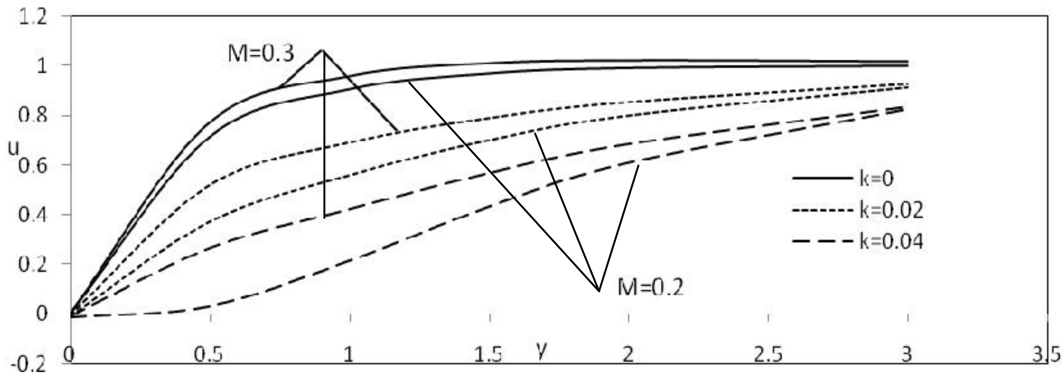

Figure 6: Velocity profile against y for Gr=7, Gm=3, Pm=2, Pr=10, Sc=6, H=0.5, λ=0.5, E=0.01.

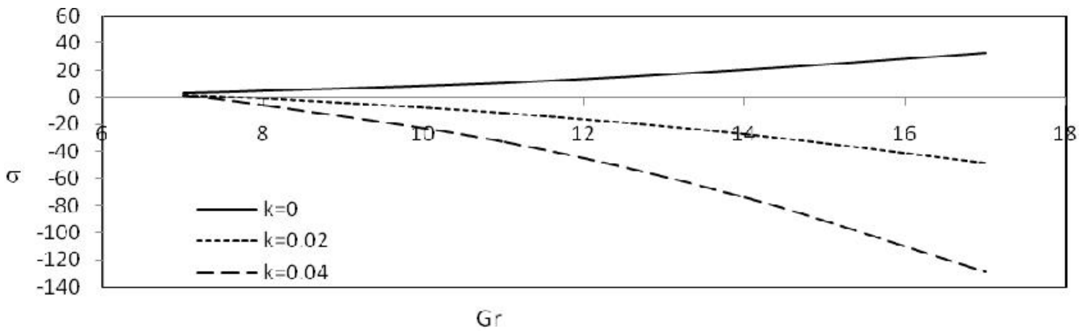

Figure 7: Shearing stress against Gr for Gm=3, Pr=10, Sc=6, Pm=2, M=0.2, H=0.5, λ=0.5, E=0.01.

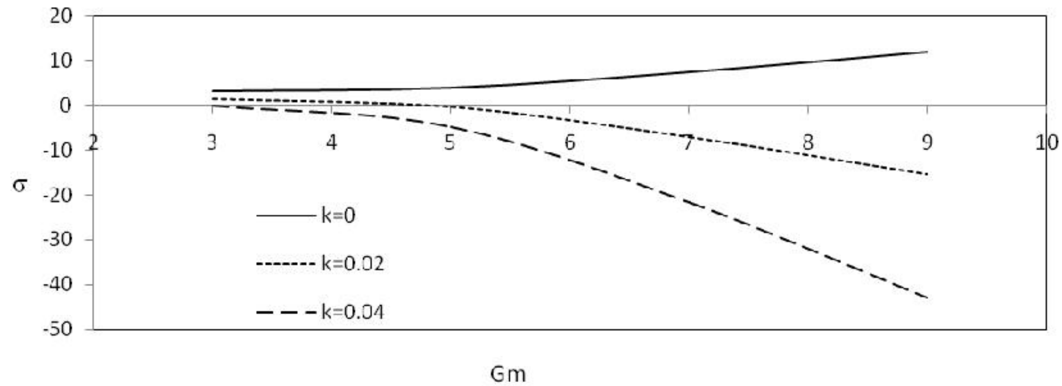

Figure 8: Shearing stress against Gm for Gr=7, Pr=10, Sc=6, Pm=2, M=0.2, H=0.5, λ=0.5, E=0.01.

