DIVERGENT CHANNEL FLOW OF A VISCO-ELASTIC HYDROMAGNETIC ELECTRICALLY CONDUCTING FLUID WITH SLIP VELOCITY

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Abstract : The two-dimensional boundary layer flow through a divergent channel of a visco-elastic electrically conducting fluid with slip velocity in presence of transverse magnetic field has been investigated analytically. Similarity solutions are obtained by considering a special form of magnetic field and the slip velocity. Expressions for velocity and approximate skin friction at the wall have been obtained and numerically worked out for different values of the flow parameters involved in the solution. The velocity and the approximate skin friction coefficient have been presented graphically to observe the visco-elastic effects for various values of the flow parameters across the boundary layer.

Key words and Phrases: Visco-elastic, divergent channel, MHD, Similarity solution, slip velocity.

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Introduction

The study of flow of a visco-elastic electrically conducting fluid through a divergent channel possesses not only a theoretical appeal but also model of many biological and engineering problems such as magnetohydrodynamic generators, nuclear reactors, industrial metal casting, plasma studies, blood flow problems, etc. The theory of such flow has many applications in aerospace, chemical, civil, environmental, mechanical and bio-mechanical engineering and also in understanding the flow of rivers and canals.

The divergent flow problem between two non-parallel planes has been analyzed by Jeffery [1]by reducing the problem to an elliptic integral equation. Srivastava [2] has extended this problem to an electrically conducting fluid in the presence of transverse magnetic field. He has found that with the application of magnetic field it is possible to have purely divergent flow without any secondary flow for greater angle between the two planes. The solution of two-dimensional incompressible laminar flow in a divergent channel with impermeable wall has been investigated by Rosenhead [3]. Terril[4] has studied the slow laminar flow in a converging or diverging channel with suction at one wall and blowing at the other wall. Hamel [5] has analyzed the preceding problem of calculating all three dimensional flows whose stream-lines are identical with those of potential flow.

The numerical calculations of Jeffery-Hamel flows between non-parallel plane walls have been performed byMillsaps andPohlhausen[6]. The two-dimensional laminar boundary layer flow of an incompressible, viscous, non uniform stream past solid obstacles has been analyzed by Falkner and Skan [7]. Phukan [8] has studied the hydromagnetic divergent channel flow of a Newtonian electrically conducting fluid. The two-dimensional laminar MHD boundary layer flow past a wedge with slip velocity has been studied by Sanyal and

Adhikari[9]. Choudhury and Dey [10] has investigated the hydromagnetic convergent channel flow of a visco-elastic electrically conducting fluid with slip velocity.

The aim of the present work is to study the two-dimensional magnetohydrodynamic boundary layer flow with slip velocity through a divergent channel of an electrically conducting visco-elastic fluid characterized by Walters liquid (Model B') in presence of transverse magnetic field. The effects of the visco-elastic fluid across the boundary layer on the dimensionless velocity component and skin friction coefficient have been shown graphically with the combination of other flow parameters involved in the solution.

The constitutive equation for Walters liquid (Model B') is

$$\sigma^{ik} = -pg_{ik} + 2\eta_0 e^{ik} - 2k_0 e'^{ik} \tag{1.1}$$

whereσ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate systemx^i, v^i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk} \tag{1.2}$$

It is the convected derivative of the deformation rate tensore^{ik} defined by

$$2e^{ik} = v^i_{,k} + v^k_{,i} \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \text{and} k_0 = \int_0^\infty \tau N(\tau)d\tau, \tag{1.4}$$

$N(\tau)$being the relaxation spectrum as introduced by Walters [11-12]. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau)d\tau, \qquad n \geq 2 \tag{1.5}$$

have been neglected.

