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HYDROMAGNETIC DIVERGENT CHANNEL FLOW OF A VISCO-ELASTIC ELECTRICALLY CONDUCTING FLUID

RITA CHOUDHURY

Department of Mathematics, Gauhati University
Guwahati-781014, Assam, India
Email: rchoudhury66@yahoo.in

KAMAL DEBNATH *

Department of Mathematics, Gauhati University
Guwahati-781014, Assam, India
Email: debnathtura@gmail.com

Abstract

A theoretical study for the two-dimensional boundary layer flow through a divergent channel of a visco-elastic electrically conducting fluid in presence of transverse magnetic field has been considered. Similarity solutions are obtained by considering a special form of magnetic field. The analytical expressions for velocity and skin friction at the wall have been obtained and numerically worked out for different values of the flow parameters involved in the solution. The velocity and the skin friction coefficient have been presented graphically to observe the visco-elastic effects for various values of the flow parameters across the boundary layer.

Key words and Phrases: Visco-elastic, divergent channel, electrically conducting fluid, MHD, similarity solution.

2000 AMS Mathematics Subject Classification: 76A05, 76 A10.

Introduction

Jeffery(1) has investigated the divergent flow problem between two non-parallel planes by reducing the problem to an elliptic integral equation. Srivastava (2) has extended this problem to an electrically conducting fluid in the presence of transverse magnetic field. He has observed that with the application of magnetic field it is possible to have purely divergent flow without any secondary flow for greater angle between the two planes. The solution of two-dimensional incompressible laminar flow in a divergent channel with impermeable wall has been presented by Rosenhead (3). Terril(4) has analyzed the slow laminar flow in a converging or diverging channel with suction at one wall and blowing at the other wall.

Hamel (5) has studied the preceding problem of calculating all three dimensional flows whose stream-lines are identical with those of potential flow. The numerical calculations of Jeffery-Hamel flows between non-parallel plane walls were performed by Millsaps and Pohlhausen (6). Phukan(7) studied the hydromagnetic divergent channel flow of a Newtonian electrically conducting fluid. Magnetohydrodynomic laminar flow of a viscous fluid in a converging or diverging channel with suction at one wall and equal blowing at the other wall has been studied by Mahapatra *et. al*(8).

The present work deals with the two-dimensional magnetohydrodynamic boundary layer flow through a divergent channel of an electrically conducting visco-elastic fluid characterized by Walters liquid (Model B') in presence of transverse magnetic field. The effect of the visco-elastic fluid across the boundary layer on the dimensional velocity component and skin friction coefficient have been presented graphically with the combination of other flow parameters involved in the solution.

The constitutive equation for Walters liquid (Model B') is

$$\sigma^{ik} = -pg_{ik} + 2\eta_0 e^{ik} - 2k_0 e'^{ik} \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v^i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}{}_{,m} - v^k{}_{,m} e^{im} - v^i{}_{,m} e^{mk} \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v^i{}_{,k} + v^k{}_{,i} \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau)d\tau \tag{1.4}$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters (9, 10). This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau)d\tau, n \geq 2 \tag{1.5}$$

have been neglected.

Mathematical Formulation

The basic equations for steady two-dimensional boundary layer flow of Walters liquid (Model B') in the presence of a magnetic field B(x) are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} - \frac{k_0}{\rho}\left[u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}\right] + \frac{\sigma B^2(x)}{\rho}(U-u) \tag{2.2}$$

subject to the boundary conditions

$$y = 0 : u = 0, v = 0$$
$$y \to \infty : u = U(x) \tag{2.3}$$

where x-axis coincides with the wall of the divergent channel and y-axis perpendicular to it. U is the velocity component outsides the boundary layer, u and v are the flow velocities in the direction of x and y respectively, ρ the fluid density, ν the kinematic viscosity, σ the electrical conductivity of the fluid and k_0 the visco-elastic parameter.

In equation (2.2), the secondary effects of magnetic induction are ignored i.e. the induced magnetic field is negligible as it is small in comparison to the applied magnetic field. Furthermore, we assume that the external electric field is zero and the eletric field due to polarization of charges is also negligible.

As in Sinha and Choudhury (11), the potential flow near the sources is taken to be

$$U(x) = \frac{u_1}{x} \tag{2.4}$$

With $u_1 > 0$ represents two dimensional divergent flow and leads to similarity solution.

