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Digital solution of temperature distribution in an electrical machine using network analogy

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Abstract. An analytical method which predicts the temperature rise of the rotor and stator packets, and the associated endwindings of an induction motor is presented. The method also enables the prediction of the temperature rise of the coolant at discrete points in its flow path through the motor.

Numerische Lösung der Temperaturverteilung in elektrischen Maschinen mittels der Netzwerk-Analogie

Zusammenfassung. Eine analytische Methode, die den Temperaturanstieg in den Rotor- und Statorwicklungen und im zugehörigen Wicklungskopf eines Induktionsmotors berechnet, wird vorgestellt. Das Verfahren erlaubt auch die Ermittlung des Temperaturanstieges des Kühlmittels an bestimmten Punkten des Motors.

Nomenclature

C_p	specific heat of air $\frac{J}{kg\,K}$
F_x	xth fluid node
I, i	current flow, Amp.
m	mass flow rate of coolant, kg/s
Q	heat flow, W
R	resistance; °C/W (thermal), ohm (electrical)
T	temperature, °C
V	electrical potential, volts.

Subscript

e	electrical
t	thermal

1 Introduction

With continuous improvement in the rating of electrical machines, greater emphasis is being given to the temperature limitation of the windings, or more correctly, of the insulations. Under all circumstances, the windings must be kept below the maximum permissible temperature of the insulations to maintain reliability and prolong durability of the machine. On the other hand, a machine with temperatures too low would imply an uneconomic use of material. From this point of view, designers as well as researchers have mostly concentrated on the temperature rise of the windings treating the coolant temperature as constant through its travel in the machine. Little attention has been paid to the possibility that temperature rise of the fluid carrying away the heat generated in the active parts of the machine could influence the local heat transfer.

Air is one of the most common coolants used in electrical machines because of its simplicity in handling. During its travel from the inlet to the exit of the machine, the air passes through every available passage and picks up heat from the machine surfaces coming in contact with it.

The total rise in temperature of the coolant can be calculated from simple heat balance, at least as a first approximation, assuming that the heat generated in the machine is completely carried away by the coolant. Knowing the volume of air delivered by the fan and the inlet air conditions, a heat balance gives the probable temperature rise of the cooling air. The information is necessary because the modern machines mostly employ a closed ventilation circuit in which a cooler (heat exchanger) is used to remove heat (by using a secondary fluid) from the coolant and bring it back to the condition at which it enters the machine. The contamination of the primary fluid, i.e., the coolant is thus avoided. The size and power required for pumping the coolant fluid are important criteria for the economic design of the equipment.

2 Importance of considering coolant temperature rise

Thermal analysis of electrical machines assuming constant coolant temperature leads to only approximate values which are somewhat different from actual values. At the inlet to the machine the coolant is comparatively cold and, so, able to pick up more heat, the rate of which continously falling with its gradual progress through the machines. Considerable error in estimation of the winding temperature could result if the coolant temperature rise is neglected.

The flow of heat from the hot machine parts to the coolant is a function of the temperature difference existing between the machine part and the coolant. If, at every location, the coolant temperature is assumed constant, the heat flow will be overestimated.

2.1 Constant vs. variable coolant temperature

Let us assume that at a particular location, the coolant is at a constant temperature T_F, which is also its temperature at inlet to the machine. Let the predicted machine surface temperature be T_M. For a given rate of heat loss Q from the machine surface,

$$Q = \alpha (T_M - T_F) \tag{1}$$

In practice, as the coolant progresses through the machine, it picks up heat and its temperature increases, i.e., the actual temperature at the location referred to is not T_F but say T_F' and obviously, $T_F' > T_F$. Let the predicted machine surface temperature under this condition be T_M'. Then for the same heat loss condition.

$$Q = \alpha (T_M' - T_F') \tag{2}$$

Since the approximation of variable coolant temperature represents the physics of the problem more accurately, the machine temperature T_M' differs very little from the actual machine temperature, within the constraints of the numerical method of analysis.

