Far East Journal of Mechanical Engineering and Physics
Volume 1, Number 1, 2010, Pages 63-78
This paper is available online at http://pphmj.com/journals/fjmep.htm

VISCO-ELASTIC MHD FREE CONVECTIVE FLOW PAST A POROUS PLATE

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Abstract

An unsteady laminar free convective flow of a visco-elastic fluid through a porous medium bounded by an infinite vertical porous plate whose temperature is oscillating with time about a constant non-zero mean value has been investigated under the influence of a uniform transverse magnetic field by taking into account the effect of energy dissipation. The fluid is assumed to be slightly conducting and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the transverse magnetic field. Under the usual Boussinesq's approximation, the governing equations are solved analytically by using a regular perturbation technique. The expressions for the velocity, temperature, skin friction and the rate of heat transfer coefficient are obtained and presented graphically to observe the visco-elastic effects in combination of other flow parameters involved in the solution.

1. Introduction

The study of free convective MHD laminar flow through porous medium has attracted the attention of many researchers because of its important role in agriculture engineering to study the underground water resources, seepage of water in river beds, etc., in chemical engineering for filtration and purification processes, in petroleum

Keywords and phrases: visco-elastic, free convective, MHD, Porous plate, energy dissipation.

Received February 10, 2010

technology to study the movement of the natural gas, oil and water through oil reservoirs.

The investigation of the flow streaming into a porous and permeable medium with arbitrary but smooth plate surface was obtained by Yamamoto and Iwamura [1]. A theoretical analysis of two dimensional flow of a viscous, compressible fluid through a porous medium bounded by a porous plate was presented by Varshney [2]. With suction at vertical plate, the unsteady free convective flow problems have been studied by Nanda and Sharma [3], and Pop [4]. Nanda and Sharma [3] have considered the suction velocity to be proportional to $t^{1/2}$, whereas Pop [4] has assumed time-dependent oscillatory type of suction velocity. The flow past an impulsively started infinite plate in an incompressible fluid was studied by Stokes [5]. Stewartson [6] has studied to Stokes problem for a semi-infinite plate by analytical method, whereas Hall [7] has studied it by finite-difference method. Taking into account the free convection effects, Stokes problem was solved in closed form by Soundalgekar [8]. The plate was assumed to be isothermal and hence the effects of heating or cooling of the plate on the flow were considered.

In view of the practical importance of the MHD flows, an unsteady two dimensional hydro-magnetic free convection flow of an incompressible, viscous fluid through a porous medium bounded by an infinite vertical plate whose temperature is oscillating with time about a constant non-zero mean value was studied by Helmy [9]. In all the above papers, viscous dissipation effect has been neglected. But Gebhart [10] has shown that the viscous dissipative heat is important when the natural convection flow field is of extreme size or the flow is at extremely low temperature or in high gravity field. Neeraja and Reddy [11] have studied the MHD unsteady free convection flow past a vertical porous plate with viscous dissipation. The object of this paper is to extend the problem studied by Neeraja et al. to the case of visco-elastic fluid characterized by Walters liquid (Model B$'$). The constitutive equation for Walters liquid (Model B$'$) is

$$\sigma^{ik} = -pg^{ik} + 2\eta_0 e^{ik} - 2k_0 e'^{ik}, \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g^{ik} is the metric tensor of a fixed coordinate system x^i, v^i is the velocity vector, the contravariant form of e'^{ik}

is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk}. \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v^i_{,k} + v^k_{,i}. \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)\,d\tau \quad \text{and} \quad k_0 = \int_0^\infty \tau N(\tau)\,d\tau, \tag{1.4}$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters [12, 13]. This idealized model is a valid approximation of Walters liquid (Model B$'$) taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)\,d\tau, \quad n \geq 2 \tag{1.5}$$

have been neglected.

2. Mathematical Formulation and Analysis

Consider an unsteady free convective hydro-magnetic flow of a visco-elastic, electrically conducting fluid through a porous medium bounded by an infinite vertical plate, whose temperature oscillates with time about a constant non-zero mean value. The x'-axis is taken in the upward direction along the plate and y'-axis normal to it. Since the plate is of infinite length, all the physical variables are functions of y' and t' only. The applied magnetic field is considered in the direction perpendicular to the plate. The fluid is assumed to be of slightly conducting and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the transverse applied magnetic field.

