2011

BUSINESS MATHEMATICS

Paper : 1.3

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer question Nos. *1* and *2* and *any five* from the rest.

1. Choose the correct alternative : $1 \times 10 = 10$

    (a) If N be a set of natural numbers, then $\{0\} - N$ is

    (i) $\{0\}$

    (ii) $\{1, 2, 3, 4, \ldots\}$

    (iii) ϕ

    (iv) $\{0, 1, 2, 3, 4, \ldots\}$

(b) If α and β be the roots of the equation $2x^2 - 5x + 2 = 0$ then the value of $\alpha^2 + \beta^2$ is

(i) $\dfrac{16}{4}$

(ii) $\dfrac{17}{4}$

(iii) 17

(iv) $\dfrac{5}{2}$

(c) If two rows (or columns) of a determinant are identical, the value of the determinant becomes

(i) 0

(ii) 1

(iii) 2

(iv) Can't say.

(d) If $\log x^2 + \log x^4 + \log x^{64} = 9$; then x is

(i) 2

(ii) 3

(iii) 0

(iv) 4

(e) In an LPP

(i) only the objective function is linear

(ii) only the constraints are linear

(iii) The objective function as well as the constraints are linear

(iv) None of the above.

(f) If $f(x) = 4x + 8$, $g(x) = 2x + 10$ and $h(x) = x + 2$ then which of the following is true ?

(i) $f(x) = g(x)$ at $x = 1$

(ii) $f(x) = h(x)$ at $x = 4$

(iii) $g(x) = h(x)$ at $x = 2$

(iv) None of the above.

(g) If one root of $2x^2 - 5x + k = 0$ be double the other root, then the value of k is

(i) $\dfrac{25}{9}$

(ii) $\dfrac{17}{2}$

(iii) $\dfrac{34}{3}$

(iv) 5

(h) The value of $\int_{-3}^{3} x^2 dx$ is

    (i) 18

    (ii) 36

    (iii) 0

    (iv) 54

(i) For the optimum solution of an LPP involving three variables, we can use —

    (i) Graphical method

    (ii) Simplex Method

    (iii) Either (i) or (ii)

    (iv) None

(j) If the roots of the equation $ax^2 + bx + c = 0$ are equal then

    (i) $b^2 - 4ac = 0$

    (ii) $b^2 - 4ac < 0$

    (iii) $b^2 - 4ac > 0$

    (iv) Any of the above

2. Answer the following : *(any five)*   2×5=10

    (a) Define one-to-one function and onto function

    (b) Prove that $log_{10}{}^{2500} = 2 + 2 \, log_{10}{}^{5}$

    (c) Find the middle terms of $\left(1 - 2x + x^2\right)^5$

    (d) Prove that

$$\left[\frac{x^b}{x^c}\right]^{b^2 + bc + c^2} \times \left[\frac{x^c}{x^a}\right]^{c^2 + ca + a^2} \times \left[\frac{x^a}{x^b}\right]^{a^2 + ab + b^2} = 1$$

    (e) If the total profit function is given by $\angle = Q^2 + 13Q + 78$ where Q is the output, find average profit when $Q = 3$.

    (f) The 3rd and 6th terms of a series in G.P. are 3 and 81 respectively, find the first term and common ratio.

    (g) Solve : $4^x = 2^y$, $27^{xy} = 9^{y+1}$

3. (a) If $p^a = q^b = r^c$ and $pqr = 1$ find
$ab + bc + ca$ 3

    (b) Find the values of k for which the equation $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ may have a common root.   3

(c) If $a^2 + b^2 = 14ab$ prove that

$$\log\left\{\frac{1}{\sqrt{3}}(a-b)\right\} = \frac{1}{2}(2\log 2 + \log a + \log b)$$

(d) If one root of the equation $ax^2 + bx + c = 0$ is five times the other, then find the relation between a, b and c. 3

4. (a) Evalute : 2×4=8

(i) $\lim\limits_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2}}{x}$

(ii) $\lim\limits_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 7x + 10}$