It can be observed from above that predicted machine temperature, T_M, assuming constant coolant temperature rise is less than machine temperature T_M' when the coolant temperature rise is neglected. This establishes the importance of considering the rise in coolant temperature along the flow passage. A review of the literature shows that this aspect of the detailed estimation of the coolant temperature at salient positions in the machine mostly went unnoticed. It is the aim of this paper to take into consideration this aspect of the fluid temperature distribution and obtain a more realistic and accurate temperature distribution in the machine.

2.2 The VCVS

By treating the coolant to be at a constant temperature throughout the machine the heat is assumed to flow from the machine to an infinite sink. The temperature rise of the coolant is dependent on the heat supplied to it, which, in turn, dependent upon the temperature differential between the machine part and the coolant. This thermal phenomenon can be translated into an equivalent electrical analogy as a Voltage Controlled Voltage Source (VCVS).

2.2.1 The machine model

A section of the machine along with the coolant flow passages is shown in Fig. 1.

The entire amount of air delivered or sucked (as the case may be) by the fan first flows over the endwindings (A, C) at inlet. It then enters the spider annulus (L) where from it continuously branches out in the radial (H) and axial (L) directions and fills up the radial ducts between packets as well as the axial spider passage. At the back of stator core (K), flow from each radial duct joins that from the previous duct and, thus, at the end of the last radial channel (KN), the flow is the same as at entry. This total volume then flows over the end windings at the exit (B, D) from where it passes on to the heat exchanger and again enters the machine after getting cooled.

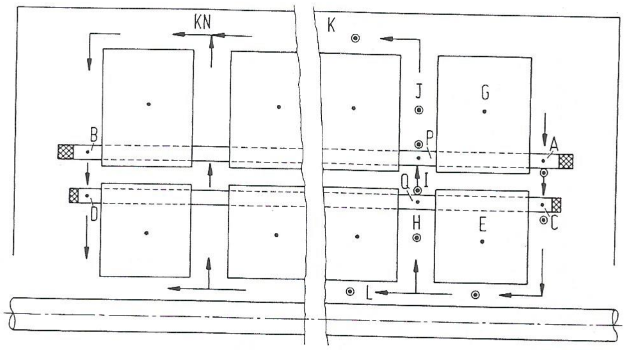

Fig. 1. The machine model; A, B – Stator endwindings, C, D – Rotor endwindings, E – Rotor packet, G – Stator packet, H – Radial channel in rotor, I – Airgap, J – Radial channel in stator, K – Back of stator core, L – Spider annulus, KN – End of last radial channel, P – Stator copper in channel, Q – Rotor copper in channel, ● – Lump nodes, o – Fluid nodes

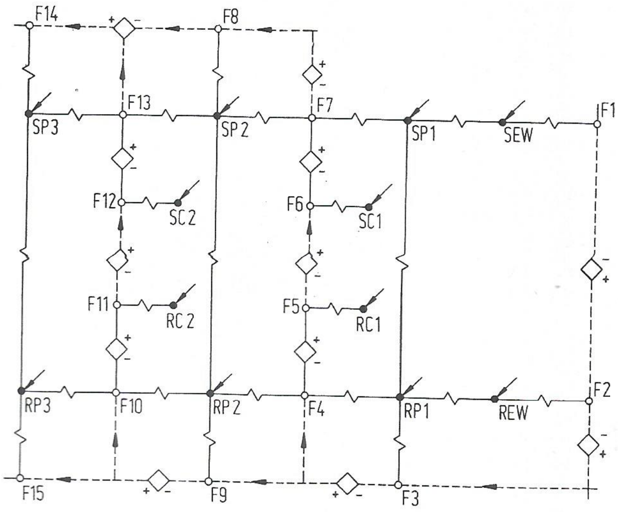

Fig. 2. Equivalent electrical circuit for the machine model; RPN – Node at the Nth rotor packet, SPN – Node at the Nth stator packet, RCN – Copper node at the Nth rotor duct, SCN – Copper node at the Nth stator duct, SEW – Node at stator endwinding, REW – Node at rotor endwinding, FX – Xth fluid node. ◇ – Voltage controlled source, – – → – – Coolant flow path, arrows indicate current injection at nodes

2.2.2 The equivalent electrical circuit

The model as envisaged in Fig. 1 shows the lumped machine components along with the air flow path for a section of the machine. This can be represented by an equivalent electrical circuit as shown in Fig. 2.

