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47 (1) BMAT 1·3

2010

BUSINESS MATHEMATICS

Paper : 1·3

Full Marks : 80

Time : Three hours

**The figures in the margin indicate full marks
for the questions.**

Answer question No. *1 & 2* and **any five** from the rest.

1. Choose the correct alternative :   1×10=10

    (a) Which of the following collections can be
         defined as a set ?

         (i)   All long rivers in the world

         (ii) All books which are fun to read

         (iii) All odd numbers < 100

         (iv) None of these.

(b) If the roots of the quadratic equation $ax^2 + bx + c = 0$ are equal then —

    (i) $b^2 - 4ac = 0$

    (ii) $b^2 - 4ac > 0$

    (iii) $b^2 - 4ac < 0$

    (iv) Any of the above.

(c) If two rows (or columns) of a determinant are identical, the value of the determinant becomes —

    (i)   0

    (ii) 1

    (iii) 2

    (iv) Can't say.

(d) If $\log_3 27 + \log_3 x = 4$; then x is

    (i)   1

    (ii) 2

    (iii) 3

    (iv) 4

(e) In an LPP

    (i)   Only the objective function is linear

    (ii) Only the constraints are linear

    (iii) The objective function as well as the constraints are linear

    (iv) None of the above.

(f) If TC, MC, AC and x represent the total cost, marginal cost, average cost and output respectively, then which of the following are true ?

    I. $\dfrac{d}{dx}(TC) = MC$

    II. $\dfrac{TC}{x} = AC$

    III. $\int MC\, dx = TC$

    (i)   I and II

    (ii) II and III

    (iii) I and III

    (iv) I, II and III.

(g) If $f(x) = 3x + 5$, $g(x) = 2x + 9$ and $h(x) = 8x + 3$, then which of the following is true ?

    (i) $f(x) = g(x)$ at $x = 3$

    (ii) $f(x) = h(x)$ at $x = 4$

    (iii) $g(x) = h(x)$ at $x = 2$

    (iv)   None of these.

(h) Which of the following equations has sum of the roots $= 13/6$ and product of the roots $= 1$

    (i) $3x^2 - \dfrac{13}{6}x + 3 = 0$

    (ii) $6x^2 - 13x - 6 = 0$

    (iii) $12x^2 - 26x + 12 = 0$

    (iv) $x^2 + \dfrac{13}{6}x + 1 = 0$

(i) Consider the following sets :

$A = \{1\}$, $B = \{0\}$, $C = \{ \ \}$, $D = \{1, 2, 3\}$
$E[0, 1, 2]$, $F[2, 3]$ $G[3, 2]$, then which of the following is true ?

    (i) A & B are singleton sets

    (ii) B & C are null sets

    (iii) D & E are equal sets

    (iv) F is a proper subset of G.

(j) For the optimum solution of an LPP involving three variables, we can use —

    (i)   Graphical method

    (ii)   Simplex method

    (iii)   Either (i) and (ii)

    (iv)   None.

2.   Answer the following : *(any five)* $2 \times 5 = 10$

(a) Define odd function and even function with examples.

(b) Prove that $\log_2 \log_2 \log_3 81 = 1$

(c) Find the middle term in the expansion of $\left(\dfrac{a}{x} + \dfrac{x}{a} \right)^{10}$.

(d) Prove that
$$\left[\dfrac{x^m}{x^n} \right]^{m+n} \times \left[\dfrac{x^n}{x^l} \right]^{n+l} \times \left[\dfrac{x^l}{x^m} \right]^{l+m} = 1$$

(e) IF the total profit function is given by $L = Q^2 + 13Q + 78$; where Q is the output, find average profit when $Q = 3$.

(f) The second term of a GP series is 9 and the fifth term is 243; find the common ratio.

(g) Solve $2^{x+3} + 2^{x+1} = 320$.

