

Total number of printed pages— 7

47 (1) BMAT 1·3

2009

BUSINESS MATHEMATICS

Paper : 1·3

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any five from the rest.

1. (a) 10×1=10

(i) Evaluate $\log_2 \log_3 \log_2 512$.

(ii) If $\log_2 = \cdot 3010$ find the number of digits in 4^{25} .

(iii) Evaluate ${}^{10}C_8$.

(iv) State *two* properties of determinants.

(v) Define unit matrix with example.

(vi) Find tenth term of the series
2, 4, 8, 16,

1

Contd.

(vii) Define odd function with example

(viii) If $y = \sqrt{x}$, find $\frac{dy}{dx}$.

(ix) Integrate $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$.

(x) Evaluate $\int_0^4 (x^2 - 3x + 1) dx$.

(b)

(i) If $x^p = y^q = (xy)^{pq}$, find $p+q$

(ii) Find middle term of $\left(x + \frac{1}{x^2}\right)^{14}$

(iii) Evaluate

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix}$$

(iv) Evaluate

$$\lim_{x \rightarrow 2} \frac{\sqrt{x+7} - 3}{(x-2)}$$

(v) The total cost $C(x)$ of a firm is
 $C(x) = 0.0005x^3 - 0.7x^2 - 30x + 3000$
where x is the output.
Determine Average Cost (AC) and
Marginal Cost (MC).

(a) If $x = 1 + 3^{\frac{2}{3}} + 3^{\frac{1}{3}}$
prove that $x^3 - 3x^2 - 6x - 4 = 0$. 3

(b) Solve : (any two) 3+3

(i) $2\sqrt{x+5} - \sqrt{2x+8} = 2$

(ii) $4^x - 3 \cdot 2^{x+2} + 32 = 0$

(iii) $2x + 3y = 1$
 $x^2 - xy = 6$.

(c) If α, β are roots of $x^2 + px + q = 0$ then
form the equation whose roots are
 $\alpha^2\beta$ and $\alpha\beta^2$. 3

(d) If $\log_2 x + \log_4 x + \log_8 x = 11$
find x . 4

(b) An enterprise produced 600 units in its third year of its existence and 700 units in its seventh year.

(i) What is the initial production in the first year?

(ii) What was the total production in the first five years? (Use A. P.)

(c) Find the coefficient of x^z in the expansion of $(2x^2 + \frac{1}{4x})^{11}$.

4. (a) Using matrices, solve the equation

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x - y + z = 2$$

(b) If

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \text{ show that}$$

$$A^2 - 4A - 5I = 0, \text{ where } I \text{ is the } 3 \times 3 \text{ identity matrix.}$$

(c) Show that

$$\begin{vmatrix} 1 & x & x^2 - yz \\ 1 & y & y^2 - zx \\ 1 & z & z^2 - xy \end{vmatrix} = 0$$

5. (a) If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 $A = \{1, 2, 3, 4\}$ $B = \{2, 4, 6\}$
 $C = \{1, 2, 5\}$ find $A' \cap (B \setminus C)$
and $A \setminus (B' \setminus C')$.

(b) If $n(H \cap E) = 11$; $n(H - E) = 22$
 $n(E - H) = 12$, find $n(H)$, $n(E)$ and
 $n(H \cup E)$.

(c) If $f(x) = \frac{3x+2}{3x-2}$ show that

$$\frac{f(x)+1}{f(x)-1} = \frac{3x}{2}$$

6. (a) State the limitation of linear programming.

(b) Maximize $z = 30x + 20y$
Subject to

$$2x + 3y \leq 36$$

$$5x + 2y \leq 50$$

$$x + 3y \leq 30; \quad x, y \geq 0.$$

7. (a) Evaluate

$$(i) \lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 + x - 6}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-3x}}{x}$$

(b) A function $f(x)$ is defined as

$$f(x) = \begin{cases} 2x + 6 & \text{when } -3 \leq x \leq 0 \\ 6 & \text{when } 0 < x < 2 \\ 2x - 6 & \text{when } 2 \leq x \leq 5 \end{cases}$$

Examine the continuity of the function at $x = 0$ and $x = 2$.

(c) Find $\frac{dy}{dx}$, (any two)

$$(i) y = x^3 (\log x)^2$$

$$(ii) y = \sqrt{3x^4 + 5}$$

2+2

$$(iii) xy + x^3 + y^3 = 5$$

8. (a) A firm has the following Total Revenue (TR) and Total Cost (TC) functions as

$$TR = 100x - x^2$$

$$TC = x^3 - \frac{57}{2}x^2$$

where x is the level of output. Determine the maximum profit.

(b) Integrate:

$$(i) \int \frac{2x+4}{2x^2+8x-10} dx$$

$$(ii) \int \sqrt{5x-3} dx$$

(c) Evaluate:

$$\int_2^4 \frac{x}{1+x^2} dx$$

(d) The Marginal Cost (MC) of a product is given by

$$MC = \text{Rs} (25 + 30x - 9x^2)$$

and the fixed cost is known to be Rs. 550. Find the total cost function.