We introduce the following change of variables (Schlichting (12))

$$\eta(x,y) = y\sqrt{\frac{U(x)}{\nu x}} = \frac{y}{x}\sqrt{\frac{u_1}{x}} \tag{2.5}$$

and the stream function

$$\varphi(x,y) = \sqrt{U(x)\nu x}F(\eta) = \sqrt{\nu u_1}F(\eta) \tag{2.6}$$

Then, we obtain the velocity component as

$$u = \frac{\partial \varphi}{\partial y} = U(x)F'(\eta) \text{ and } v = -\frac{\partial \varphi}{\partial y} = \sqrt{\nu u_1}F'(\eta) \tag{2.7}$$

The equation of continuity (2.1) is identically satisfied for the velocity component (2.7)

Similarity solution exists if the magnetic field B(x) has the special form (Chiam (13))

$$B(x) = \frac{B_0}{x} \tag{2.8}$$

Using the equation (2.4) to (2.8), the equation (2.2) takes the form as follows:

$$F''' + F'^2 + k_1\left[4F'F''' - 2F''^2\right] + M(1 - F') - 1 = 0 \tag{2.9}$$

Here prime denotes the differentiation with respect to η. $k_1 = \frac{k_0}{\rho}$ and M denote the modified non-Newtonian and hydrodynamic parameter respectively.

The corresponding boundary conditions are

$$F(0) = 0, F'(0) = 0, F'(\infty) = 1 \tag{2.10}$$

Method of Solution

We first assume

$$z = \sqrt{M}\eta, f(z) = \sqrt{M}F(\eta) \tag{3.1}$$

which implies

$$f'(z) = F'(\eta), \; f''(z) = \frac{1}{\sqrt{M}}F''(\eta), \; f'''(z) = \frac{1}{M}F'''(\eta) \tag{3.2}$$

Using the equation (3.2) in the equation (2.9), we get the following differential equation

$$f'''(z) + k_1\left[4f'(z)f'''(z) - 2f''^2(z)\right] + (1 - f'(z)) = \varepsilon(1 - f'^2(z)) \tag{3.3}$$

where $\varepsilon = \frac{1}{M}$

The corresponding boundary conditions are

$$f(0) = 0, f'(0) = 0, f'(\infty) = 1 \tag{3.4}$$

The unknown function f(z) is expanded in terms of powers of the small parameter ε as follows:

$$f(z) = f_0(z) + \varepsilon f\ (z) + \varepsilon^2 f_2(z) + \ldots\ldots\ldots\ldots \tag{3.5}$$

On substituting the equation (3.5) in the equation (3.3) and equating the like powers of ε, we get

$$f_0''' + k_1\left[4f_0'f_0''' - 2f_0''^2\right] + (1 - f') = 0 \tag{3.6}$$

$$f_1''' + k_1\left[4f_0'f_1''' + f_1'f_0''' - f_0''f_1''\right] - f' = 1 - f_0'^2 \tag{3.7}$$

The relevant boundary conditions are:

$$f_0(0) = 0, f_0'(0) = 0, f_0'(\infty) = 1$$

$$f_1(0) = 0, \; f_1'(0) = 0, \; f_1'(\infty) = 0 \tag{3.8}$$

Again, in order to solve equations (3.6) and (3.7), we consider very small values of k_1, so that f_0 and f_1 can be expressed as

$$f_0 = f_{00}(z) + k_1 f_{01}(z) + 0(k_1^2)$$

$$f_1 = f_{10}(z) + k_1 f_{11}(z) + 0(k_1^2) \tag{3.9}$$

Now substituting (3.9) into the equations (3.6) and (3.7), we get the following sets of ordinary differential equations

$$f_{00}''' - f_{oo}' = -1 \tag{3.10}$$

$$f_{01}''' - f_{o1}' = -4f_{00}'f_{00}''' + 2f_{00}''^2 \tag{3.11}$$

$$f_{10}''' - f_{10}' = f_{00}'^2 - 1 \tag{3.12}$$

$$f_{11}''' - f_{11}' = 4\left[f_{00}''f_{10}'' - f_{00}'f_{10}''' - f_{10}'f_{00}'''\right] - 2f_{00}'f_{01}' \tag{3.13}$$

The appropriate boundary conditions are

$$f_{00}(0) = 0, f_{00}'(0) = 0, f_{00}'(\infty) = 1$$

$$f_{01}(0)=0, f'_{01}(0)=0, f'_{01}(\infty)=0$$
$$f_{10}(0)=0, f'_{10}(0)=0, f'_{10}(\infty)=0$$
$$f_{11}(0)=0, f'_{11}(0)=0, f'_{11}(\infty)=0 \quad (3.14)$$