3. (a) If $a^x = b^y = c^z$ and $b^2 = ac$ then show that
$$\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$$
3

(b) Find the quadratic equation with $2 - \sqrt{3}$ as one of its root.
3

(c) If $a^2 + b^2 = 7ab$ then show that
$$2\log(a+b) = \log a + \log b + 2\log 3$$
3

(d) If one root of $x^2 - px + q = 0$ is twice the other, show that $2p^2 = 9q$.
3

4. (a) Evaluate :
$2 \times 4 = 8$

(i) $\displaystyle \lim_{x \to 2} = \frac{x^2 - 5x + 6}{x^2 - 4x + 3}$

(ii) $\displaystyle \lim_{x \to \infty} = \frac{4x^3 + 5x - 1}{7x^3 - 3x^2 + 8x}$

(iii) $\displaystyle \lim_{n \to \infty} = \frac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{n^3}$

(iv) $\displaystyle \lim_{x \to 0} = \frac{1 - \sqrt{1 - x^2}}{x^2}$

(b) The cost function $C(x)$ of a product is given by —
$$\begin{aligned} C(x) &= 2000 - x \quad \text{when } x < 500 \\ &= 1000 \quad\quad\;\; \text{when } x = 500 \\ &= 3000 - 2x \quad \text{when } x > 500 \end{aligned}$$
Is $C(x)$ continuous of $x = 500$.
4

5. (a) Find derivative dy/dx : *(any three)*
$2 \times 3 = 6$

(i) $y = \log x \left(3x^2 - 4x + 1\right)$

(ii) $x = z^2$, $y = 2/z$

(iii) $y = \left(x + 1/x\right)^5$

(iv) $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

(b) A monopolist has a demand curve $x = 106 - 2P$ and the average cost curve $AC = 5 + \frac{x}{50}$, where P is the price per unit output and x is the number of units of output. If the total revenue is $TR = xP$, determine the most profitable output and the maximum profit. 6

Or

The average cost function AC for producing x unit of a commodity is $AC = \frac{60}{x} - 12 + 2x$. Find the level of output at which the total cost (TC) is minimum. Also find the minimum total cost. 6

6. (a) What do you mean by LPP ? What are the basic assumptions of LPP ? 2+4=6

(b) Solve the LPP by graphical method

Maximise $Z = 10x_1 + 15x_2$

subject to $2x_1 + x_2 \le 26$

$2x_1 + 4x_2 \le 56$

$x_1 - x_2 \ge -5$

$(x_1, x_2) \ge 0$ 6

7. (a) Find the term independent of x in the expansion $\left(3x^2 - \frac{1}{3x}\right)^9$. 3

(b) Determine the coefficient of x^8 in the binomial expansion of $(2 - x^2)^6$. 3

(c) Using binomial theorem, find the value of $(1 \cdot 02)^5$ correct to 3 places of decimal. 3

(d) Prove that

$C_1 + 2C_2 + 3C_3 + \ldots\ldots + nC_n = n.\,2^{n-1}$ 3

8. Answer *any three* : 4×3=12

(a) Find the inverse of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$

(b) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

(c) Solve the following system of equations by elementary row operation method

$x + 2y - 2z = 1$, $3x + y + 4z = 17$

$5x - 3y + z = 2$.

(d) The table below gives the price for share of two companies A & B during the month of January and February 1990 and it also gives the amount invested by Mr. X during these two months for the purchase of shares of the two companies.

Month	Company and value per share		Total amount invested
	A	B	
JAN	10	5	125
FEB	9	12	150

Find he number of shares of A and B purchased during both the months using Cramer's rule.

9. Answer *any three* :

4×3=12

(a) Mr. X repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then he increases the payment by Rs. 15 every month. How long did he take to clear this loan ?

(b) The sum of four terms in A.P. is 40 and the sum of their squares is 480. Find the numbers.

(c) Find the sum of n terms of the series

$4 + 44 + 444 + \ldots\ldots$

(d) If a, b, c are in A.P. and x, y, z are in G.P. then prove that

$$x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$$

10. (a) If $\phi(x) = \dfrac{x-1}{x+1}$

then show that, $\dfrac{\phi(a) - \phi(b)}{1 + \phi(a) \cdot \phi(b)} = \dfrac{a-b}{1+ab}$.

3

(b) Draw the graph of the following function

3

$$f(x) = |x|$$

(c) In a hostel, out of 110 students, 80 takes tea, 50 takes coffee and 25 takes both tea and coffee. How many students take neither tea nor coffee ?

3

(d) Verify by using Venn diagram

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

3

11. (a) Integrate the following : *(any three)*

$$3 \times 3 = 9$$

(i) $\int \dfrac{5x^4 - 6x^2 + 7x - 1}{\sqrt{x}}\, dx$

(ii) $\int x \cdot e^x\, dx$

(iii) $\int \dfrac{dx}{\sqrt{7x + 2}}$

(iv) $\int \dfrac{3x + 5}{x^2 + x - 12}\, dx$

(b) If the marginal revenue function is Rs $(7x + 9)$ where x denotes the number of units sold, find the total revenue function and the demand function.

(take p to be price per unit). 3

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