The following assumptions are made in the construction of the circuit shown in Fig. 2.

1. Each packet is considered as a single lump and the temperature of the lump is uniform throughout. This holds good for both stator and rotor.

2. Total heat loss from the packet is concentrated at the packet node.

3. Heat losses from the conductors in the end windings and in the radial ducts are concentrated at the corresponding nodes.

4. Axial flow of air in the rotor-stator air gap is not considered as flow in this direction is negligible. Heat transfer takes place between the rotor and the stator through the gap by convection.

5. Temperature variation in the tangential direction is neglected because of axisymmetry, except in the end windings, where heat flow in the tangential direction is appreciable.

2.3 Single and multiple heat sources

(a) Single heat source

Figure 3 shows the equivalent electrical circuit for an electrical machine when the coolant flowing past a node receives heat from a single source only. This situation occurs when heat from each rotor packet is given to the fluid flowing through the spider annulus.

The thermal resistance offered to heat flow is characterised by the electrical resistances R_{F_1} and R_{F_2}. These resistances are the electrical equivalent of the sum of the conductive and convective thermal resistances. I_1 and I_2 are the electrical equivalents of the heat generated at the respective nodes. The fluid flow direction is as shown by the arrows. Because of I_1 at node 1, the fluid gains heat while passing from F_1 to F_2, implying a voltage gain between the nodes. The amount of this gain depends upon the quantity of coolant flow and i_1, the fraction of I_1, flowing towards the coolant through the resistance R_{F_1}.

Now,

$$Q_1 = m_1 C_p \Delta T_F \tag{3}$$

where

Q_1 = radial heat flow from the rotor packet 1 to the coolant m_1 in the spider annulus and is analogous to current i_1,

m_1 = mass flow rate of coolant while flowing past node F_1,

ΔT_F = rise in coolant temperature between coolant nodes F_1 and F_2 and is analogous to V_F.

Equation (3.10) can be written as,

$$Q_1 = \Delta T_F/(1/m_1 C_p) = \Delta T_F/R_t \tag{4}$$

where

$$R_t = 1/(m_1 C_p) \tag{5}$$

The general electrical equivalent of Eq. (4) is,

$$I = \Delta V_F/R_e \tag{6}$$

Applying Kirchhoff's current law at node F_1,

$$i_3 = i_1 = (V_1 - V_{F_1})/(R_{F_1})_e \tag{7}$$

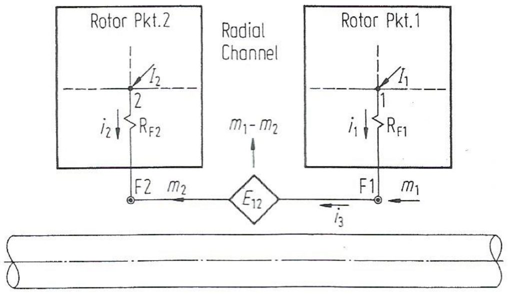

Fig. 3. Equivalent electrical circuit when the coolant receives heat from a single source. F_1, F_2 = Coolant nodes, 1, 2 = Rotor packet nodes, I, i = Current; electrical equivalent of heat generation, m_1, m_2 = Mass flow rate of coolant, E_{12} = Voltage controlled voltage source

Then E_{12} which represents the voltage gain between F_1 and F_2, can be expressed as,

$$E_{12} = V_{F_2} - V_{F_1}$$
$$= i_3 \cdot (R_{12})_e$$
$$= (R_{12})_e (V_1 - V_{F_1})/(R_{F_1})_e \tag{8}$$

where

$(R_{12})_e$ = electrical equivalent of thermal resistance between nodes F_1 and F_2, and is numerically equal to $(R_{12})_t$.