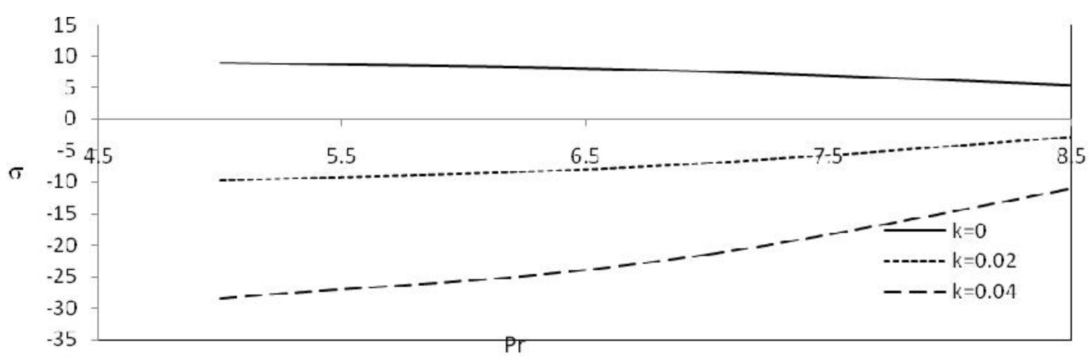

Figure 9: Shearing stress against Pr for Gr=7, Gm=3, Sc=6, Pm=2, M=0.2, H=0.5, λ=0.5, E=0.01.

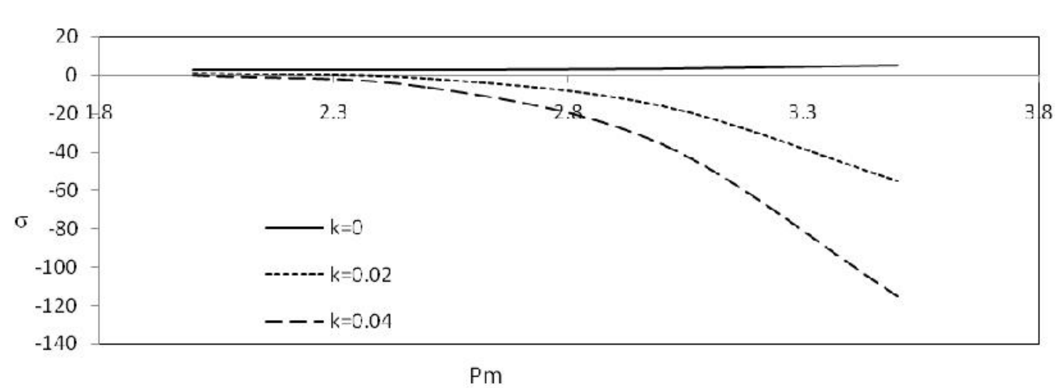

Figure 10: Shearing stress against Pm for Gr=7, Pr=10, Sc=6, Gm=3, M=0.2, H=0.5, λ=0.5, E=0.01.

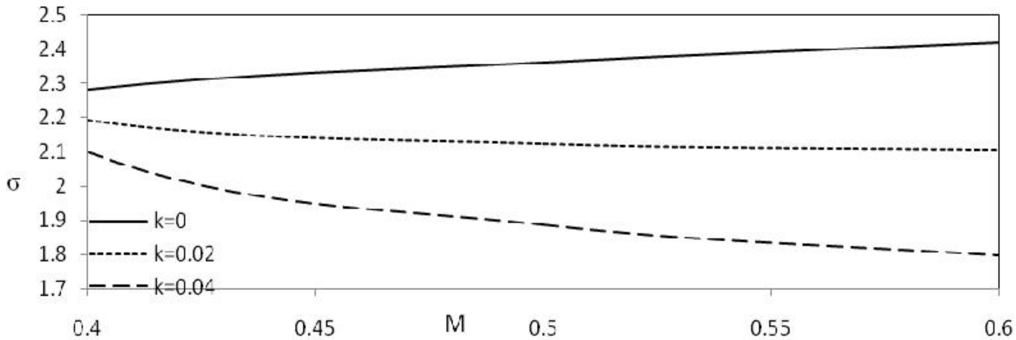

Figure 11: Shearing stress against M for Gr=7, Pr=10, Sc=6, Gm=3, Pm=2, H=0.5, λ=0.5, E=0.01.