(iii) $\lim\limits_{x \to \alpha} \dfrac{3x^2 - 4x + 6}{x^2 + 6x - 7}$

(iv) $\lim\limits_{x \to \alpha} \dfrac{1^2 + 2^2 + 3^2 + \cdots + n^2}{n^3}$

(b) A function is defined in $(0, 3)$ in the following way 4

$$f(x) = x^2 \qquad 0 < x \le 1$$
$$= x \qquad 1 < x < 2$$
$$= \frac{1}{4}x^3 \qquad 2 < x < 3$$

Is $f(x)$ continuous at $x = 2$

5. (a) Find derivative $\dfrac{dy}{dx}$ *(any three)* 2×3=6

(i) $y = \sqrt{\dfrac{1+x}{1-x}}$

(ii) $x = y\log(xy)$

(iii) $x^y = y^x$

(iv) $y = x^{\log x}$

(b) The average cost function AC for producting x unit of a commodity is $AC = \dfrac{60}{x} - 12 + 2x$.

Find the level of output at which the total cost (TC) is minimum. Also find the minimum total cost. 6

Or

The daily cost of production C for x unit of an assemble is given by

$C(x) = $ Rs. $12.5x + 6400$

(i) If each unit is sold at Rs. 25, determine the minimum number of units that should be produced and sold to ensure no loss.

(ii) If the selling price is reduced by Rs. 2.5 per unit, what would be the break-even point ?

6. (a) What do you mean by LPP ? Give the advantages and disadvantages of LPP.

$2+4=6$

(b) Solve the LPP by graphical method 6

Minimize $Z = 20x + 40y$ subject to

$6x + y \geq 18$

$x + 4y \geq 12$

$2x + y \geq 10$, $x, y \geq 0$

7. (a) Find the value of the term free from x of the expansion $\left(9x^2 - \dfrac{1}{3x}\right)^{12}$ 3

(b) Find the coefficient of x^{-17} in $\left(x^4 - 1/x^3\right)^{15}$ 3

(c) Using binomial theorem, find the value of $(1.02)^5$ correct to 3 places of decimal.   3

(d) Prove that

$$C_1 + 2C_2 + 3C_3 + \cdots + nC_n = n.2^{n-1}$$

   3

8. Answer *any three* : $4 \times 3 = 12$

(a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ -2 & 1 & -1 \end{bmatrix}$$

(b) Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$$

(c) Solve the equation by matrix inversion method

$$x + y + z = 4$$

$$2x - y + 3z = 1$$

$$3x + 2y - z = 1$$

(d) The table below gives the price for share of two companies A & B during the months of January and February 1990 and it also gives the amount invested by Mr. X during these two months for the purchase of shares of the two companies

Month	Company and Value per share		Total amount invested
	A	B	
Jan	10	5	125
Feb	9	12	150

Find the number of shares of A and B purchased during both the months using Cramer's rule.

9. Answer *any three* :   4×3=12

(a) A man borrows Rs. 5115 to be repaid in 10 monthly instalments. If each instalment is double the value of the last, find the value of 1st and last instalments.

(b) The sum of three numbers in G.P. is 26 and their product is 216. Find the numbers.

(c) Find the sum of n terms of the series

7+77+777+·········

(d) If the A. M. between a and b $(a>b)$ is twice as large as their G.M. prove that the ratio between the numbers can be written as

$$2 + \sqrt{3} : 2 - \sqrt{3}$$

10. (a) If $f(x) = b\dfrac{x-a}{b-1} + a\dfrac{x-b}{a-b}$, then

$$f(a) + f(b) = f(a+b)$$ 3

(b) Prove the graph of the following function   3

$$f(x) = |x|$$

(c) In a class of 100 students, 45 students read physics, 52 students read chemistry and 17 students read both the subjects. Find the

number of students who study neither physics nor chemistry. 3

(d) Verify by using Venn diagram

$$A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$$ 3

11. (a) Integrate the following : *(any three)*

$3 \times 3 = 9$

(i) $\int \dfrac{dx}{4 + 7x}$

(ii) $\int \left(3x^2 + 2\right)^3 x\, dx$

(iii) $\int x^2 e^x\, dx$

(iv) $\int \dfrac{3x + 5}{x^2 + x - 12}\, dx$

(b) The marginal cost of producing x dozens of pencil is Rs. $\left(5x^2 - 4x + 12\right)$. If the cost of producing 2 dozens pencil is Rs. 200, find the total cost function. 3