It is seen from Eq. (8) that the voltage gain between the two coolant nodes F_1 and F_2 is controlled by the voltage difference between nodes 1 and F_1. Thus, E_{12} is the VCVS between coolant nodes F_1 and F_2 and $(V_1 - V_{F_2})$ is the controlling voltage.

Thermally Eq. (8) is equivalent to

$$\Delta T_F = T_{F_2} - T_{F_1}$$
$$= (R_{12})_t (T_1 - T_{F_1})/(R_{F_1})_t . \tag{9}$$

2.4 Multiple heat sources

The situations inside the radial ducts and at the back of the stator core are different from that in the spider annulus, as the coolant, while flowing through these passages, picks up heat from multiple sources. In the radial duct, the coolant picks up heat from its two adjacent packets while at the back of the stator core, the coolant receives heat not only from the adjoining stator packet, but also from the coolant approaching it from the preceding radial ducts. Such a situation is shown in Fig. 4. It can be noted that,

$$m_3 = m_1 + m_2 \tag{10}$$

The mixing temperature T_3' may be estimated from,

$$T_3' = (m_1 T_1 + m_2 T_2)/(m_1 + m_2) \tag{11}$$

Substituting (10) in (11),

$$T_3' = (m_1/m_3) T_1 + (m_2/m_3) T_2 \tag{12}$$

If T_1' be the temperature of coolant m_1 before it reaches F_1, then

$$T_1 = T_1' + Q_1/(m_1 C_p) + Q_1'/(m_1 C_p) \tag{13}$$

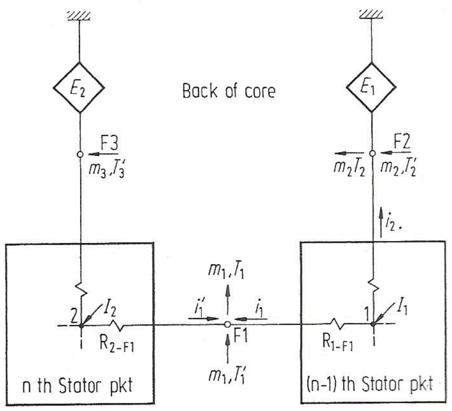

Fig. 4. Equivalent electrical circuit when the coolant receives heat from multiple sources; T_1 = Fluid temperature at node F_1, T_2 = Fluid temperature at node F_2; T_1' and T_2' = Fluid temperature just before nodes F_1 and F_2 respectively, T_3' = Fluid temperature after mixing of m_1 and m_2 just before reaching F_3, m_1, m_2, m_3 = Mass flow rate of coolant at different locations, 1, 2 = Stator packet nodes, F_1, F_2, F_3 = Coolant nodes

where

Q_1 = axial heat flow from $(n-1)^{th}$ stator packet to coolant m_1 and is analogous to i_1,

Q_1' = axial heat flow from n^{th} stator packet to coolant m_1 and is analogous to I_1'.

Similarly, if T_2' be the temperature of coolant m_2 before it reaches F_2, then

$$T_2 = T_2' + Q_2/(m_2 C_p) \tag{14}$$

where

Q_2 = radial heat flow from $(n-1)^{th}$ stator packet to coolant m_2.

