Stratified Visco-Elastic Fluid Flow in a Slip Flow Regime Past a Porous Surface in Presence of Heat Source/Sink

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Abstract- A steady stratified visco-elastic fluid flow past a porous plate in a slip flow regime has been investigated under the influence of heat source/ sink. The plate is subjected to a constant suction veloctiy. In this study, generalized boundary conditions for the slip flow regime at the plate are used. The mechanism of visco-elastic fluid flow contains both viscous and elastic responses and it is characterized by Walters Liquid (Model B´) for short relaxation memories. The governing equations are solved analytically by using perturbation technique. Velocity profile and temperature fields are presented graphically and shearing stress and Nusselt number (rate of heat transfer) are represnted in tabular form for various values of flow parameters involved in the solution with special emphasis given on the effects of visco-elasticity and stratification.

Index Terms- Heat source, Nusselt number, perturbation technique, shearing stress, slip flow regime, stratification, Walters liquid.

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1 INTRODUCTION

The study of visco-elastic fluid flows has attracted the attention of many researchers because of its wide applications in various branches of science and technology. The complex stress-strain relationship of visco-elastic fluid flow mechanism is used in geophysics, chemical engineering, petroleum engineering, hydrology, soil-physics, bio-physics, paper and pulp technology. A theoretical model for visco-elastic fluids has been proposed by Walters [1]. He [2] reported that the mixture of polymethyl methacrylate and pyridine at 25^o C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml fits very nearly to this model. Polymers are used in the manufacture of space crafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastic, engineering equipments, contact lens etc. Walters liquid (Model $B^/$) forms the basis for the manufacture of many such important and useful products.

The stability of laminar flow of a dusty gas by neglecting the volume fraction of dust particles has been studied by Saffman [3]. Nayfeh [4] has formulated the equations of motion of the fluid particles in presence of volume fraction of dust particles. Michael and Miller [5] have investigated the behaviour of plane parallel flow of a dusty gas. Gupta and Gupta [6] have studied the flow of a dusty gas through a channel with time varying pressure gradient. The unsteady flow of a dusty fluid through a rectangular channel with time dependent pressure gradient has been analysed by Singh [7]. The unsteady two dimensional flow of an electrically conducting dusty viscous fluid through a channel in presence of transverse magnetic field has been discussed by Singh and Ram [8]. Prasad and Ramacharyulu [9] have investigated the unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient. Alle et al. [10] have studied the unsteady flow of a dusty fluid through an inclined channel under the influence of pulsatile pressure gradient. An unsteady MHD flow of dusty electrically conducting viscous liquid between two parallel plates by the use of Laplace transform techniques has been investigated by Kalita [11].

Rayleigh [12] has studied the character of an incompressible heavy fluid of variable density. Channabasappa and Rangana [13] have discussed the nature of viscous stratified fluid flow past a permeable bed. The effects of stratification and slip velocity on the fluid of variable viscosity over a porous bed have been analysed by Gupta and Sharma [14]. MHD flow and heat transfer of a dusty visco-elastic stratified fluid down an inclined channel in porous medium under variable viscosity has been investigated by Chakraborty [15]. Agarwal et al. [16] have demonstrated the effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate. The heat transfer in MHD flow of dusty viscoelastic (Walters' liquid model-B´) stratified fluid in porous medium under variable viscosity has been investigated by Prakash et al. [17]. The thermal diffusion effect on MHD free convection flow of stratified viscous fluid with heat and mass transfer has been studied by Sharma et al. [18].

In this paper, a two dimensional flow of visco-elastic (Walters liquid Model B/) stratified fluid past a porous plate has been analysed under the influence of slip flow regime.

2 MATHEMATICAL FORMULATION

The steady two-dimensional stratified visco-elastic fluid characterized by Walters liquid (Model B´) in a slip flow regime past a porous surface in presence of heat source/sink has been investigated. Let $x^{\prime}$ axis be taken along the plate in the horizontal direction and $y^{\prime}$ axis be taken normal to it. Let $u^{\prime}$ and $v^{\prime}$ be the velocity components along $x^{\prime}$ axis and $y^{\prime}$ axis respectively. The problem is governed by the following system of equations:

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$$v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\eta\frac{\partial u'}{\partial y'}\right) - \frac{1}{\rho}\frac{\partial}{\partial y}\left[\alpha\left(v'\frac{\partial^2 u'}{\partial y'^2}\right)\right] - \frac{\eta}{\rho K}u' \quad (2.1)$$

$$v'\frac{\partial T'}{\partial y'} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k\frac{\partial T'}{\partial y'}\right) + \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial u'}{\partial y'}\right)^2 - \alpha\left(v'\frac{\partial^2 u'}{\partial y'^2}\frac{\partial u'}{\partial y'}\right)\right] - S(T' - T_\infty) \quad (2.2)$$

The boundary conditions of the problem are

$$u' = U_0 + L_1\left(\eta\frac{\partial u'}{\partial y'} - \alpha v'\frac{\partial^2 u'}{\partial y'^2}\right), T' = T_w + L_2\left(\frac{\partial T'}{\partial y'}\right) \text{ at } y' = 0$$
$$u' = 0, T' = 0 \text{ as } y' \to \infty \quad (2.3)$$

For stratified fluid flow, we consider

$$\rho = \rho_0 e^{-\beta y'}, \qquad \eta = \eta_0 e^{-\beta y'},$$
$$k = k_0 e^{-\beta y'}, \qquad \alpha = \alpha_0 e^{-\beta y'} \quad (2.4)$$

Here, β is the stratification parameter and for stable fluid flow, $(\beta > 0)$, y' be the displacement variable, u' be the velocity of the fluid in the direction of motion, V_0 be the velocity of the suction and its negative sign indicates that the suction is towards the plate, T' be the temperature of the fluid motion, K be the permeability, ρ_0 be the density of the fluid at the plate, η_0 be the limiting viscosity in small rate, k_0 be the thermal conductivity of the fluid, α_0 be the visco-elastic parameter and S be the strength of source/ sink.

3 METHOD OF SOLUTION

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$y = \frac{y'V_0}{\nu_0}, u = \frac{u'}{U_0}, \theta = \frac{T' - T_\infty}{T_w - T_\infty}, a = \frac{\alpha_0 V_0^2}{\rho_0 \nu_0^2}, b = \frac{\beta \nu_0}{V_0},$$
$$K_p = \frac{V_0^2}{\nu_0^2}K, Pr = \frac{\eta_0 C_p}{k_0}, Ec = \frac{U_0^2}{C_p(T_w - T_\infty)}, H = \frac{S\nu}{V_0^2} \quad (3.1)$$

where, y, u & θ be the dimensionless displacement variable, velocity and temperature of fluid motion respectively, a, b, K_p, Pr, Ec & H represent the dimensionless visco-elastic parameter, stratification parameter, permeability, Prandtl number, Eckert number and strength of heat source/ sink respectively.

Using (3.1) into (2.1) to (2.2), the non-dimensional forms of governing equations of motion are as follows:

$$a\frac{d^3u}{dy^3} + (1 - ab)\frac{d^2u}{dy^2} + (1 - ab)\frac{du}{dy} - \frac{u}{K_p} = A \quad (3.2)$$

$$\frac{1}{Pr}\frac{d^2\theta}{dy^2} + \left(1 - \frac{b}{Pr}\right)\frac{d\theta}{dy} - H\theta = -Ec\left(\frac{du}{dy}\right)^2 - aEc\,\frac{du}{dy}\frac{d^2u}{dy^2} \quad (3.3)$$

The relevant boundary conditions in the non-dimensional form are

$$u = 1 + h_1 e^{-by}\left(\frac{du}{dy} + a\frac{d^2u}{dy^2}\right), \theta = 1 + h_2\left(\frac{\partial\theta}{\partial y}\right) \text{ at } y = 0$$
$$u = 0, \qquad \theta = 0 \text{ as } y \to \infty \quad (3.4)$$

where, $h_1 = L_1\rho_0 V_0$ & $h_2 = L_2\frac{V_0}{\nu_0}$ are slip parameters.

Due to insufficient number of boundary conditions, the solution of (3.2) in closed form is not possible, so to solve (3.2), we use perturbation technique. The velocity in the neighbourhood of the plate is assumed to be

$$u = u_0 + au_1 + o(a^2) \quad (3.5)$$

Using (3.5) into (3.2) and equating the like powers of a, we get

$$u_0'' + (1 - b)u_0' - \frac{u_0}{K_p} = A \quad (3.6)$$

$$u_0''' - bu_0'' + u_1'' + (1 - b)u_1' - \frac{u_1}{K_p} = 0 \quad (3.7)$$

The relevant boundary conditions are

$$(u_0 + au_1 + a^2u_2) = 1 + h_1 e^{-b_1 y}\left\{\frac{d}{dy} + a\frac{d^2}{dy^2}\right\}(u_0 + au_1 + a^2u_2) \text{ at } y = 0$$
$$(u_0 + au_1 + a^2u_2) = 0 \text{ as } y \to \infty \quad (3.8)$$