Following Yamamoto and Iwamura [1], we regard the porous medium as an assemblage of small identical spherical particles fixed in space. Now, in the absence of any input electric field, under the usual Boussinesq's approximation, the problem is governed by the following system of equations:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{2.1}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - k_1\left[\frac{1}{2}\frac{\partial^3 \bar{u}}{\partial \bar{t}\partial \bar{y}^2} + \frac{1}{2}\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}\right] + g\beta(\bar{T} - \bar{T}_\infty) - \frac{\nu}{\bar{K}}\bar{u} - \frac{\sigma H_0^2}{\rho}\bar{u}, \tag{2.2}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\nu}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 - \frac{k_1}{2C_p}\left\{\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{t}\partial \bar{y}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right\}, \tag{2.3}$$

where $\bar{u}$, $\bar{v}$ are the velocity components in the x', y' directions, respectively, $\bar{t}$ is the time, ρ is the reference density, β is the volumetric coefficient of thermal expansions, $\bar{T}$ is the temperature of the fluid, $\bar{T}_\infty$ is the temperature of the fluid far away from the plate, g is the acceleration due to gravity, ν is the kinematic viscosity, $\bar{K}$ is the permeability of the porous medium, σ is the electric conductivity of the fluid, H is the magnetic field strength, k is the thermal conductivity of the fluid and C_p is the specific heat at constant pressure and the other symbols have their usual meanings.

The second and third terms on the right hand side of (2.3) represent the energy dissipation. As the momentum equation (2.2) and the energy equation (2.3) are coupled, the energy dissipation terms influence not only the temperature field but also the velocity field.

The boundary conditions for the velocity and temperature fields are

$$\bar{u} = 0,\ \bar{T} = \bar{T}_w + \varepsilon(\bar{T}_w - \bar{T}_\infty)e^{i\bar{w}\bar{t}} \text{ at } \bar{y} = 0,$$

$$\bar{u} = 0,\ \bar{T} = \bar{T}_\infty \text{ as } y \to \infty, \tag{2.4}$$

where $\bar{T}_w$ is the temperature of the plate, $\bar{w}$ is the frequency parameter. Equation (2.1) asserts that, the suction velocity is either a constant or a function of time. Hence the suction velocity normal to the plate is assumed in the form

$$\bar{v} = -V_0(1 + \varepsilon A e^{i\bar{w}\bar{t}}), \tag{2.5}$$

where A is a real positive constant, ε is small such that $\varepsilon << 1$, $\varepsilon A \le 1$ and V_0 is a non-zero positive constant. The negative sign indicates that the suction velocity is directed towards the plate.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced:

$$\left.\begin{array}{l} t = \dfrac{\bar{t}V_0^2}{\nu},\ y = \dfrac{V_0\bar{y}}{\nu},\ w = \dfrac{\nu\bar{w}}{V_0^2},\ u = \dfrac{\bar{u}}{V_0},\ \theta = \dfrac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty},\ K = \bar{K}\dfrac{V_0^2}{\nu^2}, \\ G = \dfrac{\nu g\beta(\bar{T}_w - \bar{T}_\infty)}{V_0^3},\ N = M + \dfrac{1}{K},\ M = \dfrac{\sigma H_0^2\nu}{\rho V_0^2},\ Pr = \dfrac{\rho\nu C_p}{k},\ E = \dfrac{V_0^2}{C_p(\bar{T}_w - \bar{T}_\infty)} \end{array}\right\} \tag{2.6}$$

Reduced forms of equation (2.2) and (2.3) in view of (2.6) are

$$\frac{\partial u}{\partial t} - Pr(1 + A\varepsilon e^{iwt})\frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} - k_2\left[\frac{\partial^3 u}{\partial t\partial y^2} - (1 + \varepsilon A e^{iwt})\frac{\partial^3 u}{\partial y^3}\right] - Nu, \tag{2.7}$$

$$Pr\frac{\partial \theta}{\partial t} - Pr(1 + A\varepsilon e^{iwt})\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PrE\left(\frac{\partial u}{\partial y}\right)^2$$

$$- PrEk_2\left[\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial t\partial y} - (1 + EAe^{iwt})\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right], \tag{2.8}$$

where G is the Grashof number, M is the magnetic field parameter, Pr is the Prandtl number and E is the Eckert number, $k_2 = \frac{k_1 V_0^2}{2\nu^2}$, the non-Newtonian parameter.

The boundary conditions for the velocity and temperature fields in the non-dimensional form are

$$u = 0,\ \theta = 1 + \varepsilon e^{iwt} \text{ at } y = 0,$$

$$u = 0,\ \theta \to 1 \text{ as } y \to \infty. \tag{2.9}$$

3. Solution of the Problem

To solve the coupled non-linear equations (2.7) and (2.8) subject to the boundary conditions (2.9), we represent the velocity and temperature of the fluid in the neighbourhood of the plate as

$$u(y, t) = u_0(y) + \varepsilon u_1(y)e^{iwt} + \cdots + \cdots,$$

$$\theta(y, t) = \theta_0(y) + \varepsilon \theta_1(y)e^{iwt} + \cdots + \cdots. \qquad (3.1)$$

