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The Assam Royal Global University, Guwahati

Royal School of Applied and Pure Sciences

B.Sc. Mathematics 6th Semester

Semester End Examination, June 2023

Course Title: Introduction to Probability & Statistics

Course Code : MAT012D601

Time: 3 Hours

Maximum Marks: 70

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt **all** questions. (Maximum word limit 50) 2 x 8
- Write the axiomatic approach to probability.
 - For any three events A, B , and C , justify the following:
$$P(A \cap \bar{B}|C) + P(A \cap B|C) = P(A|C).$$
 - What is meant by discrete random variable? Give example.
 - Write the probability mass function of binomial probability distribution?
 - Interpret correlation coefficient $r = +1, 0$ and -1 .
 - Write the normal equations for estimating the constants in fitting a second-degree parabola.
 - Mention the characteristics of point estimation.
 - “Level of significance is associated with type I error” - Justify.

Section – B

2. Attempt **any two** of the following: 6 x 2
- A factory produces three types of products: Product X, Product Y, and Product Z. The probability of a defective unit in Product X is 2%, in Product Y is 3%, and in Product Z is 5%. The factory produces 50% of Product X, 30% of Product Y, and 20% of Product Z. If a defective unit is found, what is the probability that it is from Product X?
 - Two computers A and B are to be marketed. A salesman who is assigned the job of finding customer for them as 60% and 40% chances respectively of succeeding for computer A and B. The two computers can be sold independently. Given that he was able to sell at least one computer.
 - What is the probability that computer A has been sold?
 - What is the probability that computer B has been sold?
 - A clinic offers two types of blood tests for a certain disease: Test A and Test B. Test A has a sensitivity of 95% and a false positive rate of 5%. Test B has a sensitivity of 90% and a false positive rate of 2%. If a patient tests positive for both tests, what is the probability that they have the disease?
3. Attempt **any two** of the following: 7 x 2
- If a random variable X has the following Probability distribution:

x	0	1	2	3	4	5	6	7
P(x)	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2 + a$

Find the (i) a (ii) $P(1 \leq x \leq 4)$.

b. The probability density function of a continuous random variable X is given as

$$f(x) = a(x - 1)(2 - x), 1 \leq x \leq 2. \text{ Find (i) } a \text{ (ii) } P\left(\frac{5}{4} \leq x \leq \frac{3}{2}\right)$$

c. Find the mean and variance of binomial distribution.

4. Attempt **any two** of the following:

7 x 2

a. Fit a parabola of second degree $Y = a + bx + cx^2$ to the following data:

X	1	2	3	4	5	6	7
Y	2.3	5.2	9.7	16.5	29.4	35.5	54.4

b. Compute the rank correlation coefficient between the marks in Linear Algebra and Mathematical Statistics from the following:

Linear Algebra	56	62	73	62	75	56	78
Mathematical Statistics	72	72	92	92	92	63	54

c. The regression lines are given as: $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$. Obtain (i) Regression line of x on y and regression line of y on x (ii) means of x and y and (iii) correlation coefficient between x and y.

5. Attempt **any two** of the following:

7 x 2

a. Let $\{T_n\}$ be a sequence of estimators such that for $\theta \in \theta$,

(i) $E_{\theta}(T_n) \rightarrow \gamma(\theta)$ as $n \rightarrow \infty$

(ii) $Var_{\theta}(T_n) \rightarrow 0$ as $n \rightarrow \infty$

Then show that T_n is a consistent estimator of $\gamma(\theta)$.

b. The manufacturer of television tubes knows from experience that the average life of a tube is 2,000 hours with a standard deviation of 200 hours. A sample of 100 tubes has an average life of 1950 hours. Test at the 0.05 level of significance if this sample came from a normal population of mean 2,000 hours. State your null and alternative hypothesis and indicate clearly whether a one tail or a two-tail test is used and why? Is the result of the test significant?

c. The information of two samples are as follows:

Mean of 1st sample = 82cm, sample size = 1200,

Mean of 2nd sample = 80cm, sample size = 1800.

Test whether the samples have been drawn from the same population of standard deviation 3.2cm.
