Roll No: | | | | | | | | | |

The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

B.Sc. Mathematics 1st Semester

Semester End Examination, January 2023

Course Title: Calculus (Differential & Integral)

Course Code: MAT012C101

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt all questions: **2 x 8 = 16**

a. If $y = x^2 e^{ax}$, find y_n by using Leibnitz's theorem.

b. If $u = x^2 + y^2 + z^2$, show that $xu_x + yu_y + zu_z = 2u$.

c. If $\phi(\alpha) = \int_{\alpha}^{\alpha^2} \frac{\sin \alpha x}{x} dx$, find $\phi'(\alpha)$ where $\alpha \neq 0$.

d. Show that the subnormal at any point of the parabola $y^2 = 4ax$ is constant and the subtangent varies as the abscissa of the point of contact.

e. Define multiple point and node of a curve.

f. Find the curvature of the parabola $y^2 = 12x$ at the point $(0,0)$.

g. Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^4 x dx$.

h. Find the area of the region enclosed by the circle $x^2 + y^2 = 4$.

Section – B

2. Attempt any two of the following questions: **6 x 2=12**

a. If $f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}, (x,y) \neq (0,0)$, show that $\lim_{x\to 0}\lim_{y\to 0} f(x,y) \neq \lim_{y\to 0}\lim_{x\to 0} f(x,y)$.

Also Show that $\lim_{(x,y)\to 0} \frac{xy}{x^2 + y^2}$ does not exist.

P.T.O.

b. Show that if $z(x+y)=x^2+y^2$, then $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$.

c. If $z=f(u,v)$, where $u=x^2-2xy-y^2, v=y$, prove that $(x+y)\frac{\partial z}{\partial x}+(x-y)\frac{\partial z}{\partial y}=0$, can be transformed into $\frac{\partial z}{\partial v}=0$.

3. Attempt any two of the following questions: 7 x 2=14

a. Find the subtangent, subnormal, length of the tangent and length of the normal at the point 't' on the cycloid $x=a(t+\sin t), y=a(1-\cos t)$.

b. Find the saddle points, relative maxima and minima of $f(x,y)=8x^3-24xy+y^3$.

c. Find the points of intersection of the curves $2x^2+y^2=20, 4y^2-x^2=8$ and find the angle of intersection of the curves at any two of those points of intersection.

4. Attempt any two of the following questions: 7 x 2=14

a. Find the equation of the circle of curvature of $2xy+x+y=4$ at the point $(1,1)$.

b. Find the radius of curvature at any point of the curve $x=a(t+\sin t), y=a(1-\cos t)$ at $t=\frac{\pi}{2}$.

c. Trace the Cardioid $r=a(1-\cos\theta)$.

5. Attempt any two of the following questions: 7 x 2=14

a. Evaluate $\int \frac{xdx}{\sqrt{x^2+2x+5}}$

b. Prove that $\beta(m,n)=2\int_0^{\frac{\pi}{2}} \sin^{2m-1}x\cos^{2n-1}xdx$ and hence find the value of $\Gamma\left(\frac{1}{2}\right)$.

c. Show that $\int_{x=1}^{2}\int_{y=\sqrt{x}}^{x} \sin\frac{\pi x}{2y}dydx+\int_{x=2}^{4}\int_{y=\sqrt{x}}^{2} \sin\frac{\pi x}{2y}dydx=\frac{4(\pi+2)}{\pi^3}$.

*****