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EFFECTS OF VISCO-ELASTIC MHD UNSTEADY FREE CONVECTION FLOW PAST A POROUS PLATE

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Abstract

The free convective MHD unsteady laminar flow of a visco-elastic fluid through a porous medium bounded by an infinite vertical porous plate whose temperature is oscillating with time about a constant non-zero mean value in the presence of a uniform transverse magnetic field is considered. The governing equations are solved analytically by using a regular perturbation technique. The expressions for the velocity, temperature, skin-friction and the rate of heat transfer coefficient are obtained. The velocity profile and the skin-friction have been presented graphically to observe the visco-elastic effects in combination of other flow parameters involved in the solution.

1. Introduction

In recent years, the requirements of modern technology have stimulated interest in fluid flows, which involve the interaction of several phenomena. One such study is related to the effects of free convection flows through porous medium which plays an important role in industrial, biophysical and hydrological problems, particularly in petroleum, chemical and nuclear industries.

Key Words : *Visco-elastic, Free convective, MHD, Porous plate.*
2000 AMS Subject Classification : 76A05, 76A10.

The study of such flow was initiated by Yamamoto and Iwamura (1) who investigated the flow streaming into a porous and permeable medium with arbitrary but smooth plate surface. Varshney (2) has presented the theoretical analysis of two dimensional flow of a viscous compressible fluid through a porous medium bounded by a porous plate. With suction at vertical plate, the unsteady free convective flow problems have been studied by Nanda and Sharma (3), and Pop (4). The flow past an impulsively started infinite plate in an incompressible fluid was studied by Stokes (5). Stewartson (6) has studied to Stokes problem for a semi-infinite plate by analytical method, whereas Hall (7) has studied it by finite -difference method. Taking into account the free convection effects, Stokes problem was solved in closed form by Soundalgekar (8). Neeraja and Reddy (9) have studied the MHD unsteady free convection flow past a vertical porous plate with viscous dissipation.

In this paper, the unsteady laminar free convective MHD flow of a Walters liquid (Model B) through a porous medium has been investigated.

The constitutive equation for Walters liquid (Model B') is

$$\sigma^{ik} = -pg_{ik} + 2\eta_0 e^{ik} - 2k_0 e^{'ik} \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v^i is the velocity vector, the contravariant form of $e^{'ik}$ is given by

$$e^{'ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk} \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v^i_{,k} + v^k_{,i}. \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \quad \text{and} \quad k_0 = \int_0^\infty \tau N(\tau)d\tau, \tag{1.4}$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters (10, 11). This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau)d\tau, \quad n \geq 2 \tag{1.5}$$

have been neglected.

2. Mathematical Formulation

Consider an unsteady free convective hydro-magnetic flow of a visco-elastic electrically conducting fluid through a porous medium bounded by an infinite vertical plate, whose temperature oscillates with time about a constant non-zero mean value. The $\overline{x}$-axis is taken in the upward direction along the plate and $\overline{y}$-axis normal to it. Since the plate is of infinite length, all physical variables are functions of $\overline{y}$ and $\overline{t}$ only. The applied magnetic field is considered in the direction perpendicular to the plate. The fluid is assumed to be of slightly conducting and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the transverse applied magnetic field.

Following Yamamoto and Iwamura (1) we regard the porous medium as an assemblage of small identical spherical porous particles fixed in space. Now, in absence of any input electric field, under the usual Boussinesq's approximation, the problem is governed by the following system of equations:

$$\frac{\partial \overline{v}}{\partial \overline{y}} = 0 \tag{2.1}$$

$$\frac{\partial \overline{u}}{\partial \overline{t}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = v\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} - k_1\left[\frac{1}{2}\frac{\partial^3 \overline{u}}{\partial \overline{t}\partial \overline{y}^2} + \overline{v}\frac{\partial^2 \overline{u}}{\partial \overline{y}^3}\right] + g\beta(\overline{T} - \overline{T}_\infty) - \frac{v}{k}\overline{u} - \frac{\sigma H_0^2}{\rho}\overline{u} \tag{2.2}$$

$$\frac{\partial \overline{T}}{\partial \overline{t}} + \overline{v}\frac{\partial \overline{T}}{\partial \overline{y}} = \frac{k}{\rho C_p}\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} \tag{2.3}$$

where $\overline{u}, \overline{v}$ are the velocity components in the $\overline{x}, \overline{y}$ directions respectively, $\overline{t}$ is the time, ρ is the reference density, β is the volumetric coefficient of thermal expansion, $\overline{T}$ is the temperature of the fluid, $\overline{T}_x$ is the temperature of the fluid far away from the plate, g is the acceleration due to gravity, v is the kinematic viscosity, $\overline{K}$ is the permeability of the porous medium, σ is the electric conductivity of the fluid, H_0 is the magnetic field strength, k is the thermal conductivity of the fluid and C_p is the specific heat at constant pressure and the other symbols have their usual meanings.