Mathematical Formulation

The basic equations for steady two-dimensional boundary layer flow of Walters liquid (Model B') in the presence of a magnetic field B(x) are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \mathrm{v}\frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2} - \frac{\mathrm{k}_0}{\rho}\left[\mathrm{u}\frac{\partial^3 \mathrm{u}}{\partial x\,\partial \mathrm{y}^2} + \mathrm{v}\frac{\partial^3 \mathrm{u}}{\partial \mathrm{y}^3} - \frac{\partial u}{\partial y}\frac{\partial^2 \mathrm{u}}{\partial x\,\partial \mathrm{y}} - \frac{\partial v}{\partial y}\frac{\partial^2 \mathrm{u}}{\partial \mathrm{u}^2}\right] + \frac{\sigma \mathrm{B}^2(\mathrm{x})}{\rho}(-\mathrm{u}) \tag{2.2}$$

subject to the boundary conditions

$$\mathrm{y} = 0 : \mathrm{u} = \mathrm{U}_0(\mathrm{x}), \qquad \mathrm{v} = 0$$
$$y \to \infty := U(x) \tag{2.3}$$

where x-axis coincides with the wall of the divergent channel and y-axis perpendicular to it. U is the velocity component outside the boundary layer, u and v are the flow velocities in the direction of x and y respectively, ρ the fluid density, ν the kinematic viscosity, σ the electrical conductivity of the fluid and k_0 the visco- elastic parameter.

In equation (2.2), the secondary effects of magnetic induction are ignored i.e. the induced magnetic field is negligible as it is small in comparison to the applied magnetic field. Furthermore, we assume that the external electric field is zero and the electric field due to polarization of charges is also negligible.

As in Sinha and Choudhury [13], the potential flow near the sources is taken to be

$$U(x) = \frac{u_1}{x} \tag{2.4}$$

with$u_1 > 0$represents two dimensional divergent flow and leads to similarity solution.

To obtain similarity solutions, we assume

$$\mathrm{U_0(x)} = cU(x) \tag{2.5}$$

We introduce the following change of variables (Schlichting [14])

$$\eta(\mathrm{x,y}) = \mathrm{y}\sqrt{\frac{\mathrm{U(x)}}{\mathrm{\nu x}}} = \frac{\mathrm{y}}{\mathrm{x}}\sqrt{\frac{\mathrm{u_1}}{\mathrm{x}}} \tag{2.6}$$

and the stream function

$$\psi(x,y) = \sqrt{\mathrm{U(x)\nu x}}\,F(\eta) = \sqrt{\nu u_1}F(\eta) \tag{2.7}$$

Then, we obtain the velocity component as

$$u = \frac{\partial\psi}{\partial y} = \mathrm{U(x)}F'(\eta)$$
$$v = -\frac{\partial\psi}{\partial y} = \sqrt{\nu u_1}F'(\eta) \tag{2.8}$$

The equation of continuity (2.1) is identically satisfied for the velocity component (2.8).

Similarity solution exists if the magnetic field B(x) has the special form (Chiam [15])

$$B(x) = \frac{B_1}{x} \tag{2.9}$$

Using the equation (7.2.4) to (7.2.8), the equation (7.2.2) become

$$F''' + F'^2 + k_1\left[4F'F''' - 2F''^2\right] + M(1 - F') - 1 = 0 \tag{2.10}$$

Here prime denotes the differentiation with respect toη. $k_1 = \frac{k_0}{\rho}$and M denote the modified non-Newtonian and hydromagnetic parameters respectively.

The corresponding boundary conditions are

$$F'(0) = \mathrm{c}, \qquad F'(\infty) = 1\,, \qquad F''(\infty) = 0 \tag{2.11}$$

Method of solution

We first assume

$$z = \sqrt{\mathrm{M}}\,\eta\,, f(\mathrm{z}) = \sqrt{\mathrm{M}}\,F(\eta) \tag{3.1}$$

which implies

$$f^{'}(z) = F^{'}(\eta), f^{''}(z) = \frac{1}{\sqrt{M}}F^{''}(\eta), f^{'''}(z) = \frac{1}{\mathrm{M}}\mathrm{F}^{'''}(\eta) \tag{3.2}$$