The solution of equations (3.10) to (3.13) satisfying the respective boundary conditions (3.14) are

$$f_{00} = z + e^{-z} - 1 \quad (3.15)$$

$$f_{01} = \frac{1}{3}\left(6ze^{-z} + 4e^{-z} + e^{-2z} - 5\right) \quad (3.16)$$

$$f_{10} = -\frac{1}{6}\left(6ze^{-z} + 4e^{-z} + e^{-2z} - 5\right) \quad (3.17)$$

$$f_{11} = \frac{4}{45}\left(90z^2e^{-z} + 3e^{-z} - e^{-3z} - 3\right) \quad (3.18)$$

Substituting (3.15) to (3.18) into (3.5) and after differentiation with respect to z, we obtain

$$f'(z) = f'_0(z) + \varepsilon f'_1(z) \quad (3.19)$$

where $f'_0(z) = \left(1-e^{-z}\right) + k_1\left\{-\frac{4}{3}e^{-z} + 2e^{-z}(1-z) - \frac{2}{3}e^{-2z}\right\}$

$$f'_1(z) = \frac{2}{3}e^{-z} - e^{-z}(1-z) + \frac{1}{3}e^{-2z} + k_1\left\{-\frac{4}{15}e^{-z} + \frac{4}{15}e^{-3z} + 16ze^{-z} - 8z^2e^{-z}\right\}$$

From (3.2) and (3.19) we can easily find the dimensionless velocity $F'(\eta)$ across the boundary layer.

Results and Discussion

The approximate skin friction coefficient is given by

$$\tau = f''(0) = f''_0(0) + \varepsilon f''_1(0) \quad (3.20)$$

where $f''_0(0) = 1 - \frac{1}{3}k_1$ and $f''_1(0) = \frac{2}{3} + \frac{232}{15}k_1$

The effects of visco-elastic parameter on the two-dimensional laminar MHD boundary layer flow in a divergent channel have been analyzed in this study The visco-elastic effect is exhibited through the non-dimensionl parameter k_1. The corresponding results for Newtonian fluid are obtained by setting $k_1 = 0$.

The figures 1,2 and 3 demonstrate the variations of dimensionless velocity $F'(\eta)$ against the variable η across the boundary layer for different flow parameters. The figures depict that the velocity decreases with the increasing values of the variable in both Newtonian and non-Newtonian cases. Also, the figures show that an increase in the visco-elastic parameter in comparison with the Newtonian fluid reduce the flow velocity at all corresponding points in the flow field. Further, it is noticed from the figures that $F'(\eta)$ decreases when the magnetic parameter M increases with the increase of visco-elastic parameter.

Figure 4 depicts the variation of the shearing stress τ at the wall of the divergent channel against the magnetic parameter M. It shows that the shearing stress decreases with the increasing values of the magnetic parameter in both Newtonian and non-Newtonian cases. Further, the increase of visco-elastic parameter enhances the shearing stress at all corresponding points in the flow field.