Substituting (3.1) in equations (2.7) and (2.8) and equating harmonic and non-harmonic terms, neglecting the coefficient of ε^2, we get

$$u_0'' + u_0' - Nu_0 = -G\theta_0 - k_2 u_0''', \qquad (3.2)$$

$$u_1'' + u_1' - (N + iw)u_1 = -G\theta_1 - Au_0' - k_2\lfloor Au_0''' + u_1''' \rfloor, \qquad (3.3)$$

$$\theta_0'' + Pr\theta_0' = -PrEu_0'^2 - PrEk_2 u_0' u_0'', \qquad (3.4)$$

$$\theta_1'' + Pr\theta_1' - iwPr\theta = -APr\theta_0' - 2PrEu_0'u_1' - PrEk_2[u_0^1 u_1'' + u_1' u_0'' + Au_0' u_0'']. \qquad (3.5)$$

Here the primes denote differentiation with respect to y.

The corresponding boundary conditions become

$$u_0 = 0,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 1 \text{ at } y = 0,$$

$$u_0 \to 0,\ u_1 \to 0,\ \theta_0 \to 1,\ \theta_1 = 0 \text{ as } y \to \infty. \qquad (3.6)$$

The equations (3.2)-(3.5) are still coupled non-linear equations, whose exact solutions are not possible. So we expand u_0, u_1, θ_0, θ_1 in terms of E in the following way, as the Eckert number for the incompressible fluid is always very small:

$$u_0(y) = u_{01}(y) + Eu_{02};\ u_1(y) = u_{11}(y) + Eu_{12}(y),$$

$$\theta_0(y) = \theta_{01}(y) + E\theta_{02}(y);\ \theta_1(y) = \theta_{11}(y) + E\theta_{12}(y). \qquad (3.7)$$

Substituting (3.7) in equations (3.2)-(3.5), equating the coefficients of E to zero and neglecting the terms in E^2 and higher orders, we get

$$u_{01}'' + u_{01}' - Nu_{01} = -G\theta_{01} - k_2 u_{01}''', \qquad (3.8)$$

$$u_{02}'' + u_{02}' - Nu_{02} = -G\theta_{02} - k_2 u_{02}''', \qquad (3.9)$$

$$u_{11}'' + u_{11}' - (N + iw)u_{11} = -G\theta_{11} - Au_{01}' - k_2\lfloor Au_{01}''' + u_{11}''' \rfloor, \qquad (3.10)$$

$$u_{12}'' + u_{12}' - (N + iw)u_{12} = -G\theta_{12} - Au_{02}' - k_2\lfloor Au_{02}''' + u_{12}''' \rfloor, \qquad (3.$$

$$\theta_{01}'' + Pr\theta_{01}' = 0, \qquad (3.$$

$$\theta_{02}'' + Pr\theta_{02}' = -Pru_{01}'^2 - Prk_2 u_{01}' u_{01}'', \qquad (3.$$

$$\theta_{11}'' + Pr\theta_{11}' - iwPr\theta_{11} = -APr\theta_{01}', \qquad (3.$$

$$\theta_{12}'' + Pr\theta_{12}' - iwPr\theta_{12} = -APr\theta_{02}' - 2Pru_{01}'u_{11}'$$

$$- Prk_2(u_{01}' u_{11}'' + u_{11}' u_{01}'' + Au_{01}' u_{01}''). \qquad (3.$$

The corresponding boundary conditions are

$$u_{01} = 0,\ u_{02} = 0,\ u_{11} = 0,\ u_{12} = 0,\ \theta_{01} = 1,\ \theta_{02} = 0,\ \theta_{11} = 1,\ \theta_{12} = 0 \text{ at } y = 0$$

$$u_{01} \to 0,\ u_{02} \to 0,\ u_{11} \to 0,\ u_{12} \to 0,\ \theta_{01} \to 0,\ \theta_{02} \to 0,\ \theta_{11} \to 0,\ \theta_{12} \to 0$$

$$\text{as } y \to \infty. \qquad (3.1$$

The equations (3.8) to (3.11) and (3.13), (3.15) are still coupled non-line equations, whose exact solutions are not possible. So, again we expand u_{01}, u_0 u_{11}, u_{12}, θ_{01}, θ_{02}, θ_{11}, θ_{12} in terms of k_2 since $k_2 << 1$:

$$u_{01} = u_{010} + k_2 u_{011};\ u_{02} = u_{020} + k_2 u_{021},$$

$$u_{11} = u_{110} + k_2 u_{111};\ u_{12} = u_{120} + k_2 u_{121},$$

$$\theta_{02} = \theta_{020} + k_2\theta_{021};\ \theta_{12} = \theta_{120} + k_2\theta_{121}. \qquad (3.1$$