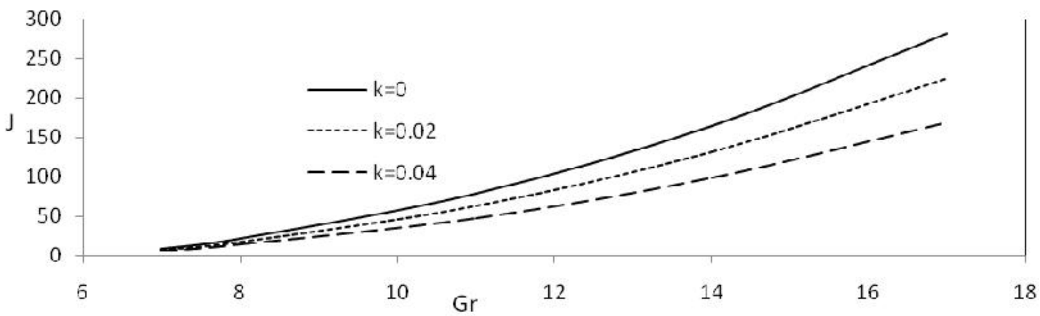

Figure 12: Current density against Gr for Gm=3, Pr=10, Sc=6, Pm=2, M=0.2, H=0.5, λ=0.5, E=0.01.

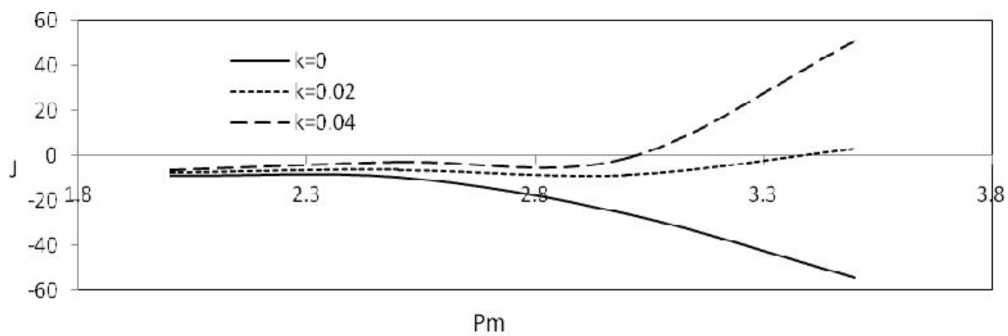

Figure 13: Current density against Pm for Gr=7, Gm=3, Pr=10, Sc=6, M=0.2, H=0.5, λ=0.5, E=0.01.

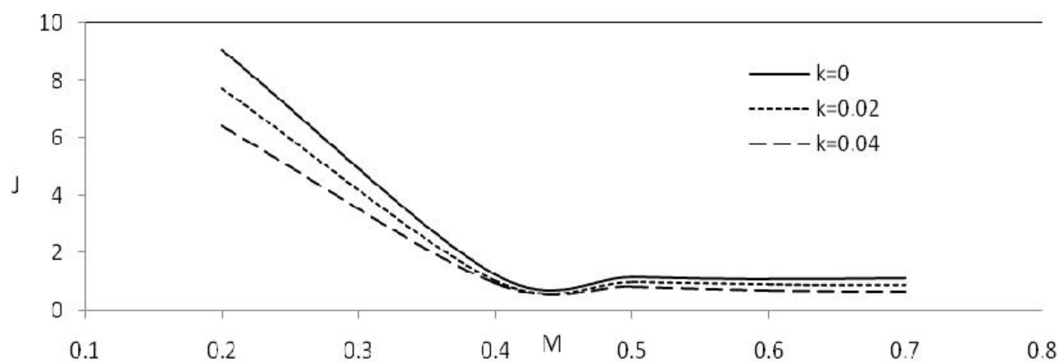

Figure 14: Current density against M for Gm=3, Pr=10, Sc=6, Pm=2, Gr=7, H=0.5, λ=0.5, E=0.01.