Solving this, we get

$$\left.\begin{aligned} u_0 &= C_1 e^{-A_1 y} - K_p A \\ u_1 &= C_3 e^{-A_1 y} + A_3 y e^{-A_1 y} \\ u &= C_1 e^{-A_1 y} - K_p A + a_1(C_3 e^{-A_1 y} + A_3 y e^{-A_1 y}) \end{aligned}\right\} \quad (3.9)$$

Then solving (3.3), we get

$$\theta = C_5 e^{-A_5 y} + A_{10}e^{-2A_1 y} + A_{11}ye^{-2A_1 y} + A_{12}y^2e^{-2A_1 y} \quad (3.10)$$

4 RESULTS AND DISCUSSIONS

After knowing the velocity profile and temperature field, now we determine the shearing stress and Nusselt number (rate of heat transfer) at the plate, and are obtained as

$$\sigma = \left[e^{-\beta y}\left(\frac{\partial u}{\partial y} + a\frac{\partial^2 u}{\partial y^2}\right)\right]_{y=0} \quad (4.1)$$

$$Nu = \left[-\frac{\partial\theta}{\partial y}\right]_{y=0} \quad (4.2)$$

The purpose of this study is to bring out the effects of visco-elastic and stratification parameter on the steady two-dimensional stratified visco-elastic fluid past a porous surface in a slip flow regime in presence of heat source/sink. The visco-elastic effect is exhibited through the non-dimensional parameter a and the stratification parameter by b.

Figures 1-2 depicts the velocity profile u against y for various values of visco-elastic and stratification parameters with the combination of other flow parameters involved in the solution. From figure 1 it is observed that the speed of the fluid enhances with the growth of visco-elastic parameter. Also it is noticed that the velocity profile accelerates with the rising values of stratification parameter. Figures 3-5 illustrates the temperature profiles for various values of of visco-elastic parameter, stratification parameters and the strength of heat source/sink with other flow parameters involved in the solution. Figures 3-4 reveal that the temperature of the fluid diminishes with the accelerated values of visco-elastic and stratification parameters. It is also observed from figure 5 that the temperature rises with the increasing values of strength of heat source/sink. Table 1 represents the shearing stress and the rate of heat transfer at the plate for various values of of visco-elastic parameter, stratification parameters and the strength of heat source/sink. It is observed that the shearing stress enhances with the accelerated values of stratification parameters (cases I,II and III). But the decay of strength of heat source/sink does not bring any significant change in the shearing stress (cases II, IV and V). It is also noticed the rate of heat transfer diminished with the growth of stratification parameter (cases I,II and III). The rate of heat transfer is not significantly affected with the reduce values of strength of heat source/sink.

5 CONCLUSIONS

The steady two-dimensional stratified visco-elastic fluid in a slip flow regime past a porous surface in presence of heat source/sink has been investigated for different values of values visco-elastic parameter, stratification parameters and the strength of heat source/sink with the combination of other flow parameters involved in the solution. In the analysis, the following conclusions are made:

- The speed of the fluid accelerates with the growth of visco-elastic and stratification parameter.
- The temperature profile diminishes with the increasing values of visco-elastic and stratification parameters.
- The temperature rises with the increasing values of strength of heat source/sink.
- The shearing stress enhances with the accelerated values of stratification parameters
- The decay of strength of heat source/sink does not bring any significant change in the shearing stress.
- The rate of heat transfer reduces with the growth of stratification parameter
- The rate of heat transfer is not significantly affected with the decay of strength of heat source/sink.