Substituting (3.17) in equations (3.8)-(3.11) and (3.13), (3.15); equating coefficient of k_2 to zero and neglecting the terms in k_2^2 and higher orders, we get

$$u_{010}'' + u_{010}' - Nu_{010} = -G\theta_{01}, \qquad (3.1$$

$$u_{011}'' + u_{011}' - Nu_{011} = -u_{010}''', \qquad (3.1$$

$$u_{020}'' + u_{020}' - Nu_{020} = -G\theta_{02}, \qquad (3.2$$

$$u_{021}'' + u_{021}' - Nu_{021} = -u_{020}''', \qquad (3.2$$

$$u''_{110} + u'_{110} - (N + iw)u_{110} = -G\theta_{11} - Au'_{010}, \tag{3.22}$$

$$u''_{111} + u'_{111} - (N + iw)u_{111} = -Au'_{011} - Au'''_{010} - u'''_{110}, \tag{3.23}$$

$$u''_{120} + u'_{120} - (N + iw)u_{120} = -G\theta_{12} - Au'_{020}, \tag{3.24}$$

$$u''_{121} + u'_{121} - (N + iw)u_{121} = -Au'_{021} - Au'''_{020} - u'''_{120}, \tag{3.25}$$

$$\theta''_{020} + Pr\theta'_{020} = -Pru'^{2}_{010}, \tag{3.26}$$

$$\theta''_{021} + Pr\theta'_{021} = -2Pru'_{010}u'_{011} - Pru'_{010}u''_{010}, \tag{3.27}$$

$$\theta''_{120} + Pr\theta'_{120} - iwPr\theta_{120} = -APr\theta'_{020} - 2Pru'_{010}u'_{110}, \tag{3.28}$$

$$\theta''_{121} + Pr\theta'_{121} - iwPr\theta_{121} = -APr\theta'_{021} - 2Pr(u'_{010}u'_{111} + u'_{011}u'_{110})$$
$$- Pr(u'_{010}u''_{110} + u'_{110}u''_{010} + Au'_{010}u''_{010}). \tag{3.29}$$

The corresponding boundary conditions are

$$u_{010} = 0,\ u_{011} = 0,\ u_{020} = 0,\ u_{021} = 0,$$
$$u_{110} = 0,\ u_{111} = 0,\ u_{120} = 0,\ u_{121} = 0,$$
$$\theta_{020} = 0,\ \theta_{021} = 0,\ \theta_{120} = 0,\ \theta_{121} = 0 \text{ at } y = 0,$$
$$u_{010} \to 0,\ u_{011} \to 0,\ u_{020} \to 0,\ u_{021} \to 0,$$
$$u_{110} \to 0,\ u_{111} \to 0,\ u_{120} \to 0,\ u_{121} \to 0,$$
$$\theta_{020} \to 0,\ \theta_{021} \to 0,\ \theta_{120} \to 0,\ \theta_{121} \to 0 \text{ at } y \to \infty. \tag{3.30}$$

Solving the equations (3.18) to (3.29) subject to boundary conditions (3.30), we get the solutions for velocity and temperature. The solutions and the constants of the differential equations are obtained but not presented here for the sake of brevity.

The skin friction at the plate in the non-dimensional form is given by

$$\tau = \left[\frac{\partial u}{\partial y} - \frac{1}{2}k_2\left\{\left(\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right) - (1 + \varepsilon Ae^{iwt})\frac{\partial u}{\partial y}\right)\right\}\right]_{y=0}. \tag{3.31}$$

The rate of heat transfer coefficient at the plate, which in the non-dimensional for in terms of Nusselt number is given by

$$Nu = \frac{1}{Pr}\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \frac{1}{Pr}\left(\frac{\partial\theta_0}{\partial y} + \varepsilon e^{iwt}\frac{\partial\theta_1}{\partial y}\right)_{y=0}. \quad (3.3$$

The purpose of this study is to bring out the effects of visco-elastic parameter the MHD free convective flow past a porous plate with energy dissipation as t effects of other parameters have been discussed by Neeraja and Reddy. The visc elastic effect is exhibited through the parameter k_2. The corresponding results f Newtonian fluid are obtained by setting $k_2 = 0$ and it is worth mentioning that the results coincide with Neeraja and Reddy.

Figures 1-12 represent the profiles of velocity, temperature and skin frictic coefficient for various values of the visco-elastic parameter k_2 with the combinati of other flow parameters. The convection velocity is upwards in the case of cooli of the boundary, i.e., $G > 0$ (Figures 1 and 2) while it is downwards when t boundary is heated, i.e., $G < 0$ (Figures 3 and 4) in both Newtonian and no Newtonian cases. It is also noted that the convection velocity u which attai maximum in the vicinity of the boundary plate, rapidly lies down in the transver direction to the boundary at a distance y greater than 0.25, also the maximum of | is attained in the case of cooling $(G > 0)$ as well as heating $(G < 0)$ at about 0.25 both Newtonian and non-Newtonian cases with Prandtl number $Pr = 9$, the magnet field parameter $M = 2$, Eckert number $E = 0.001$, the thermal conductivity $K = 1.$ $\varepsilon = 0.1$, $A = 0.5$ and $wt = \pi$ or $wt = \pi/2$. Figures 5-8 depict the profiles of t non-dimensional temperature in the flow field. It is noticed that, in both the cases cooling and heating of the boundary, the temperature profiles increase in no Newtonian cases in comparison with Newtonian fluid for fixed values of other flo parameters Pr, M, E, ε, K, A and $wt = \pi$ or $wt = \pi/2$. The profiles of the sk friction are depicted in Figures 9-12. From Figures 9-12, it is noticed that for $wt =$ or $\pi/2$, the skin friction profiles decrease in non-Newtonian cases in comparis with Newtonian fluid for $G > 0$ but the reverse pattern is observed for $G < 0$ wh the other flow parameters are kept fixed.