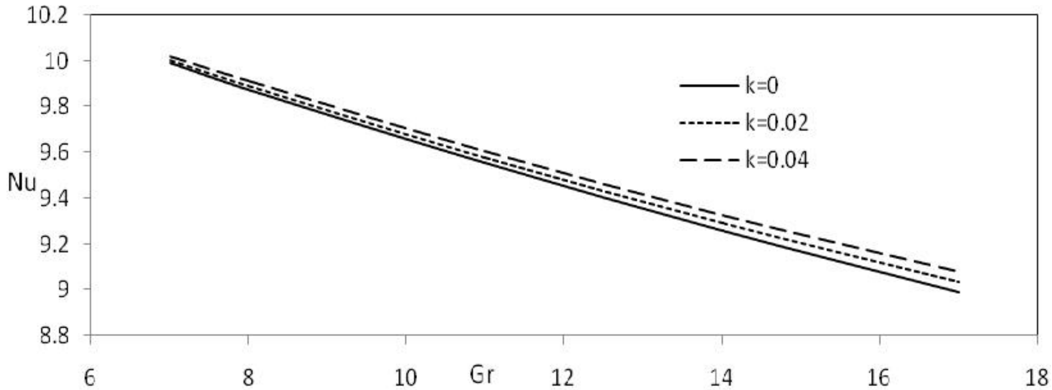

Figure 15: Nusselt number against Gr for Gm=3, Pr=10, Sc=6, Pm=2, M=0.2, H=0.5, λ=0.5, E=0.01.

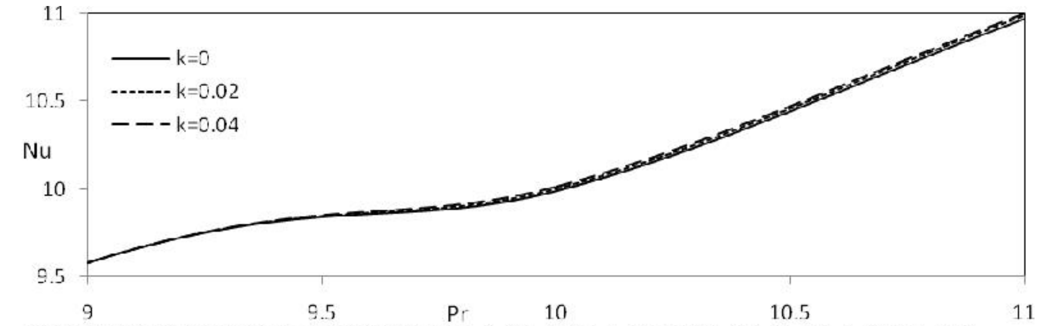

Figure 16: Nusselt number against Pr for Gm=3, Gr=7, Sc=6, Pm=2, M=0.2, H=0.5, λ=0.5, E=0.01.

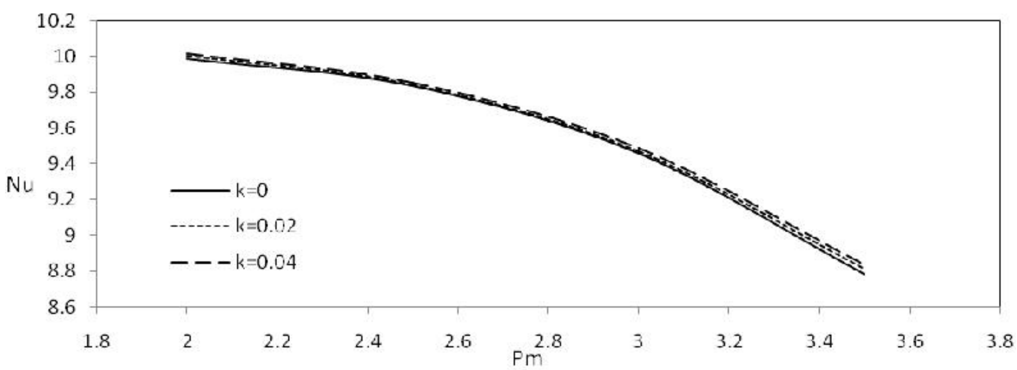

Figure 17: Nusselt number against Pm for Gm=3, Pr=10, Sc=6, Gr=7, M=0.2, H=0.5, λ=0.5, E=0.01.