The boundary conditions for the velocity and temperature fields are

$$\overline{u} = 0, \overline{T} = \overline{T}_w + \epsilon(\overline{T}_w - \overline{T}_\infty)e^{i\overline{w}\overline{t}} \text{ at } \overline{y} = 0, \quad \overline{u} = 0, \overline{T} = \overline{T}_\infty \text{ as } \overline{y} \to \infty \tag{2.4}$$

where $\overline{T}_\omega$ the temperature of the plate, $\overline{\omega}$ is the frequency parameter. Equation (2.1) asserts that, the suction velocity is either a constant or a function of time. Hence the suction velocity normal to the plate is assumed in the form

$$\overline{v} = -V_0(1 + \epsilon A e^{i\overline{\omega}\overline{t}}) \tag{2.5}$$

where A is a real positive constant, ϵ is small such that $\epsilon \ll 1, \epsilon A \leq 1$ and V_0 is a non-zero positive constant. The negative sign indicates that the suction is directed towards the plate.

3. Methods of Solution

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$t = \frac{\overline{t}V_0^2}{v}, \quad y = \frac{V_0\overline{y}}{v}, \quad w = \frac{v\overline{w}}{V_0^2}, \quad u = \frac{\overline{u}}{V_0}, \quad \theta = \frac{\overline{T} - \overline{T}_\infty}{\overline{T}_w - \overline{T}_x}, \quad K = \frac{\overline{K}V_0^2}{v^2}, \tag{3.1}$$

$$G = \frac{vg\beta(\overline{T}_w - \overline{T}_\infty)}{V_0^3}, \quad N = M + \frac{1}{K}, \quad M = \frac{\sigma H_0^2 v}{\rho V_0^2}, \quad P_r = \frac{\rho v C_p}{k}, \quad E = \frac{V_0^2}{C_p(\overline{T}_w - \overline{T}_\infty)}.$$

Reduced forms of equation (2.2) and (2.3) in view of (3.1) are

$$\frac{\partial u}{\partial t} - P_r(1 + A\epsilon e^{iwt})\frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} - k_2\left[\frac{\partial^3 u}{\partial t \partial y^2} - (1 + A\epsilon e^{iwt})\frac{\partial^3 u}{\partial y^3}\right] - Nu \tag{3.2}$$

$$P_r\frac{\partial \theta}{\partial y} - P_r(1 + A\epsilon e^{iwt})\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \tag{3.3}$$

where G is the Grashoff number, M is the magnetic field parameter, P_r is the Prandtl number and E is the Eckert number, $k_2 = \frac{k_1 V_0^2}{2v^2}$ is the the non-Newtonian parameter.

The boundary conditions for the velocity and temperature fields in the non-dimensional form are

$$u = 0, \theta = 1 + \epsilon e^{iwt} \text{ at } y = 0, \quad u = 0, \theta \to 1 \text{ as } y \to \infty. \tag{3.4}$$

To solve the coupled non-linear equations (3.2) and (3.3) subject to the boundary conditions (3.4), we represent the velocity and temperature of the field in the neighbourhood of the plate as

$$\begin{aligned} u(y,t) &= u_0(y) + \epsilon u_1(y)e^{iwt} + \cdots \\ \theta(y,t) &= \theta_0(y) + \epsilon \theta_1(y)e^{iwt} + \cdots \end{aligned} \tag{3.5}$$

Substituting (3.5) in equations (3.2) and (3.3) and equating harmonic and non-harmonic terms, neglecting the coefficient of ϵ^2, we get

$$u_0'' + u_0' - Nu_0 = -G\theta_0 - k_2 u_0''' \tag{3.6}$$

$$u_0'' + u_0' - (N + iw)u_0 = -G\theta_1 - Au_0 - k_2[Au_0''' + u_1'''] \tag{3.7}$$

$$\theta_0'' + Pr\theta_0' = 0 \tag{3.8}$$

$$\theta_0'' + Pr\theta_1' - iwPr\theta_1 = -Apr\theta_0'. \tag{3.9}$$

Here the primes denote differentiation with respect to y.