Graphs for Velocity Distribution and Skin friction Co-efficient

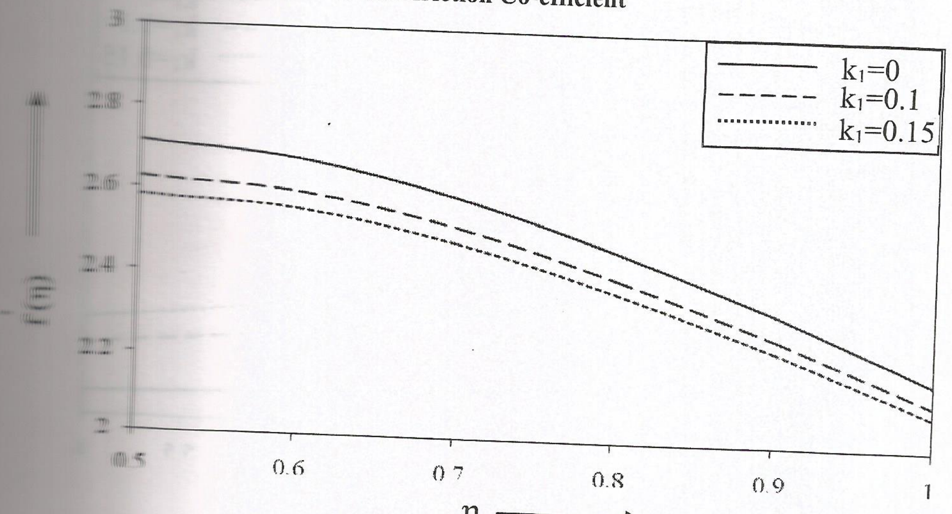

Figure 1: Velocity distribution against the variable η for M = 2

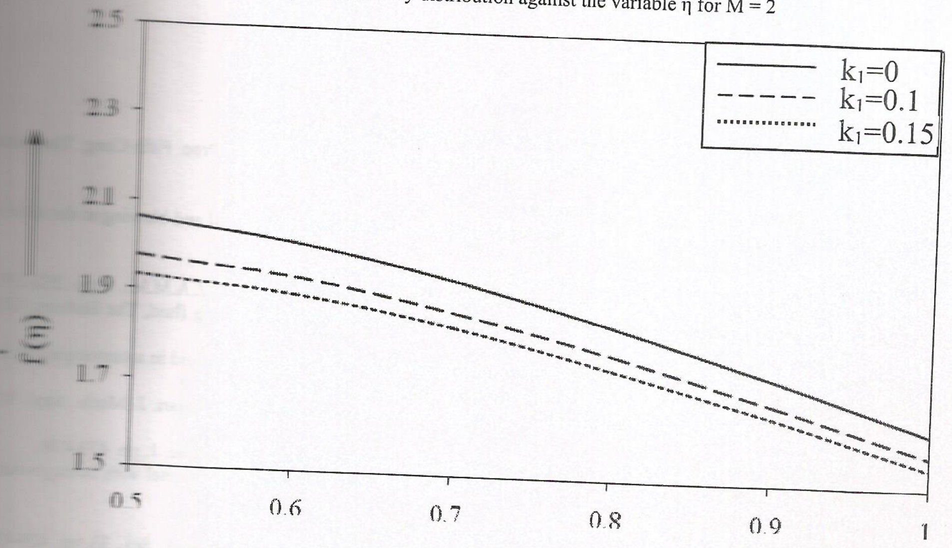

Figure 2: Velocity distribution against the variable η for M = 3

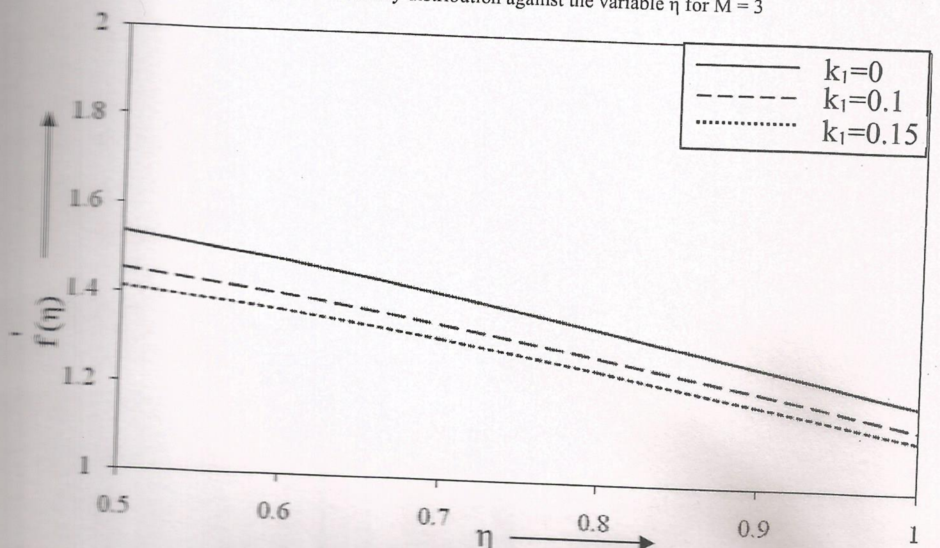