Using the equation (3.2) in the equation (2.10), we get the following differential equation

$$f^{'''}(z) + k_1\left[4f^{'}(z)f^{'''}(z) - 2f^{''2}(z)\right] + \left(1 - f^{'}(z)\right) = \varepsilon\left(1 - f^{'2}(z)\right) \tag{3.3}$$

where$\varepsilon = \frac{1}{M}$

The corresponding boundary conditions are

$$f^{'}(0) = \mathrm{c},\ f^{'}(\infty) = 1\,, \qquad f^{''}(\infty) = 0 \tag{3.4}$$

The unknown function f(z) is expanded in terms of powers of the small parameterε as follows:

$$f(\mathrm{z}) = \mathrm{f}_0(\mathrm{z}) + \varepsilon \mathrm{f}_1(\mathrm{z}) + \varepsilon^2\mathrm{f}_2(\mathrm{z}) + \cdots \tag{3.5}$$

substituting the equation (3.5) in the equation (3.3) and equating the like powers of ε, we get

$$\mathrm{f}_0^{'''} + k_1\left[4f_0^{'}f_0^{'''} - 2\mathrm{f}_0^{''2}\right] + \left(1 - f_0^{'}\right) = 0 \tag{3.6}$$

$$\mathrm{f}_1^{'''} + 4k_1\left[4f_0^{'}f_1^{'''} + f_1^{'}f_0^{'''} - f_0^{''}f_1^{''}\right] - \mathrm{f}_1^{'} = 1 - \mathrm{f}_0^{'2} \tag{3.7}$$

The relevant boundary conditions are:

$$\mathrm{f}_0^{'}(0) = \mathrm{c},\ \ \mathrm{f}_0^{'}(\infty) = 1, \mathrm{f}_0^{''}(\infty) = 1$$

$$\mathrm{f}_1^{'}(0) = 0, \mathrm{f}_1^{'}(\infty) = 0\,, \mathrm{f}_0^{''}(\infty) = 0 \tag{3.8}$$

Again, in order to solve equations (3.6) and (3.7), we consider very small values ofk_1, so that f_0 and f_1 can be expressed as

$$\mathrm{f}_0 = \mathrm{f}_{00}(\mathrm{z}) + k_1\mathrm{f}_{01}(\mathrm{z}) + 0\left({k_1}^2\right)$$

$$\mathrm{f}_1 = \mathrm{f}_{10}(\mathrm{z}) + k_1\mathrm{f}_{11}(\mathrm{z}) + 0\left({k_1}^2\right) \tag{3.9}$$

Now substituting (7.3.9) into the equations (7.3.6) and (7.3.7), we get the following sets of ordinary differential equations

$$\mathrm{f}_{00}^{'''} - \mathrm{f}_{00}^{'} = -1 \tag{3.10}$$

$$\mathrm{f}_{01}^{'''} - \mathrm{f}_{01}^{'} = -4\mathrm{f}_{00}^{'}\mathrm{f}_{00}^{'''} + 2\mathrm{f}_{00}^{''2}(3.11)$$

$$\mathrm{f}_{10}^{'''} - \mathrm{f}_{10}^{'} = \mathrm{f}_{00}^{'2} - 1 \tag{3.12}$$

$$\mathrm{f}_{11}^{'''} - \mathrm{f}_{11}^{'} = 4\left[\mathrm{f}_{00}^{''}\mathrm{f}_{10}^{''} - \mathrm{f}_{00}^{'}\mathrm{f}_{10}^{''} - \mathrm{f}_{10}^{'}\mathrm{f}_{10}^{'''}\right] - 2\mathrm{f}_{00}^{'}\mathrm{f}_{01}^{'} \tag{3.13}$$