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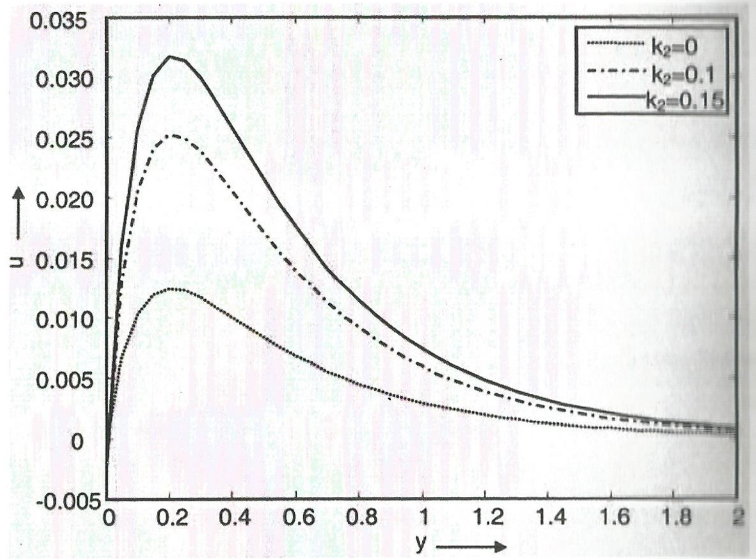

Figure 1. Variation of u against y for $G = 2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi$.

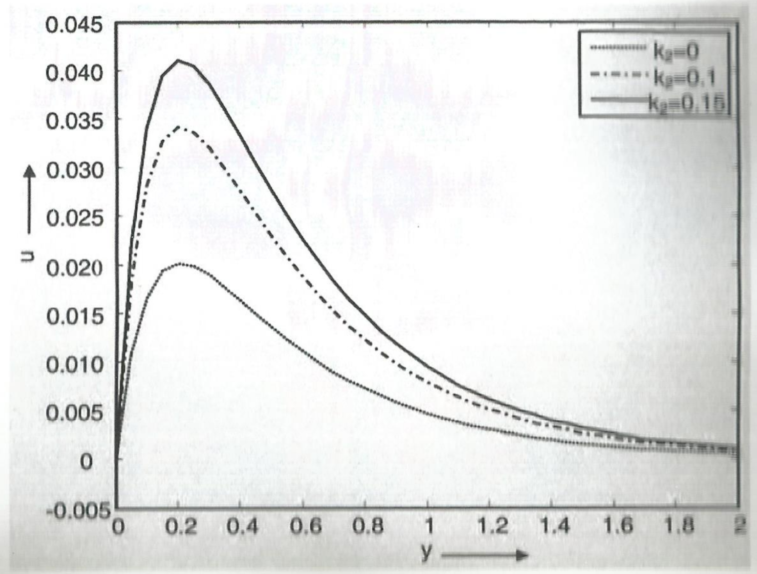

Figure 2. Variation of u against y for $G = 2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi/2$.

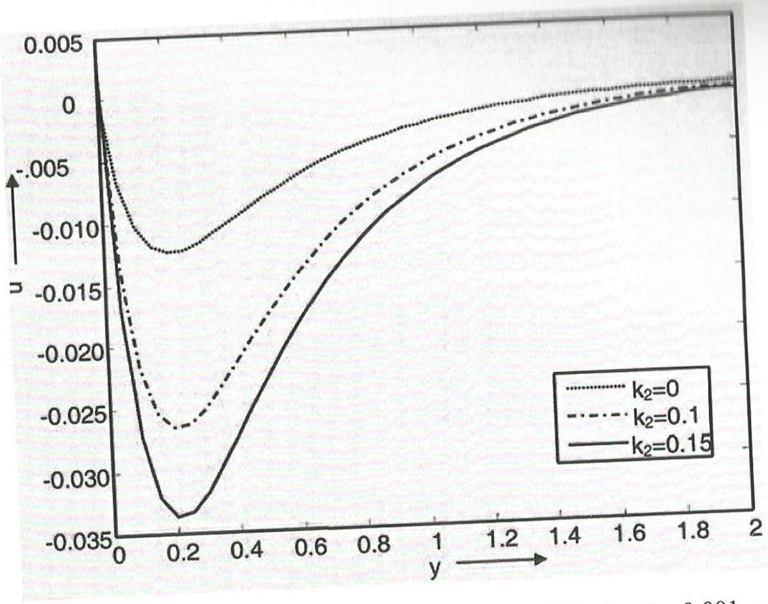

Figure 3. Variation of u against y for $G = -2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi$.