The relevant boundary conditions becomes

$$u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 1 \ \text{ at } \ y = 0 \tag{3.10}$$

$u_0 \to 0, u_1 \to 0, \theta_0 \to 1, \theta_1 \to 1$ as $y \to \infty$.

The solutions of the equations (3.8) and (3.9) subject to boundary conditions (3.10) are given by

$$\theta_0 = e^{-P_r y} \tag{3.11}$$

$$\theta_1 = \left(1 - \frac{AP_r}{\omega} i\right) e^{-m_1 y} + \frac{AP_r}{\omega} i e^{-P_r y} \tag{3.12}$$

The equations (3.6) and (3.7) are still coupled non-linear equations, whose exact solutions are not possible. So, we expand u_0 and u_1 in terms of k_2 (since $k_2 \ll 1$) as follows:

$$u_0(y) = u_{01}(y) + k_2 u_{02}(y), \quad u_1(y) = u_{11}(y) + k_2 u_{12}(y). \tag{3.13}$$

Substituting (3.13) in (3.6) and (3.7), equating the coefficient of k_2 to zero and neglecting the terms in k_2^2 and higher orders, we get

$$u_{01}'' + u_{01}' - Nu_{01} = -G\theta_0 \tag{3.14}$$

$$u_{02}'' + u_{02}' - Nu_{02} = -u_{01}''' \tag{3.15}$$

$$u_{11}'' + u_{11}' - (N + iw)u_{11} = -G\theta_1 - Au_{01}' \tag{3.16}$$

$$u_{12}'' + u_{12}' - (N + iw)u_{12} = -Au_{02}' - Au_{01}''' - u_{11}''' \tag{3.17}$$

subject to boundary conditions:

$$u_{01} = 0, u_{02} = 0, u_{11} = 0, u_{12} = 0 \text{ at } y = 0$$
$$u_{01} \to 0, u_{02} \to 0, u_{11} \to 1, u_{12} \to 1 \text{ as } y \to \infty. \tag{3.18}$$

Using the solutions (3.11) and (3.12) in the equations (3.14) to (3.17) and solving them subject to the boundary conditions (3.18), we get the solution for velocity. The solutions are given by

$$u_{01} = A_2(e^{-P_r y} - e^{-n_1 y}) \tag{3.19}$$

$$u_{02} = A_8 e^{-n_1 y} + A_5 e^{-P_r y} \tag{3.20}$$

$$u_{11} = A_{15} e^{-p_1 y} + A_{12} e^{-m_1 y} + A_{13} e^{-n_1 y} + A_{14} e^{-P_r y} \tag{3.21}$$

$$u_{12} = A_{25} e^{-p_1 y} + A_{20} e^{-m_1 y} + A_{21} e^{-n_1 y} + A_{23} e^{-P_r y} \tag{3.22}$$

where

$$m_1 = \frac{P_r}{2}\left(1 + \sqrt{1 + \frac{i4\omega}{P_r}}\right)$$

$$n_1 = \frac{1}{2}(1 + \sqrt{1 + 4N})$$

$$p_1 = \frac{1}{2}(1 + \sqrt{1 + 4N + i4\omega}).$$

The other constants involved in the solutions are obtained but not presented here for the sake of brevity.

The skin friction coefficient at the plate in the non-dimensional form is given by

$$\tau = \left[\frac{\partial u}{\partial y} - \frac{1}{2}k_2\left\{\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right) - (1 + \epsilon A e^{i\omega t})\frac{\partial u}{\partial y}\right\}\right]_{y=0}. \tag{3.23}$$

The non-dimensional form of the rate of heat transfer coefficient at the plate is given in terms of Nusselt number as

$$Nu = \frac{1}{P_r}\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \frac{1}{P_r}\left(\frac{\partial \theta_0}{\partial y} + \epsilon e^{i\omega t}\frac{\partial \theta_1}{\partial y}\right)_{y=0}. \tag{3.24}$$

4. Discussion

The effects of the visco-elastic parameter on the MHD free convective flow past a porous plate are discussed analytically in this study. The solution of this problem is obtained by the application of the perturbation technique. Based on this solution, we have carried out numerical computations for the velocity, temperature, skin friction and Nusselt number for various values of material parameters. The visco-elastic effect is exhibited through the parameter k_2. The corresponding results for Newtonian fluid are obtained by setting $k_2 = 0$ and the real parts of the results are implied throughout the discussion.