The appropriate boundary conditions are:

$$\mathrm{f}_{00}^{'}(0) = \mathrm{c}, \mathrm{f}_{00}^{'}(\infty) = 1, \mathrm{f}_0^{''}(\infty) = 0$$

$$\mathrm{f}_{01}^{'}(0) = 0, \mathrm{f}_{01}^{'}(\infty) = 0, \mathrm{f}_0^{''}(\infty) = 0$$

$$\mathrm{f}_{10}^{'}(0) = 0, \mathrm{f}_{10}^{'}(\infty) = 0, \mathrm{f}_0^{''}(\infty) = 0$$

$f_{11}'(0) = 0, f_{11}'(\infty) = 0, f_0''(\infty) = 0$ (3.14)

The solution of equations (3.10) to (3.13) satisfying the respective boundary conditions (3.14) are obtained but not presented here for the sake of brevity.

Results and Discussion

The approximate skin friction coefficient is given by

$$\tau = f''(0) = f_0''(0) + \varepsilon f_1''(0) \quad (4.1)$$

where $f_0''(0) = 1 - \frac{1}{3}k_1$ and $f_1''(0) = \frac{2}{3} + \frac{232}{15}\frac{1}{3}k_1$

The effects of visco-elastic parameter on the two-dimensional laminar MHD boundary layer flow in a divergent channel have been analyzed in this study The visco-elastic effect is exhibited through the non-dimensionalparameterk_1. The corresponding results for Newtonian fluid are obtained by setting $k_1 = 0$.

The Fig1-4 demonstrate the variations of dimensionless velocity $F'(\eta)$ against the variable η across the boundary layer for different flow parameters. The figuresdepict that the velocity increases with the increasing values of the visco-elastic parameter in comparison with the Newtonian fluid for fixed magnetic parameter M and c.

It is observed from the Fig1 and 2 that an increase in the magnetic parameter does not bring any significant change in the flow pattern. Fig 1 and 3 shows that the velocity increases with the increasing values of c.When both M and c increases, Fig 1 and 4 shows that the velocity increases with the increasing values of the visco-elastic parameter k_1 .

Figure 5 depicts the variation of the shearing stress τ at the wall of the divergent channel against the magnetic parameter M. It shows that the shearing stress decreases with the increasing values of visco-elastic parameter k_1 (=0, 0.02, 0.04) with the fixed value of c (=0.5). Also it shows the shearing stress decreases with the increasing values of the magnetic parameter M in both Newtonian and non-Newtonian cases. Figure 6 displays the shearing stress τ at the wall of the divergent channel against various values of c. It is observed that the shearing stressτ decreases with the increasing values of c but it increases gradually with the increasing values of visco-elastic parameter $k_1$in both Newtonian and non-Newtonian fluids.

Conclusion

The steady of two-dimensional MHD boundary layer flow of a visco-elastic fluid through a divergent channel with slip velocity has been investigated for different values of non-Newtonian parameter. In the analysis, the following conclusions are made:

- The speed of both Newtonian and non-Newtonian fluids through divergent channel enhances with the increasing values of visco-elastic parameter.
- The growth of magnetic parameter does not bring any significant change in the flow pattern of the viscoelastic fluid.
- Shearing stress at the wall of the divergent channel diminishes with the increasing values of visco-elastic parameter.
- Increasing value of slip velocity parameter reduces the shearing stress at the wall of the divergent channel.

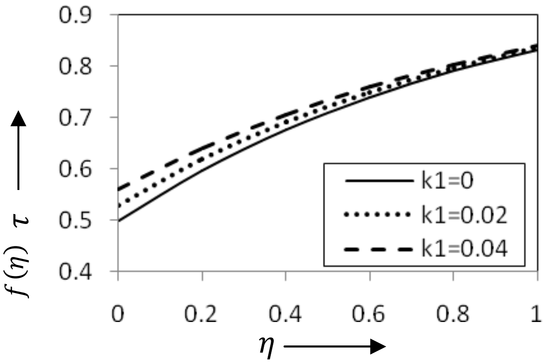

Fig 1: Velocity distribution against $\eta$for M=10 and c=0.5.