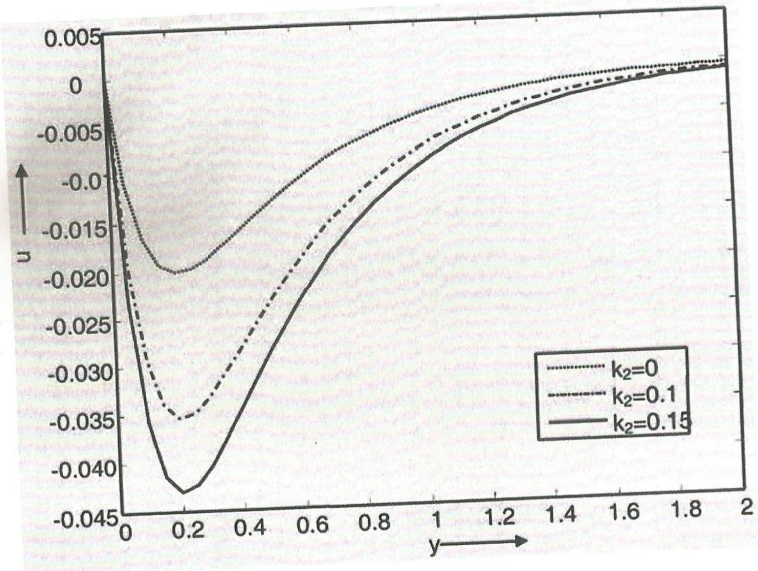

Figure 4. Variation of u against y for $G = -2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi/2$.

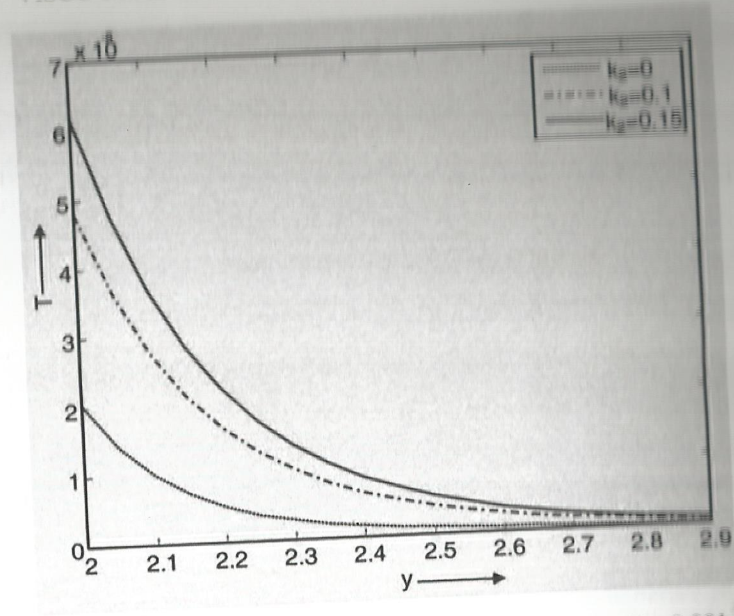

Figure 5. Variation of T against y for $G = 2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon =$

$K = 1.5$, $A = 0.5$, $\omega t = \pi$.

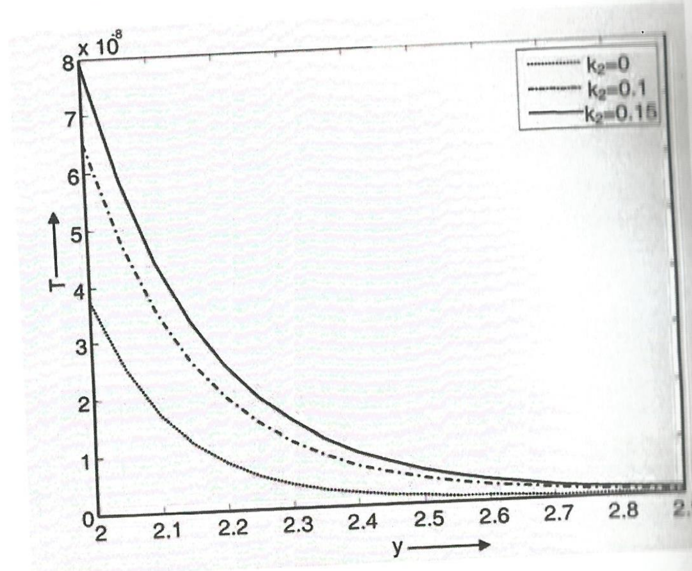

Figure 6. Variation of T against y for $G = 2.0$, $Pr = 9$, $M = 2$, $E = 0.001$,

$K = 1.5$, $A = 0.5$, $\omega t = \pi/2$.

