Roll No: | | | | | | | | | |

The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

B.Sc. Mathematics / Mathematics & Computing, 3rd Semester

Special Supplementary Examination, September 2023

Course Title : Real Analysis

Course Code : MAT012C302/MAC012C301

THE ASSAM ROYAL GLOBAL UNIVERSITY CENTRAL LIBRARY GUWAHATI

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt **all questions**. (Maximum word limit 50) **2 x 8**

a. Show that the set of all natural numbers $\mathbb{N}$ is not bounded above.

b. Is the set of all real numbers $\mathbb{R}$ countable or uncountable? Justify.

c. Is product of two divergent sequences always divergent? Justify.

d. Is the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ convergent or divergent? Explain.

e. Write the $\epsilon - \delta$ definition for continuity of a function. Using this definition, show that $f(x) = x^3$ is continuous in $[0,2]$.

f. Find $\lim_{x\to 0} x^2 \sin\frac{1}{x}$.

g. Give the geometrical interpretation of mean value theorem.

h. Check the differentiability of the function $f(x) = x|x|$ at $x = 0$.

Section – B

2. Attempt **any two** of the following: **6 x 2**

a. Define countably infinite set. Prove that every infinite set contains a countably infinite subset.

b. Define bounded subset of $\mathbb{R}$ with an example. Show that $lub\left\{1 - \frac{1}{n^2} : n \in \mathbb{N}\right\} = 1$.

c. Show that $lub(A + B) = lub(A) + lub(B)$ for any two subsets A and B of $\mathbb{R}$.

3. Attempt **any two** of the following: **7 x 2**

a. Prove that if (x_n) and (y_n) are two sequences of real numbers such that $x_n \to x$ and $y_n \to y$ then $x_n y_n \to xy$ for some $x, y \in \mathbb{R}$. Is the converse true? Justify.

b. Define absolutely convergent series. Prove that every absolutely convergent series is convergent. Is the converse true? Justify.

c. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$.

4. Attempt **any two** of the following: **7 x 2**

a. Show that a uniformly continuous function is continuous. Is the converse true? Justify.

b. Using $\epsilon - \delta$ definition, show that $\lim_{x\to a} \frac{1}{x} = \frac{1}{a}$ for $a > 0$.

P.T.O.

c. Define Lipschitz function. Prove that every Lipschitz function is continuous. Is the converse true? Justify.

5. Attempt **any two** of the following: **7 x 2**

a. Prove that if $f: [a, b] \to \mathbb{R}$ is continuous then f is bounded.

b. Find the value of a and b for which the function $f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable at every point.

c. Let $f: A \to \mathbb{R}$ be differentiable on A. Then prove that f is monotone increasing on A if and only if $f'(x) \ge 0 \ \forall \ x \in A$.

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