47 (1) BMAT 1·3

2014

BUSINESS MATHEMATICS

Paper : 1·3

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer **Question Nos.** 1 and 2 and **any five** from the rest.

1. Choose the correct alternative :   1×10=10

   (a) The characteristic of logarithm of the number 45·61 is

   (i)   1

   (ii) 2

   (iii) 0

   (iv) 3.

(b) The 5th term of the series 1, 3, 9, 27, ...

(i) 81

(ii) 80

(iii) 27

(iv) 2.

(c) If $\log_3 27 + \log_3 x = 4$; then x is

(i) 1

(ii) 2

(iii) 3

(iv) 20.

(d) Which of the following equations has sum of the roots $= \dfrac{13}{6}$ and product of the roots = 1 ?

(i) $3x^2 - \dfrac{13}{6}x + 3 = 0$

(ii) $6x^2 - 13x + 6 = 0$

(iii) $12x^2 - 26x + 12 = 0$

(iv) $x^2 + \dfrac{13}{6}x + 1 = 0$.

(e) If $f(x) = 4x + 8$, $g(x) = 2x + 10$ and $h(x) = x + 2$ then which of the following is true ?

(i) $f(x) = g(x)$ at $x = 1$

(ii) $f(x) = h(x)$ at $x = 4$

(iii) $g(x) = h(x)$ at $x = 2$

(iv) None of the above.

(f) If two rows (or columns) of a determinant are identical, the value of the determinant becomes

(i) 0

(ii) 1

(iii) 2

(iv) can't say.

(g) For the optimum solution of an LPP involving three variables, we can use —

    (i)   Graphical Method

    (ii)   Simplex Method

    (iii) Either (i) or (ii)

    (iv) None.

(h) G.M. of 2 and 18

    (i)   6

    (ii)   4

    (iii) 2

    (iv) 0.

(i) A pure quadratic equation is of the form

    (i) $ax^2 + bx + c = 0$

    (ii) $ax^2 + c = 0$

    (iii) $ax^2 + bx = 0$

    (iv) $bx + c = 0$.

(j) The value of $\int_{2}^{3} \log x \, dx$ is

    (i) $\log \dfrac{3}{2}$

    (ii) $\log 3$

    (iii) $\log 2$

    (iv) $3 \log 3 - 2 \log 2 - 1$.

Answer the following : *(Any five)*   2×5=10

(a) Define even function and odd function.

(b) The 3rd and 6th terms of a series in G.P. are 3 and 81 respectively, find the first term and common ratio.

(c) Find the 7th term is expansion of

$$\left(4x - \frac{1}{25x}\right)^{10}.$$

(d) Find the base about which logarithm of 64 is 4.

(e) The first term of an A.P. is 6 and common difference is 2, find the 15th term.

(f) The total cost $C(x)$ of a firm is

$$C(x) = 0 \cdot 0005x^3 - 0 \cdot 7x^2 - 30x + 3000,$$

where x is the output. Determine average cost (AC) and marginal cost (MC).

3. (a) The difference between a proper fraction and its reciprocal is $\dfrac{9}{20}$. Find the fraction.　　3

(b) If one root of the quadratic equation $x^2 - px + q = 0$ be twice the other, show that

$$2p^2 = 9q.$$　　　3

(c) If x, y, z be the pth, qth and rth terms respectively of a series in G.P. ; show that

$$x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = 1$$　　3

(d) Simplify :

$$\frac{\log\sqrt{27} + \log 8 + \log\sqrt{1000}}{\log 120}$$　　3

(a) Evaluate :　　　2×4=8

(i) $\displaystyle\lim_{x\to 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10}$

(ii) $\displaystyle\lim_{x\to 0} \frac{a - \sqrt{a^2 - x^2}}{x^2}$

(iii) $\displaystyle\lim_{x\to\infty} \frac{2x^2 + 7x + 5}{4x^2 + 3x - 1}$

(iv) $\displaystyle\lim_{x\to 0} \frac{\lim 2x + 2\lim x}{x}$

(b) A function f is defined by

$$f(x) = \frac{\sin^{-1} x}{x} + e^x \qquad x \neq 0$$

$$= 2 \qquad x = 0$$

Show that f is continuous at $x = 0$　　4

5. (a) Find derivative $\dfrac{dy}{dx}$: *(Any three)* $2 \times 3 = 6$

    (i) $y = \dfrac{x-1}{x+1}$

    (ii) $y = 5^x x^5$

    (iii) $y = (2x+5)^4$

    (iv) $y = (x-1)^2 (x+2)$

  (b) The total cost of output x given by

    $C = \dfrac{2}{3}x + \dfrac{35}{2}$ Find :

    (i)   cost when output is 4 units

    (ii)   average cost of output of 10 units

    (iii)   Marginal cost when output is 3 units.
                                                  6

6. (a) Solve the following L.P.P. graphically
    Minimize

    $Z = 20x + 40y$ Subject to
    $6x + y \geq 18$
    $x + 4y \geq 12$
    $2x + y \geq 10$
    $x,\ y \geq 0.$
                                   6

(b) What do you mean by LPP ? Give the advantage and disadvantage of LPP.
                                           $2 + 4 = 6$

7. (a) In the expansion $\left(3x - \dfrac{1}{x^2}\right)^{15}$ find the term free from x.   3

  (b) Find the co-efficient of x^8 in $\left(1+x^2\right)^{10}$ 3

  (c) Expand the following :

    $(1+x)^4$ 3

  (d) Prove that the middle term of

    $\left(x + \dfrac{1}{2x}\right)^{2n}$ is

    $\dfrac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{n!}$ 3

8. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$
                                           4

(b) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$$

(c) Solve by Cramer's Rule

$$2x + y + z = 1$$
$$x - y + 2z = -1$$
$$3x + 2y - z = 4$$

    4

9. (a) Find the sum of n terms of the following series

$$4 + 44 + 444 + \ldots$$

    4

(b) The sum of three integers in A.P. is 21 and their product is 280, find them.

    4

(c) A man borrows Rs. 4,500 and promises to pay back in 30 instalments, each of value Rs. 10 more than the last. Find the 1st and last instalments.

    4

10. (a) If $f(x) = \dfrac{3x+2}{3x-2}$ show that

$$\frac{f(x)+1}{f(x)-1} = \frac{3x}{2}$$

    3

(b) Draw the graph of the following function

$$f(x) = |x|$$

    3

(c) In a class of 100 students, 45 students read physics, 52 students read chemistry and 17 students read both the subjects. Find the number of students who study neither physics nor chemistry.

    3

(b) For any two sets A and B, prove that

$$(A \cup B)' = A' \cap B'$$

    3

11. (a) Integrate the following :

    $3 \times 3 = 9$

(i) $\int xe^x \, dx$

(ii) $\int \dfrac{\log(\tan^{-1} x)}{1+x^2} \, dx$

(iii) $\int\limits_{2}^{4} \dfrac{x}{1+x^2}\, dx$

(b) The marginal cost of producing x dozens of pencil is Rs. $\left(5x^2 - 4x + 12\right)$. If the cost of producing 2 dozens pencil is Rs. 200, find the total cost functions. 3

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