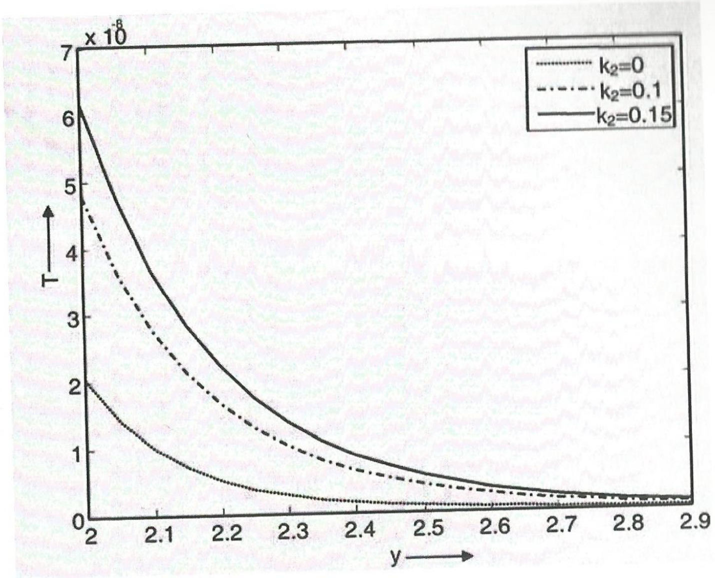

Figure 7. Variation of T against y for $G = -2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi$.

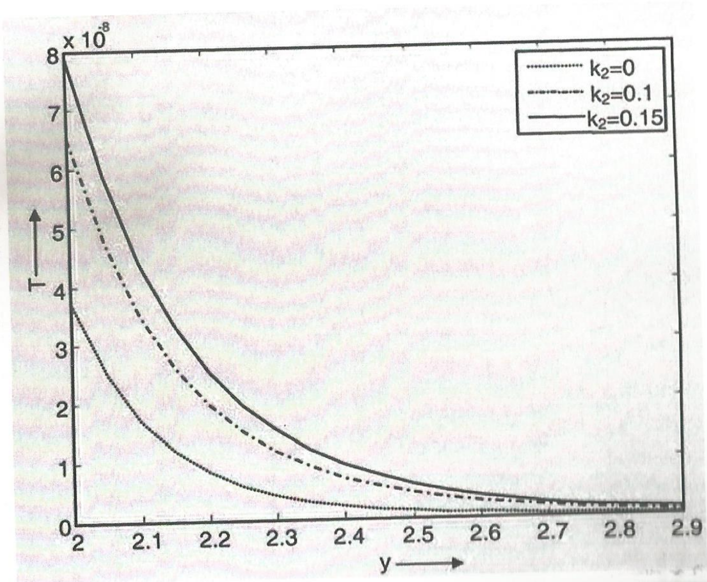

Figure 8. Variation of T against y for $G = -2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi/2$.

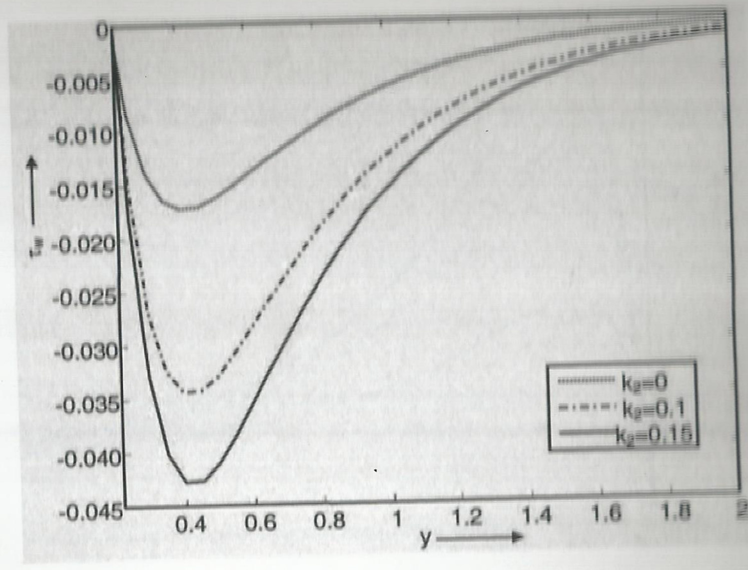

Figure 9. Variation of τ_w against y for $G = 2.0$, $Pr = 9$, $M = 2$, $E = 0.0$ $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi$.