Figures 1 to 5 represent the profiles of velocity and skin friction coefficient for various values of the visco-elastic parameter k_2 with the combination of other flow parameters. The convection velocity u is upwards in the cases of cooling of the boundary i.e. $G > 0$ (figures 1), while it is downwards when the boundary is heated i.e. $G < 0$ (figures 2) in both Newtonian and non-newtonian cases. It is also noted the convection velocity u which attains maximum (figures 1) or minimum (figures 2) in the vicinity of the boundary plate, rapidly lies down (figures 1) or goes up (figures 2) in the transverse direction to the boundary at a distance y greater than 0.45. Also, the maximum of $|u|$ is attained in the case cooling ($G > 0$) as well as heating ($G < 0$) at about 0.45 in both Newtonian and non-Newtonian cases when Prandtl number $P_r = 3$, the magnetic field parameter $M = 2$, the permeability of the parameter $K = 1.54$ with $A = 0.5$, $\omega t = \frac{\pi}{2}$. The effect of visco-elastic parameter is to decrease the velocity profile when $G > 0$ (figures 1) and enhance the same when $G < 0$ (figures 2).

When $\omega t = \pi$ then fixing the other parameters as in figure 1, a decrease in the magnetic parameter reduces the convection velocity at all corresponding points in the flow field (figure 3) in both Newtonian and non-Newtonian cases but the reverse trend is observed in figure 4 when we consider $M = 1$ and keep the other parameters same as in figure 2.

Also, the figure 3 shows that an increase in the visco-elastic parameter enhances the convection velocity at corresponding points in the flow field in case of cooling i.e. $G > 0$ but the figure 4 reveals the opposite pattern in case of heating i.e. $G < 0$.

The figure 5 shows the variation of the skin friction coefficient τ against the Grashoff number G. It is seen that the skin-friction increases for the increasing values of the thermal buoyancy G in both Newtonian and non-Newtonian cases. Further, the increase of visco-elastic parameter reduces the skin friction at all corresponding points in the flow field for fixed values of other flow parameters. Also, it is noticed that the rate of heat transfer is not significantly affected by the visco-elastic parameter.

5. Graphs for Velocity and Skin Friction Coefficient

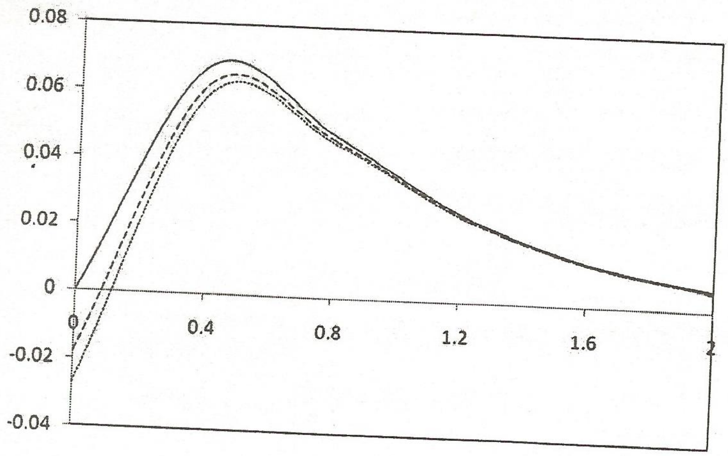

Figure 1. Variation of u against y for
$G = 2.0, P = 3, M = 2, \epsilon = 0.1, K = 1.5, A = 0.5, \omega t = \pi/2$

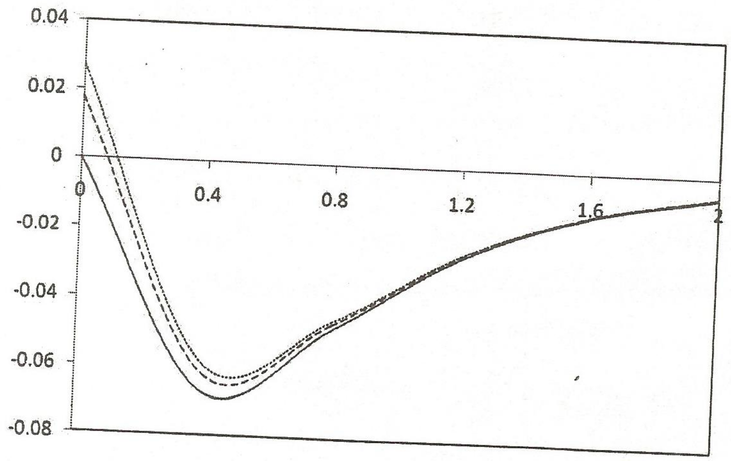

Figure 2. Variation of u against y for
$G = -2.0, P = 3, M = 2, \epsilon = 0.1, K = 1.5, A = 0.5, \omega t = \pi/2$