Using Eq. (13) and (14) in Eq. (12), one gets,

$$T_3' = (m_1/m_3)[T_1' + Q_1/(m_1 C_p) + Q_1'/(m_1 C_p)] + (m_2/m_3)[T_2' + Q_2/(m_2 C_p)]. \tag{15}$$

The electrical equivalent of Eq. (15) will be

$$V_3' = (\beta_3/\beta_1)[V_1' + \beta_1(i_1 + i_1')] + (\beta_3/\beta_2)[V_2' + \beta_2 i_2] \tag{16}$$

where V_3', V_1' and V_2' are potentials at F_3, F_1 and F_2 respectively with respect to ground, and

$$\beta_n = 1/(m_n C_p), \quad n = 1, 2, 3 \ldots \tag{17}$$

From Fig. 4, one can write,

$$i_1 = (V_1 - V_{F_1})/(R_{1-F_1})_e, \tag{18}$$

$$i_1' = (V_2 - V_{F_1})/(R_{2-F_1})_e, \tag{19}$$

$$i_2' = (V_1 - V_{F_2})/(R_{1-F_2})_e. \tag{20}$$

Since $V_{F_1} = V_1'$ and $V_{F_2} = V_2'$, equation (16) becomes

$$V_3' = (\beta_3/\beta_1)[V_{F_1} + \beta_1(V_1 - V_{F_1})/(R_{1-F_1})_e + (V_2 - V_{F_1})/(R_{2-F_1})_e] + (\beta_3/\beta_2)[V_{F_2} + \beta_2(V_1 - V_{F_2})/(R_{1-F_2})_e]. \tag{21}$$

In Fig. 4, E_2 is the voltage controlled voltage source that represents the potential of V_3'. This potential plus the drop between nodes 2 and F_3 will be the energy of the coolant beyond node F_3 in the direction of coolant flow. It is clear from Eq. (21) that the VCVS E_2 is linearly controlled by five controlling voltages, namely, those between V_{F_1} and ground, V_1 and V_{F_1}, V_2 and V_{F_1}, V_{F_2} and ground, and V_1 and V_{F_2} with their respective gains as β_3/β_1, $\beta_3/(R_{1-F_1})_e$, $\beta_3/(R_{2-F_1})_e$, β_3/β_2 and $\beta_3/(R_{1-F_2})_e$.

Thermally, Eq. (21) is equivalent to,

$$T_3' = (\beta_3/\beta_1)T_{F_1} + \beta_3(T_1 - T_{F_1})/(R_{1-F_1})_t + \beta_3(T_2 - T_{F_1})/(R_{2-F_1})_t + (\beta_3/\beta_2)T_{F_2} + \beta_3(T_1 - T_{F_2})/(R_{1-F_2})_t. \tag{22}$$

2.5 *Solution*

The network for the complete machine based on the above formulation is shown in Fig. 5. It has been solved by using a package programm SPICE (Simulated Program with Integrated Circuit Emphasis) which is a versatile program used to solve electronic circuits. The use of this software package in solving thermal networks for electrical machines is unique.

3 Results and discussions

The temperature distribution for the complete machine is shown in Fig. 6. It gives the rise in temperature of the packet and endwindings nodes above the coolant temperature. The temperature of each successive packet increases in the direction of air flow. This is due to the decrease in the driving potential for heat transfer between the packet element and the cooling medium. The possible reason for the first stator packet being at a much lower temperature than the second is due to a considerable flow of heat from this packet to the relatively colder associated endwindings.

Temperature distribution for the rotor packets follows an almost similar pattern as that for the stator packets. For both, the trend of increasing temperature for successive packets continues upto the last but one packet. In other words contrary to normal expectation, it is the last but one packet which is the hottest and not the last one. As the endwindings are washed away by the entire volume of fluid flowing through the machine, each endwinding layer is at a lower temperature than the corresponding adjoining packet. This confirms axial flow of heat in the copper conductors. The endwindings at outlet are higher than those at inlet. This too is due to decrease in the driving potential between the endwindings element and the cooling medium.

