47 (1) BMAT 1·3

2013

BUSINESS MATHEMATICS

Paper : 1·3

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks
for the questions.

Answer question Nos. *1* and *2* and
any five from the rest.

1. Choose the correct alternative : $1 \times 10 = 10$

    (a) Let $A = \{1, 3, \{1\}, \{1, 3\}\}$. Find which of the
         following statements are correct.

         (i) $\{3\} \in A$

         (ii) $\{3\} \subset A$

         (iii) $\{1\} \in A$

         (iv) $2 \in A$

(b) If α and β be the roots of the equation $2x^2 - 5x + 2 = 0$ then the value of $\alpha^2 - \beta^2$ be

(i) $17/4$

(ii) 4

(iii) 0

(iv) $\dfrac{2}{3}$

(c) The rank of the matrix $A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{pmatrix}$ will be

(i) 1

(ii) 3

(iii) 2

(iv) None of these.

(d) If $\log_3 27 + \log_3{}^x = 4$ then x is

(i) 1

(ii) 2

(iii) 3

(iv) 20

(e) In an LPP

(i) Only the objective function is linear

(ii) Only the constraints are linear

(iii) The objective function as well as the constraints are linear.

(iv) None of the above.

(f) The standard equation of Hyperbola is

(i) $x^2 + y^2 = a^2$

(ii) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

(iii) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

(iv) $y^2 = 4ax$

(g) Let $F : A \to B$. Which of the following is always true ?

(i) $F(A) \subset B$

(ii) $F(A) = B$

(iii) $F(A) \supset B$

(iv) $F(B) \subset A$.

(h) The characteristics of logarithm of the number $45 \cdot 61$ is

(i) 1

(ii) 2

(iii) 0

(v) 3

(i) The 5th term of the series 1, 3, 9, 27, ...

(i) 81

(ii) 80

(iii) 27

(iv) 2

(j) The co-efficient of x^8 in the expansion $(1+x)^8$ is

(i) 28

(ii) 29

(iii) 23

(iv) 21

Answer the following : (any five) $2 \times 5 = 10$

(a) Define constant function and identity function.

(b) Find the value of $\log_{1000} 2$, given $\log_{10} 2 = 0 \cdot 30103$.

(c) Find the 9th term in the expansion of $(2a + x)^{10}$.

(d) The total cost $C(x)$ of a firm is $C(x) = 0 \cdot 0005 x^3 - 0 \cdot 7 x^2 - 30x + 3000$, where x is the output. Determine average cost (AC) and marginal cost (MC).

(e) If $x^p = y^q = (xy)^{p+q}$, find $p + q$.

(f) Find the 9th term of a series in G.P. whose 4th term is 1 and 7th term is $\dfrac{1}{8}$.

3. (a) If $\log \dfrac{x+y}{3} = \dfrac{1}{2}(\log x + \log y)$, then show that $\dfrac{x}{y} + \dfrac{y}{x} = 7$.   3

(b) If $x = 1 + 3^{2/3} + 3^{1/3}$ prove that

$x^3 - 3x^2 - 6x - 4 = 0$.

(c) If a, b, c are in G.P. and if p is the A.M. between a, b and q is the A.M. between b, c. Show that $\dfrac{a}{p} + \dfrac{c}{q} = 2$.

(d) If α, β be the roots of $2x^2 - 6x + 2 = 0$ from

the equation whose roots are $\dfrac{1-\alpha}{1+\alpha}$ and

$\dfrac{1-\beta}{1+\beta}$.   3

4. (a) Evaluate   2×4=8

(i) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+2x} - \sqrt{1-2x}}{x}$

(ii) $\lim\limits_{x \to 3} \dfrac{\sqrt{x^2+7}-4}{x^2-5x+6}$

(iii) $\lim\limits_{x \to \infty} \dfrac{4x^3+5x-1}{6x^3+7x^2+4}$

(iv) $\lim\limits_{h \to 0} \dfrac{f(2+h)-f(2)}{h}$

where $f(x) = 2x^2 - 7x + 1$

(b) A function f is defined by

$f(x) = \dfrac{sin^{-1}x}{x} + e^x,$ $x \neq 0$

$= 2$ $x = 0$

show that f is continues at $x = 0$.   4

5. (a) Find derivative $\dfrac{dy}{dx}$: *(any three)*   2×3=6

(i) $y = \left(x^2 + \dfrac{1}{x}\right)^5$

(ii) $y = (x-1)^2(x+2)$

(iii) $y = log\left(\sqrt{x+1} - \sqrt{x-1}\right)$

(iv) $y = \dfrac{3x-1}{\sqrt{2x^2+1}}$

(b) A firm produces x tons valuable metal per month a total cost C gives by

$$C = Rs\left(\frac{1}{3}x^3 - 5x^2 + 75x + 10\right).$$

Find at what level of output the marginal cost attains its minimum ? 6

6. (a) The total cost of output x given by

$$C = \frac{2}{3}x + \frac{35}{2}$$

Find (i) Cost when output is 4 units

(ii) Average cost of output of 10 units.

(iii) Marginal cost when output is 3 units. 6

(b) Solve the following L.P.P. graphically

Maximize $Z = 7x + 5y$ subject to

$x + 2y \le 6$. $4x + 3y \le 12$, $x, y \ge 0$. 6

7. (a) Find the 7th term in expansion of

$$\left(4x - \frac{1}{2\sqrt{x}}\right)^{10}$$ 3

(b) Show that the middle term in the expansion of $(1+x)^{2n}$ is $\dfrac{1.3.5. \ldots\ldots(2n-1)}{n!} \cdot 2^n \cdot x^n$. 3

(c) Evaluate $\sqrt{99}$ to 6 places of decimals. 3

(d) Expand the following : $\left(x - \frac{1}{x}\right)^4$. 3

(a) Find the inverse of the matrix :

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ 4

(b) Show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = a^2(a+3)$$ 4

(c) A retailer purchased for his shop 10 kg butter, 20 kg ghee and 40 kg cooking oil. Cost prices are Rs. 45, Rs. 40 and Rs. 15 per kg respectively. Find the amount he spent for the articles. 4

9. (a) A man secures an interest-free loan of Rs. 14,500 from a friend and agrees to repay it in 10 instalments. He pays Rs. 1000 as first instalment and then increases each instalment by equal amount over the preceding instalment. What will be his last instalment ?   4

(b) The sum of three numbers in G.P. is 26 and their product is 216. Find the numbers.   4

(c) A man borrows Rs. 5115 to be repaid in 10 monthly instalments. If each instalment is double the value of the last, find the value of 1st and last instalments.   4

10. (a) If $f(x) = \dfrac{3x+2}{3x-2}$ show that

$$\frac{f(x)+1}{f(x)-1} = \frac{3x}{2}$$ 3

(b) Draw the graph of the following function
$$f(x) = |x|.$$ 3

(c) In a survey of 150 students it was found that 40 students studied Physics, 60 students studied Chemistry, 50 students Mathematics and 15 students studied all the three subjects. 27 students Physics and Chemistry, 35 students studied Chemistry and Mathematics and 25 students studied Physics and Mathematics. Find the number who studied only Physics and the number who studied none of these subjects.   3

(d) A, B and C are any three sets, prove that
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$ 3

11. (a) Integrate the following :   3×3=9

(i) $\int x e^x \, dx$

(ii) $\int \dfrac{\log\left(\tan^{-1} x\right)}{1+x^2} \, dx$

(iii) $\int_2^4 \dfrac{x}{1+x^2} \, dx$

Contd.