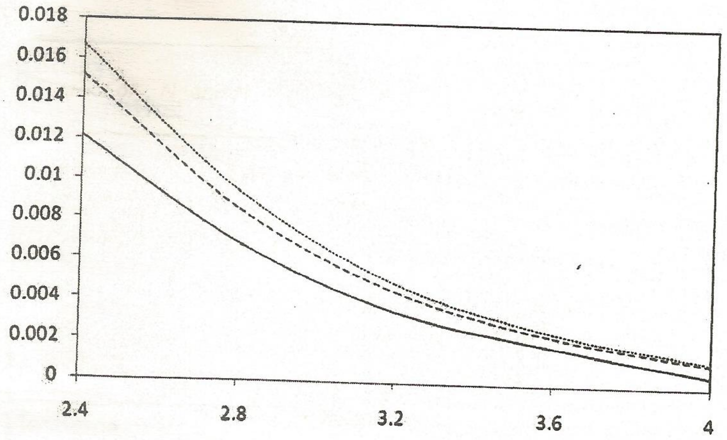

Figure 3. **Variation of** u **against** y **for**
$G = 2.0, P = 2, M = 1, \epsilon = 0.1, K = 1.5, A = 0.5, \omega t = \pi$

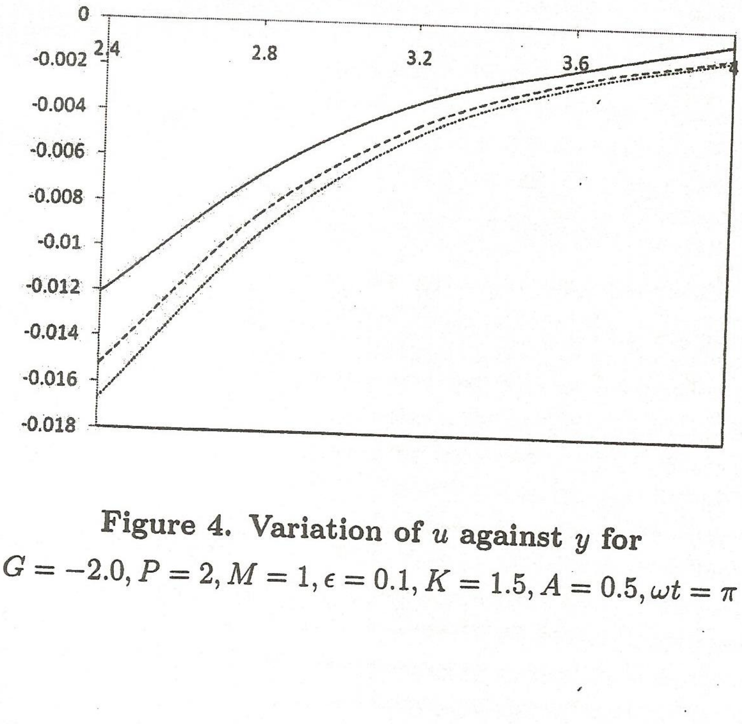

Figure 4. **Variation of** u **against** y **for**
$G = -2.0, P = 2, M = 1, \epsilon = 0.1, K = 1.5, A = 0.5, \omega t = \pi$

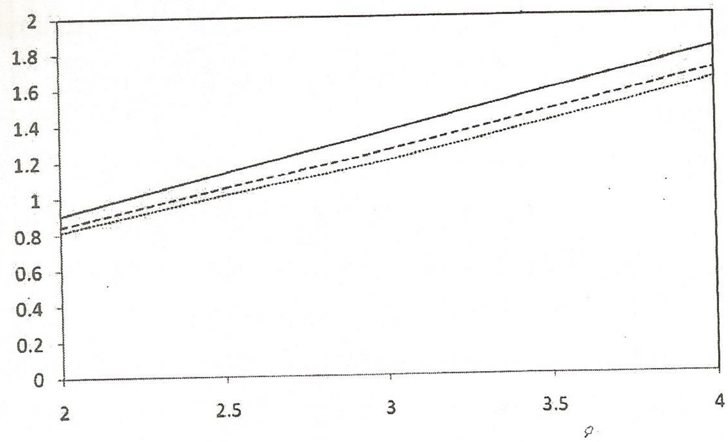

Figure 5. Variation of τ against G for
$P = 3, M = 2, ? = 0.1, K = 1.5, A = 0.5, \omega t = \pi/2$

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