The temperature distribution (rise over machine ambient) of the cooling medium has been plotted in Fig. 7. The curve (a) gives the temperature rise of the coolant during its passage through the axial spider annulus. A point on the

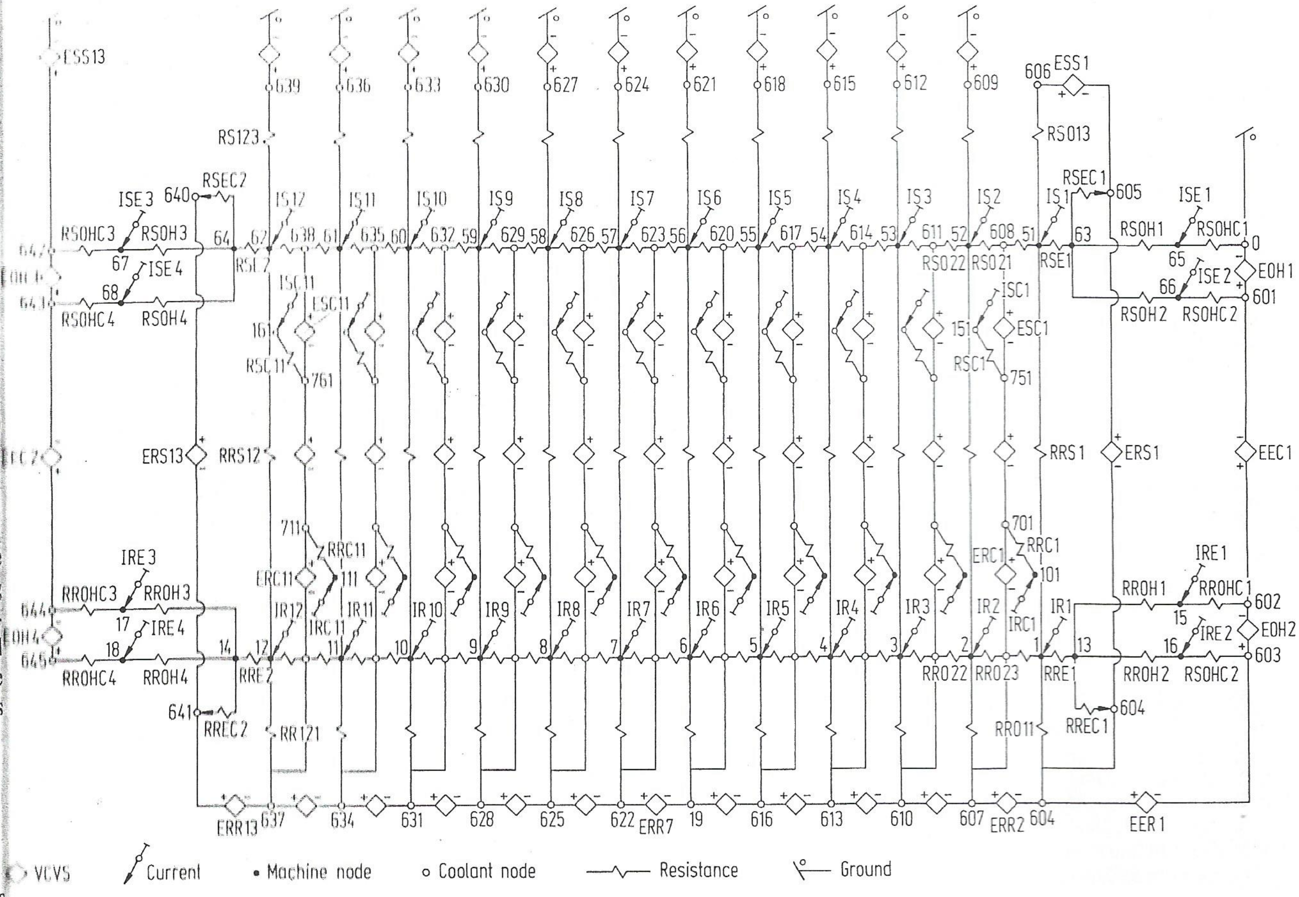

Fig. 5. 2-D network for the complete machine

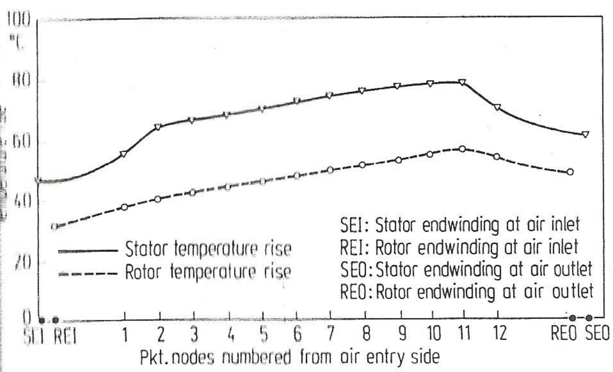

Fig. 6. Temperature distribution in the complete machine

urve corresponds to the coolant temperature below a rotor packet after the fluid has picked up radial heat from the packet. Curve (b) gives the coolant temperature in the radial channels after it has removed the axial heat from the rotor packets. ΔT between curves (a) and (b) continuously increases because of gradually decreasing mass flow of air through the successive radial channels. The coolant also removes heat from the bare conductors in the channels. The temperature of the coolant after it has moved past the bare rotor and stator conductors are given by lines (c) and (d) respectively. The graphs (b), (c) and (d) are similar because they give the coolant temperature at different locations in the radial channels and mass flow of coolant