Total No. of printed pages = 6

MA 131301 OR

Roll No. of candidate

2018

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B.Tech. 3rd Semester End-Term Examination

MATHEMATICS — III

(Old Regulation)

Full Marks – 100   Time – Three hours

The figures in the margin indicate full marks
for the questions.

Answer Question No. 1 and any *six* from the rest.

1. Answer the following :

$$(10 \times 1 = 10)$$

(i) The particular integral of
$(D^2 - 7DD' + 5D'^2)Z = e^{x-y}$ is

  (a) xe^{x-y} (b) $-\dfrac{1}{12}e^{x-y}$

  (c) $\dfrac{1}{13}e^{x-y}$ (d) $\dfrac{1}{13}xe^{x-y}$

(ii) A solution of $(px + qy - Z)^2 = p^2 + q^2$ is

  (a) $Z = ax + by + a^2 + b^2$

  (b) $Z = (ax + by)^2 + a^2 + b^2$

  (c) $Z = ax + by - \sqrt{a^2 + b^2}$

  (d) $Z = \sqrt{ax + by} + a^2 + b^2$

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(iii) The solution of The Wave equation $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$ is

(a) $y = (c_1 \cos px + c_1 \sin px)\left(c_3\, e^{pt} + c_4\, e^{-pt}\right)$

(b) $y = (c_1 \cos px + c_1 \sin px)$
$\qquad\qquad \left(c_3 \cos pct + c_4 \sin pct\right)$

(c) $y = (c_1\, e^{px} + c_2\, e^{-px})\left(c_3 \cos pct + c_4 \sin pct\right)$

(d) $y = (c_1\, e^{px} + c_2\, e^{-px})\left(c_3\, e^{pct} + c_4\, e^{-pct}\right)$

(iv) The relation $|3 - Z| + |3 + Z| = 5$ represents a

(a) Circle   (b) Parabola

(c) Ellipse   (d) Hyperbola

(v) For a complex function $f(Z) = u + iv$ where $u = u(r, \theta)$, $v = v(r, \theta)$

(a) $\dfrac{\partial u}{\partial r} = \dfrac{\partial v}{\partial \theta}$ (b) $\dfrac{\partial u}{\partial r} = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}$

(c) $\dfrac{\partial u}{\partial \theta} = \dfrac{1}{r}\dfrac{\partial v}{\partial r}$ (d) $\dfrac{\partial u}{\partial \theta} = \dfrac{\partial v}{\partial r}$

(vi) If $f(Z)$ is analytic within and on a closed curve C and 'a' is a point inside C, then

(a) $\oint_C f(Z)\, dZ = f(a)$

(b) $\oint_C f(Z)\, dZ = f^n(a)$

(c) $\oint_C \dfrac{f(Z)\, dZ}{Z - a} = 2\pi i\, f^n(a)$

(d) $\oint_C \dfrac{f(Z)\, dZ}{Z - a} = 2\pi i\, f(a)$

(vii) For any two events A and B, $P(A - B)$ is equal to ————.

(viii) The mean and variance of a binomial distribution are 8 and 4 respectively, then $P(X = 1) =$ ————.

(ix) The inverse Laplace Transform of $\dfrac{1}{(S + 3)^3}$ is

(a) $\dfrac{1}{2} e^{-3t} t^2$ (b) $\dfrac{1}{2} e^{3t} t^2$

(c) $\dfrac{1}{3} e^{-3t} t^3$ (d) $\dfrac{1}{3} e^{3t} t^3$

(x) If $L\{f(t)\} = \bar{f}(s)$, Then $L\{f'(t)\} =$

(a) $\bar{f}(s) - s\, f(0)$ (b) $s\bar{f}(s) - f(0)$

(c) $s^2 \bar{f}(s) - s f(0)$ (d) $s\bar{f}(s) + f(0)$

2. (a) Form a partial differential equation from

$$2Z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}.$$ (3)

(b) Solve the equation $Zpq = p + q$.   (5)

(c) Solve the equation

$$x^2(y - Z)\, p + y^2(Z - x)\, q = Z^2(x - y).$$ (7)

3. (a) Prove that the function $u(x, y) = \log(x^2 + y^2)$ is harmonic. (3)

(b) Determine the analytic function whose real part is $e^x(x \cos y - y \sin y)$. (7)

(c) Evaluate $\int\limits_{0}^{2+i} (\bar{Z})^2 \, dZ$ along the line $2y = x$. (5)

4. (a) Urn I contains 3 green and 5 red balls. Urn II contains 2 green, 1 red and 2 yellow balls. One urn is selected at random and then a ball is drawn from it. What is the probability that the ball is green? (5)

(b) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of total output. Of these 5%, 4% and 2% respectively are defective. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by machine C? (5)

(c) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. (5)

5. (a) Find the Laplace Transform of (2 + 3 = 5)

(i) $f(t) = 5t^4 - 4 \cos 3t$,

(ii) $f(t) = e^{-3t} \sin 2t$.

(b) Prove that $\int\limits_{0}^{\infty} \dfrac{e^{-t} - e^{-3t}}{t} \, dt = \log 3$. (5)

(c) Evaluate $L^{-1} \left\{ \dfrac{s+2}{s^2 - 4s + 13} \right\}$. (5)

6. (a) Solve: $\left(D^2 - 3DD' + 2D'^2\right) Z = \sin(2x + y)$. (5)

(b) Expand $\dfrac{\sin Z}{Z - \pi}$ about $Z = \pi$. (4)

(c) Find the mean and variance of binomial distribution. (6)

7. (a) Solve the following equation by Charpit's method: $\left(p^2 + q^2\right) y = q^Z$. (7)

(b) Evaluate $\oint\limits_{C} \dfrac{Z \, dZ}{(Z-1)(Z-3)}$ where C is the circle $|Z| = 3$. (5)

(c) Evaluate : $L^{-1} \left\{ \dfrac{1}{s(s+1)^2} \right\}$. (3)

8. (a) Find the image of the infinite strip $\dfrac{1}{4} \leq y \leq \dfrac{1}{2}$ under the transformation $w = \dfrac{1}{Z}$. (4)

(b) A company claims that the mean life time of tube lights is 500 hours. Is the claim of the company tenable if a random sample of 25 tube lights produced by the company has mean 518 hours and standard deviation 40 hours? (4)

(c) Solve the equation by Laplace Transform method: $\dfrac{d^2 x}{dt^2} - 3 \dfrac{dx}{dt} + 2x = e^{-t}$; where $x(0) = -3$, $x'(0) = 5$. (7)

9. (a) Determine the residue at each pole of $f(Z) = \dfrac{2Z+1}{Z^2 - Z - 2}$. (3)

(b) Determine the solution of the one-dimensional heat equation $\dfrac{\partial u}{\partial t} = C^2 \dfrac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u(0, t) = u(l, t) = 0$ for $t > 0$ and $u(x, 0) = u_0$. (7)

(c) In 60 throws of a die, number one is obtained 6 times, two or three 18 times, four or five 24 times and number six is obtained 12 times. Test at 1% level if the die is a fair one. (5)

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