The purpose of the present study is to bring out the effects of elastico-viscous parameter on mixed convective MHD flow of a visco-elastic fluid past an infinite vertical porous surface by imposing the effect of induced magnetic field in the governing fluid flow system. The elastico-viscous effect is exhibited through the non-dimensional parameter k. The non zero values of the parameter k characterize the visco-elastic fluid and k=0 represents the Newtonian fluid flow phenomenon. We have computed the result for $Gr < 0$ corresponds to an externally heated plate and $Gr > 0$ corresponds to an externally cooled plate. The species concentration of the visco-elastic fluid is assumed to be of smaller order. $M < 1$characterizes weak magneto-hydrodynamic flow system. Figures 2 to 6 represent the pattern of velocity profile u against the distance y for various values of other flow parameters. The graphs show that the velocity profile boosts up considerably in the neighbourhood of the plate and then it starts to converge to free stream velocity for both Newtonian and visco-elastic fluid but when the fluid flow is experienced through an externally heated plate, a download 'hill' is observed for visco-elastic fluids (Figure 2). The elasticity factor of Walters liquid (Model $B^{/}$) diminishes the speed of the fluid in comparison with a Newtonian fluid. Grashoff number studies the behaviour of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in both heat and mass transfer mechanisms. *Gr* characterizes the free convection parameter for heat transfer and *Gm* characterizes the free convection parameter for mass transfer. Figure 1 represent the nature of velocity profile for cooled plate ($Gr=7$) and heated plate ($Gr=-7$). The impact of elasticity factor present in the Walters liquid is seen more during the flow past a heated plate in comparison with the flow past a cooled plate. The Newtonian fluid will enhance its speed when it passes through a heated plate but the visco-elastic fluids ($k=0.02$, 0.04) experience a downward trend in speed. The magnetic Prandtl number (Pm) signifies the relative importance of momentum diffusion and magnetic diffusion as it is defined as the ratio of momentum diffusivity to magnetic diffusivity. In our study, Pm is considered to be greater than 1. The effects of *Pm* on Newtonian and non-Newtonian fluids are shown in figure 3. It shows that the rising value of *Pm* accelerates the fluid flow but due to the presence of elasticity, the visco-elastic fluid flow experiences a declined trend during the enhancement of magnetic Prandtl number. Prandtl number (*Pr*) plays an important role in heat transfer problems as it studies the simultaneous behaviour of momentum diffusion and thermal diffusion. The consequences of *Pr* are shown in figure 4. The increasing value of Prandtl number enlarges the viscosity of the non-Newtonian fluid and the fluid will experience a diminishing behaviour in the speed but the visco-elastic fluids experience an opposite trend in the speed due to the presence of the elasticity factor. Prandtl number is analogous to the Schmidt number in convection mass transfer. Schmidt number signifies the ratio of momentum diffusivity to concentration diffusivity. The role of Schmidt number on the velocity profile is illustrated in figure 5. Increasing value of Schmidt number increases the velocity of the Newtonian fluid but the velocity of the visco-elastic fluid subdues with the enhancement of Schmidt number. The effect of Hartmann number is analysed on figure 6. In a weak magneto-hydrodynamic flow, the Lorentz force which is produced due to the applied magnetic field will not act as drag force, rather it will behave like a body force of both visco-elastic fluid and Newtonian fluid systems. Thus it will accelerate the fluid flows of both systems. The maximum effect of applied magnetic field of visco-elastic fluid and Newtonian fluid is seen in the neighbourhood of the plate.