remains unchanged in a channel. The curve (e) gives the coolant temperature at the back of the stator core. In fact, this is the coolant temperature after it has removed heat from the stator packets in both axial and radial direction.

The coolant temperature distribution along a radial duct is plotted in Fig. 8. Due to repetitive nature, only three curves have been plotted for the first, fifth & eleventh radial duct. The temperature along any duct has a positive gradient due to continuous removal of heat by the cooling medium. The major contributions to the coolant temperature rise come from the rotor and stator packet elements. Heat given up by bare conductors in the ducts has very little effect on the coolant temperature rise.

The temperature rise obtained for the packets and the coolant were compared with those obtained from the industry on an actual 600 kW Induction motor, and the agreement found was fairly good.

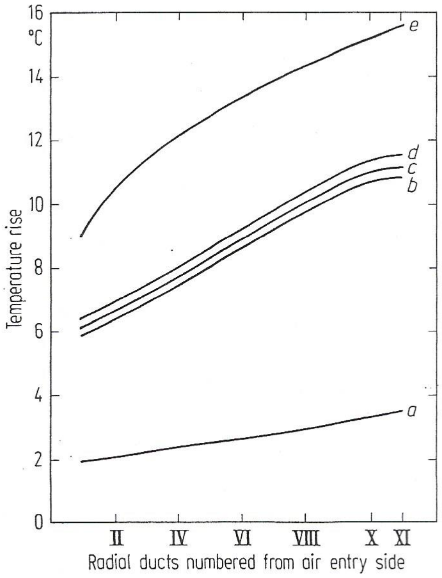

Fig. 7. Axial distribution of coolant temperature

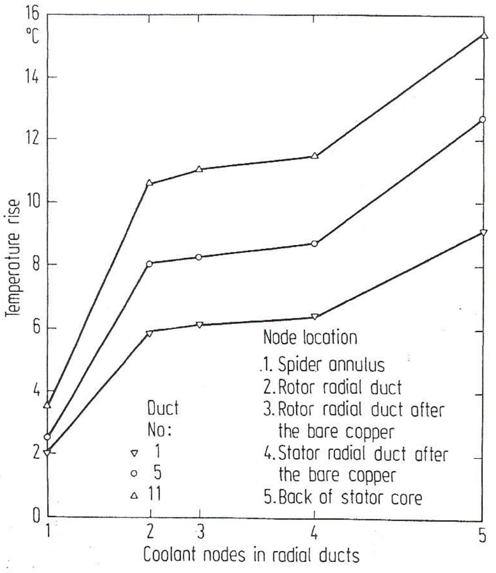

Fig. 8. Radial distribution of coolant temperature

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