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47(1) BMAT 1.3

2012

BUSINESS MATHEMATICS

Paper : 1.3

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question Nos. 1 and 2 and any *five* from the rest.

1. Choose the correct alternative: $1 \times 10 = 10$

(a) Which of the following collection can be defined as a set?

(i) All rivers in the world

(ii) All books which are fun to read

(iii) All odd numbers < 100

(iv) None of these

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(b) If α and β be the roots of equation $x(x-3)=4$ then the value of $\alpha^2 + \beta^2$ is

- (i) 17 (ii) 16 (iii) 2 (iv) 8

(c) The product AB of the two matrices

$$A = (1 \ 2 \ 3 \ 4) \quad B = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \text{ is}$$

- (i) 2 (ii) 30 (iii) 3 (iv) 4

(d) If $\log_3^{27} + \log_3^x = 4$; then x is

- (i) 1 (ii) 2 (iii) 3 (iv) 20

(e) In an LPP

- (i) Only the objective function is linear
(ii) Only the constraints are linear
(iii) The objective function as well as the constraints are linear
(iv) None of the above.

(f) If TC, MC, AC and x - represent the total cost, marginal cost, average cost and output respectively, then which of the following are true?

(i) $\frac{d}{du} TC = MC$ (ii) $\frac{TC}{x} = A.C.$

(iii) $\int Mcdx = TC$ (iv) i, ii and iii.

(g) The quadratic equation $ax^2 + bx + c = 0$ where a, b, c are real, the roots are imaginary and unequal then

- (i) $b^2 > 4ac$ (ii) $b^2 = 4ac$ (iii) $b^2 < 4ac$
(iv) $b^2 - 4ac$ is positive and perfect square.

(h) Which of the following equations has sum of

the roots = $\frac{13}{6}$ and product of the roots = 1

(i) $3x^2 - \frac{13}{6}x + 3 = 0$

(ii) $6x^2 - 13x - 6 = 0$

(iii) $12x^2 - 26x + 12 = 0$

(iv) $x^2 + \frac{13}{6}x + 1 = 0$

(i) If N be a set of natural numbers the value of $\{0\} - N$ is

- (i) N (ii) $\{0\}$ (iii) ϕ (iv) 0

(j) The 10th term of the series $2, 0, -2, -4, \dots$

- (i) -16 , (ii) 2 , (iii) 3 , (iv) 5

2. Answer the following. (any five): $2 \times 5 = 10$

(a) Define constant function and identity function.

(b) Simplify: $\log_3 \{ \log_2 (\log_5 25) \}$

(c) In the expansion $\left(3x - \frac{1}{x^2} \right)^{15}$ find the term free from x .

(d) The total cost $c(x)$ of a firm is $c(x) = 0.0005x^3 - 0.7x^2 - 30x + 3000$. Where x is the output. Determine average cost (AC) and marginal cost (MC)

(e) If $x^p = y^q = (xy)^{pq}$, find $p + q$.

(f) Find the 9th term of a series in G.P. whose 4th term is 1 and 7th term is $\frac{1}{8}$.

3. (a) If $x = 1 + 3^{2/3} + 3^{1/3}$,

prove that $x^3 - 3x^2 - 6x - 4 = 0$ 3

(b) If x, y, z be the p th q th and r th terms respectively of a series in G.P., show that

$$x^{q+r} \cdot y^{r-p} \cdot z^{p-q} = 1 \quad 3$$

(c) Simplify: 3

$$\frac{\log \sqrt{27} + \log 8 + \log \sqrt{1000}}{\log 120}$$

(d) If α and β be the roots of $px^2 + qx + q = 0$, then show that 3

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$$

4. (a) Evaluate $2 \times 4 = 8$

(i) $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x} + 2x}{x}$

(ii) $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-2x}}{x}$

$$(iii) \lim_{x \rightarrow \alpha} \frac{3x^2 - 4x + 6}{x^2 + 6x - 7}$$

$$(iv) \lim_{x \rightarrow 0} \frac{3 \lim x - \lim 3x}{x^3}$$

(b) A function is defined in (0, 3) in the following way

$$f(x) = x^2 \quad 0 < x \leq 1$$

$$= x \quad 1 < x < 2$$

$$= \frac{1}{4x^3} \quad 2 < x < 3$$

If $f(x)$ continuous at $x = 2$.

5. (a) Find derivative $\frac{dy}{dx}$ (any three): $2 \times 3 = 6$

$$(i) y = \frac{x-1}{x+1}$$

$$(ii) y = 5^x x^5$$

$$(iii) y = (2x + 5)^4$$

$$(iv) y = (x-1)^2 (x+2)$$

(b) A firm produces x tons valuable metal per month at a total cost C given by $C = \text{Rs} \left(\frac{1}{3}x^3 - 5x^2 + 75x + 10 \right)$. Find at what level of output the marginal cost attains its minimum. 6

(a) What do you mean by LPP? Give the advantages and disadvantages of LPP. 2+4=6

(b) Solve the following L.P.P. graphically 6

$$\text{Maximize } Z = 5x_1 + 7x_2$$

$$\text{Subject to } 2x_1 + 3x_2 \leq 13$$

$$3x_1 + 2x_2 \leq 12, \quad x_1, x_2 \geq 0$$

(a) Find the 7th term in expansion of

$$\left(4x - \frac{1}{2\sqrt{x}} \right)^{10}$$

3

(b) Find the co-efficient of x^8 in $(1+x^2)^{10}$. 3

(c) Using binomial theorem, find the value of $(1.02)^5$ correct to 3 places of decimal. 3

(d) Prove that the middle term of $\left(x + \frac{1}{2x}\right)^{21}$

$$\frac{1, 3, 5, \dots, (2x-1)}{x!}$$

8. (a) Find the inverse of the matrix.

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

(b) Show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-c)(c-a)(a-b)$$

(c) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ show that

$A^2 - 4A - 5I = 0$ where I is the 3×3 unit matrix.

9. (a) An enterprise produced 600 units in the third year of its existence and 700 units in its seven years 4

(i) What is the initial production in the first year?

(ii) What was the total production in the first five years?

(b) The sum of three numbers in G.P. is 26 and their product is 216. Find the numbers. 4

(c) Find the sum of n terms of the series 4

$$4 + 44 + 444 + \dots$$

10. (a) If $f(x) = \frac{3x+2}{3x-2}$ show that

$$\frac{f(x)+1}{f(x)-1} = \frac{3x}{2}$$

3

(b) Draw the graph of the following function

$$f(x) = |x|$$

3

(c) In a hostel out of 110 students, 80 takes tea, 50 takes coffee and 25 takes both tea and coffee. How many students take neither tea nor coffee? 3

(d) For any two sets A and B, prove that

$$(A \cup B)' = A' \cap B'$$

3

11. (a) Integrate the following :

3×3=9

(i) $\int \frac{2x+4}{2x^2+8x-10} dx$

(ii) $\int \sqrt{5x-3} dx$

(iii) $\int_2^4 \frac{x}{1+x^2} dx$

(b) The Marginal Cost (MC) of a product is given by

MC = Rs. $(25 + 30x - 9x^2)$ and the fixed cost is known to be Rs. 550. Find the total cost function.

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