6 GRAPHS AND TABLES

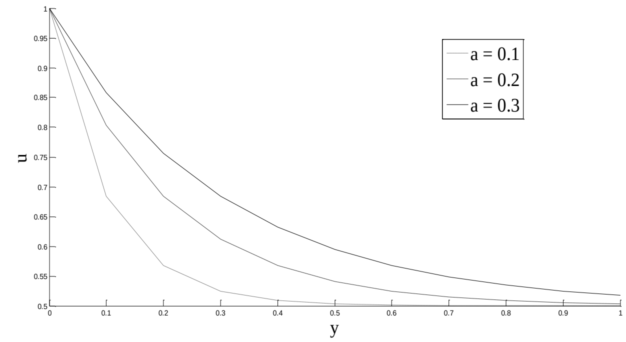

Figure 1: Velocity profile against y for K=2, Ec=0.01, b=0.5, h1=0.1, h2=1, A=-0.5, Pr=5, H=2

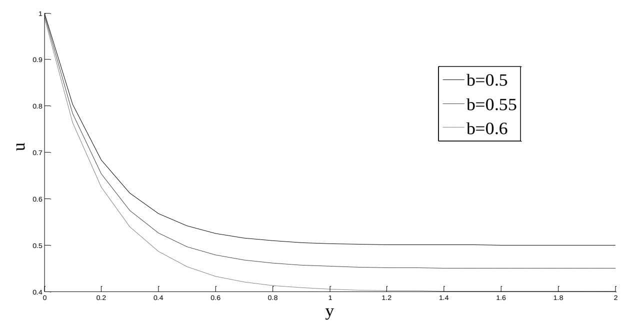

Figure 2: Velocity profile against y for K=2, Ec=0.01, a=0.2, h1=0.1, h2=1, A=-0.5, Pr=5, H=2.

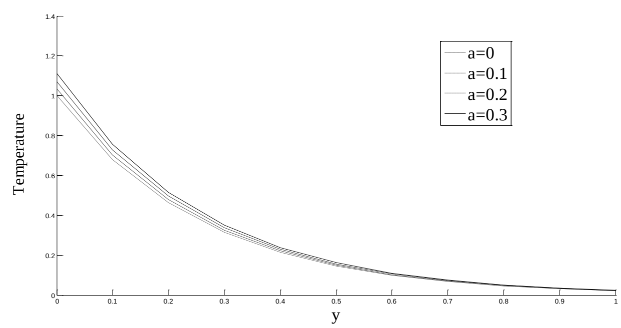

Figure 3: Temperature against y for K=2, Ec=0.01, b=0.5, h1=0.1, h2=1, A=-0.5, Pr=5, H=2.

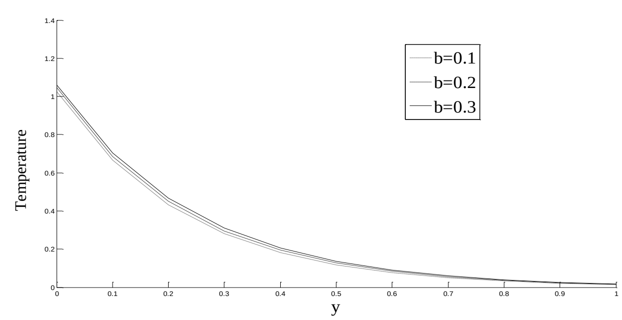

Figure 4: Temperature against y for K=2, Ec=0.01, a1=0.2, h1=0.1, h2=1, A=-0.5, Pr=5, H=2.

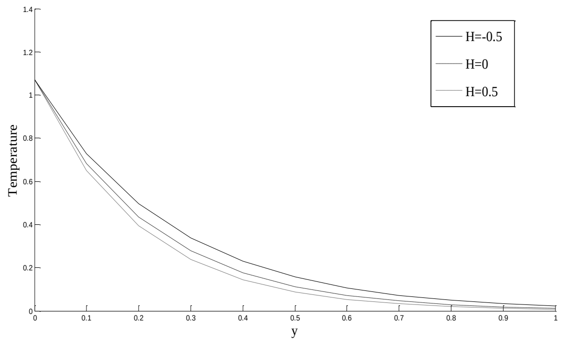

Figure 5: Temperature against y for K=2, Ec=0.01, a1=0.2, b1=0.5, h1=0.1, h2=1, A=-0.5, Pr=5.

Cases	b	H	Shearing Stress		Nusselt number	
			a= 0.1	a= 0.2	a= 0.1	a= 0.2
I	0.1	0.5	-0.0908	-0.1525	-0.0128	-0.0257
II	0.2	0.5	-0.0790	-0.1321	-0.0223	-0.0455
III	0.3	0.5	-0.0685	-0.1139	-0.0288	-0.0592
IV	0.2	0	-0.0790	-0.1321	-0.0223	-0.0453
V	0.2	-0.5	-0.0790	-0.1321	-0.0222	-0.0449

Table: Shearing stress and Nusselt number for K=2, Ec=0.01, h1=0.1, h2=1, A= - 0.5, Pr=5.

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