Figure 3: Velocity distribution against the variable η for M = 5

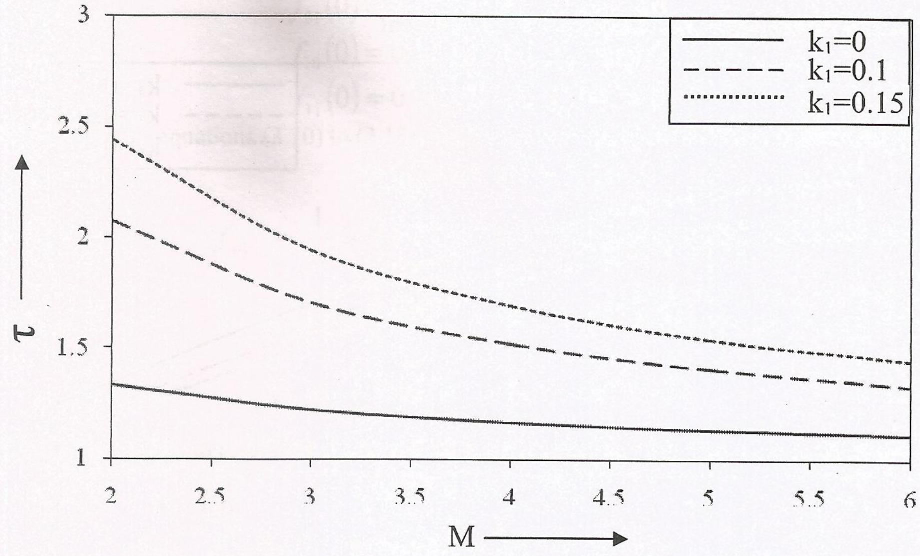

Figure 4: Skin friction co-efficient for various values of M

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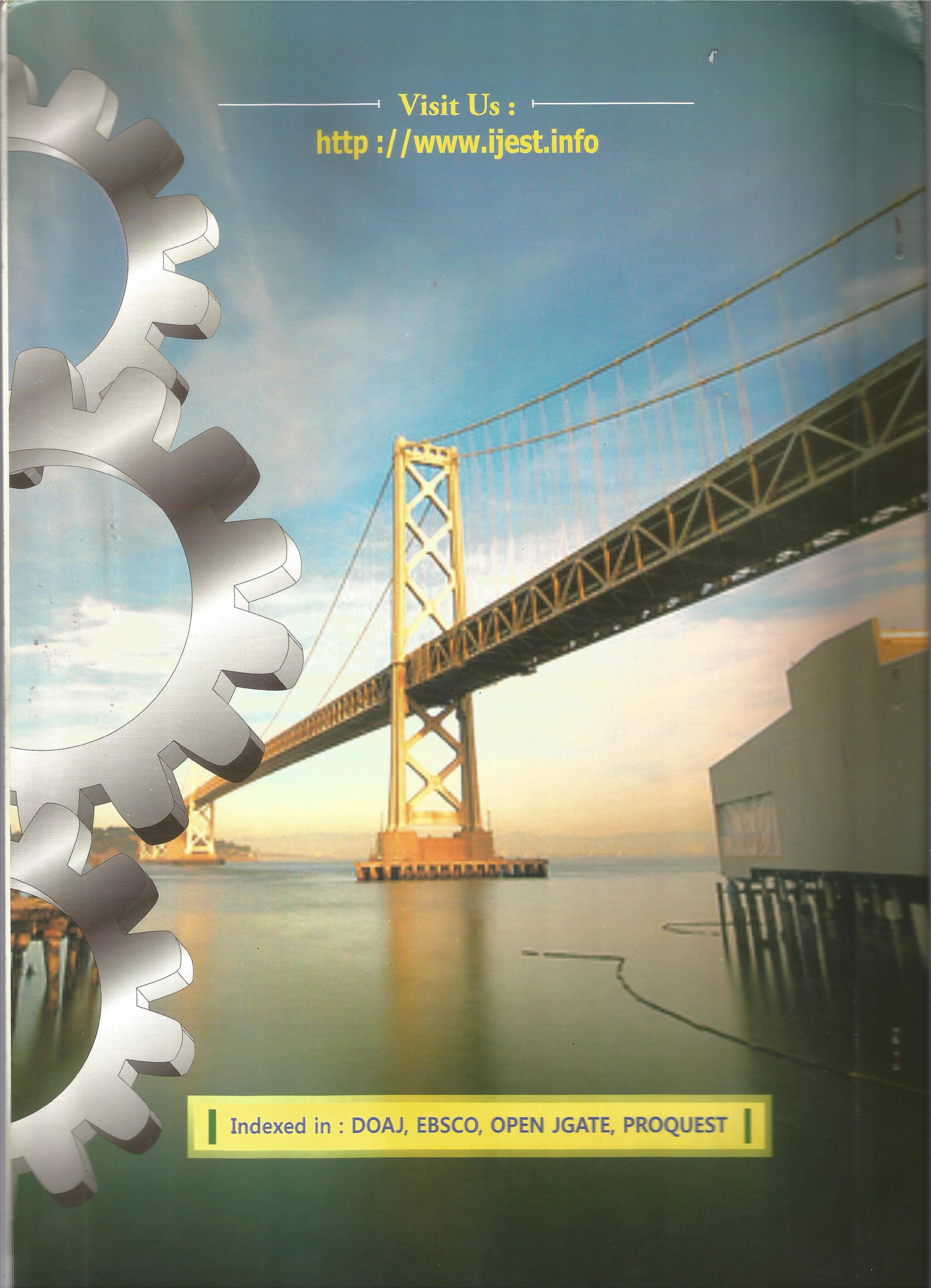
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