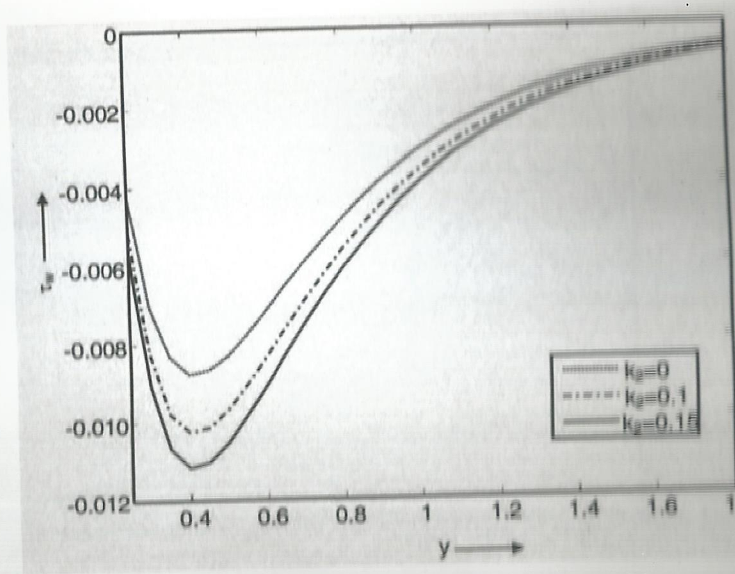

Figure 10. Variation of τ_w against y for $G = 2.0$, $Pr = 9$, $M = 2$, $E = 0$ $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi/2$.

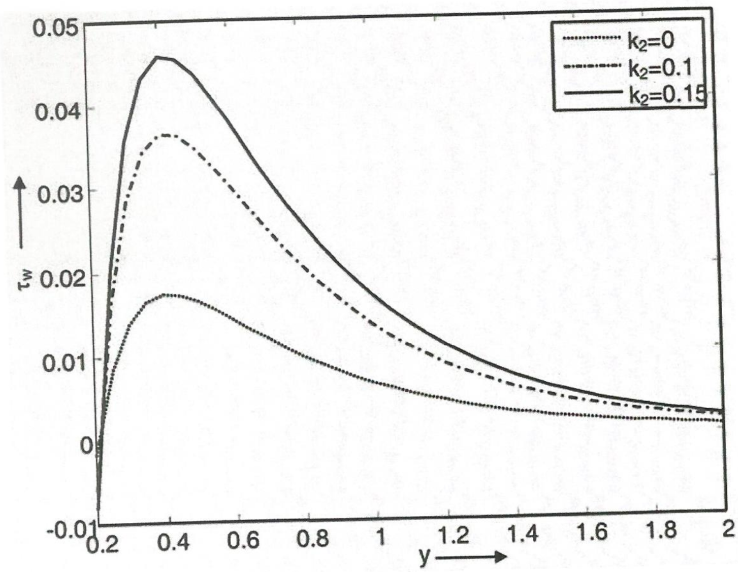

Figure 11. Variation of τ_w against y for $G = -2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi$.

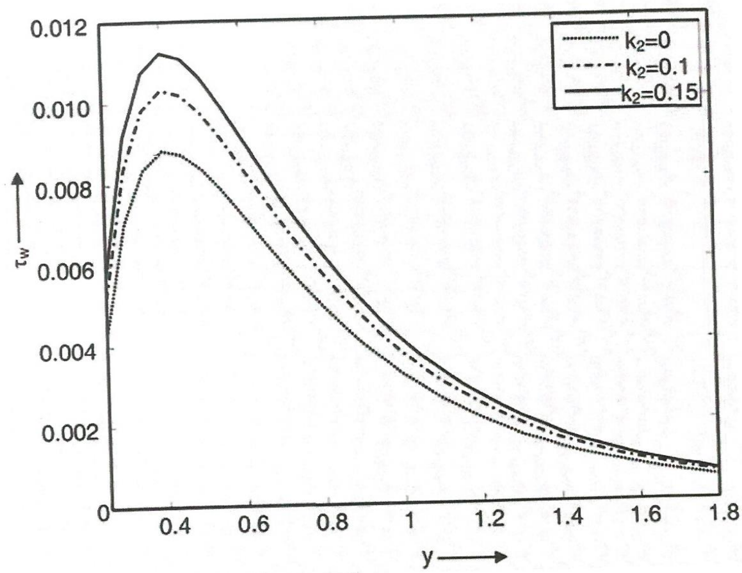

Figure 12. Variation of τ_w against y for $G = -2.0$, $Pr = 9$, $M = 2$, $E = 0.001$, $\varepsilon = 0.1$, $K = 1.5$, $A = 0.5$, $\omega t = \pi/2$.