Figures 7 to 11, depict the nature of viscous drag formed by the fluids at the plate. The elasticity factor present in the visco-elastic fluid subdues the shearing stress at the plate in comparison with Newtonian fluid. It is also noticed that the shearing stress formed by the visco-elastic fluid flow is negative, which interprets that the viscous drag experiences a reverse direction. Figure 7 & 8, characterize the variations of shearing stress against *Gr* and *Gm*. The increasing values of *Gr* interpret the degree of coolness of the plate from externally. The increasing values of free convection parameter for both heat and mass transfer lessen the shearing stress formed by visco-elastic fluid along with the increasing values of visco-elastic parameter but it shows reverse effect during the Newtonian fluid flow mechanism. The effects of Prandtl number (*Pr*) and magnetic Prandtl number (*Pm*) are analyzed in figures 9 and 10 respectively. The positive values of Prandtl number signify the dominant effect of viscosity. The growth *Pr* enhances the viscous drag formed by visco-elastic fluid at the plate but a decreasing pattern is noticed during Newtonian fluid. An opposite behaviour is experienced during the growing behaviour of magnetic Prandtl number (*Pm*). The intensity of transverse magnetic field is shown in figure 11. Hartmann number subdues the shearing stress of visco-elastic fluid in comparison with the Newtonian fluid.

Current density measures the density of flow of conserved charge. Figures 12 to 14 illustrate the variations of current density for various values of flow parameters of both Newtonian fluid and visco-elastic fluid. Figure 12 discusses the behaviour of Grashof number on current density for various fluid flow mechanisms. The rising nature of free convection from an externally heated plate enhances the current density of Newtonian as well as visco-elastic fluid and also it is notified that the elasticity factor present in the visco-elastic fluid flow mechanism diminishes the current density in comparison with simple Newtonian fluid. Effect of magnetic Prandtl number on the current density is analysed on figure 13. Higher values of *Pm* indicate that the viscosity is dominant over magnetic diffusivity. Enlargement of *Pm* declines the current density in case of Newtonian fluid but a reverse nature is occurred during the complex visco-elastic fluid flow system. The effect of Hartmann number on the current density is observed in figure 14. A steep declination is experienced in weak hydromagnetic flow for various values of visco-elastic parameter but when the *M* increases, the current density decreases steadily.

Nusselt number studies the rate of heat transfer through the fluid system. Here we have investigated the nature of Nusselt number on the flat plate. The graphical presentations of rate of heat transfer are given in figures 15 to 17. Visco-elasticity factor present in the complex fluid flow system grows the rate of heat transfer in comparison with the Newtonian fluid flow system. Diminishing impacts of free convection parameter on the Nusselt number are glimpsed on visco-elastic fluid flow and Newtonian fluid flow concepts (Figure 15). Figure 16 characterizes the pattern of rate of heat transfer against *Pr*. It shows that increasing values of *Pr* modify the rate of heat transfer of Newtonian as well as visco-elastic fluids. A reverse nature is witnessed during the rising concept of magnetic Prandtl number (*Pm*) in figure 17.

The rate of mass transfer is not significantly affected by visco-elastic parameter.

5. CONCLUSIONS:

The steady two dimensional hydro-magnetic mixed convective flow of an electrically conducting Walters liquid (Model $B^{/}$) past an infinite vertical plate in presence of induced magnetic field has been investigated. The effect of simultaneous heat and mass transfer are also studied in this paper. Some of the important points are enlisted as below:

- The velocity profile shows an enhancement trend in the neighbourhood of the plate.
- A downward hill is noticed in the velocity profile of visco-elastic fluid for the flow past a heated plate.
- The visco-elasticity factor decelerates the speed of fluid flow in comparison with the Newtonian fluid.
- A back flow is noticed in visco-elastic fluid flow system during the rise of magnetic Prandtl number.
- The shearing stress formed at the plate is subdued with the growing trend of visco-elastic parameter.
- The increasing values of *Gr* and *Gm* lessen the shearing stress formed by visco-elastic fluid.
- Current density distribution declines with the increase of visco-elastic parameter.
- The rate of mass transfer is not significantly affected by visco-elastic parameter.

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