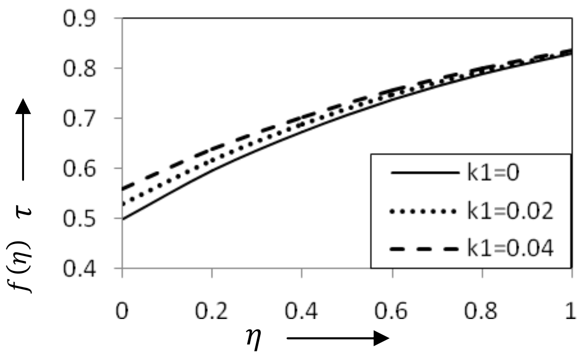

Fig 2: Velocity distribution against $\eta$for M=12 and c=0.5.

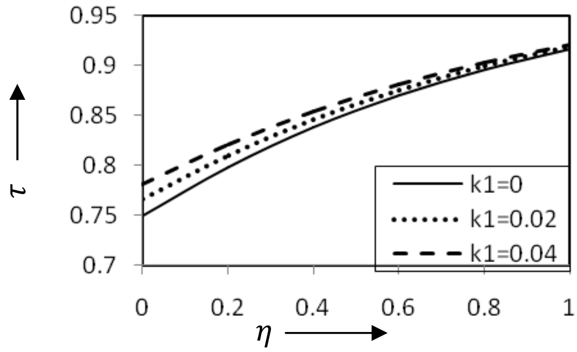

Fig 3: Velocity distribution against $\eta$for M=10 and c=0.75.

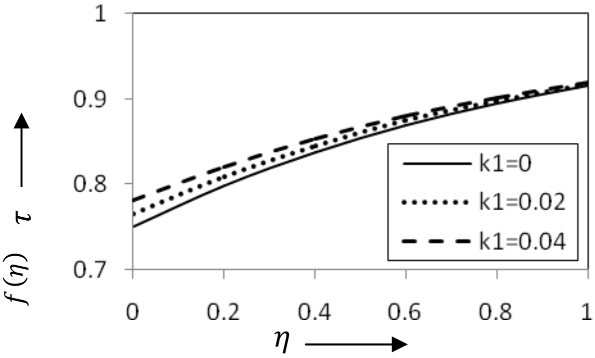

Fig 4: Velocity distribution against $\eta$for M=12 and c=0.75.

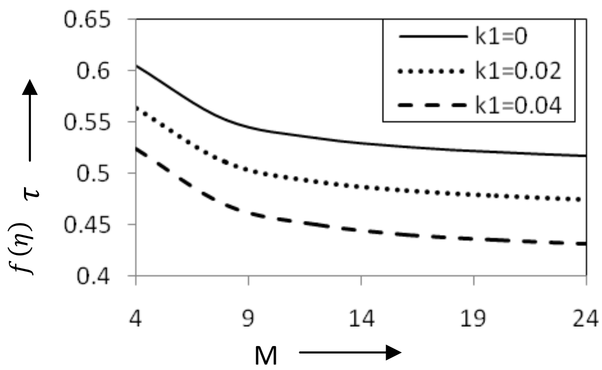

Fig 5: Skin friction co-efficient for various values M and c=0.5.

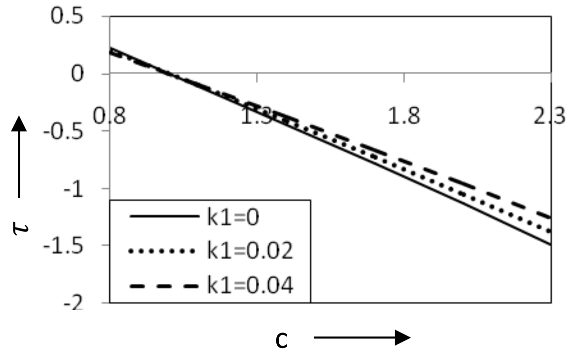

Fig 6: Skin friction co-efficient for various values c and M=10.

M

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