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The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

M.Sc. Mathematics 1st Semester

Semester End Examination, January 2023

Course Title: Ordinary Differential Equations

Course Code: MAT014C104

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt all questions: **2 x 8 = 16**

a) Prove that the functions $1, x, x^2$ are linearly independent.

b) Test the exactness of the following differential equation:

$$\sin x\, y'' - \cos x\, y' + 2 \sin x\, y = \tan^2 x$$

c) Find the nature of the critical point (0,0) of the system

$$\frac{dx}{dt} = 2x - 3y, \quad \frac{dy}{dt} = 4x - 6y\,.$$

d) Reduce the following initial-value problem into matrix form :

$$\ddot{x} + 2\dot{x} - 8x = 0;\ x(1) = 2, \dot{x}(1) = 3$$

e) Determine the interval of convergence of the power series $\sum\left\{\left(\frac{1}{n}\right)(-1)^{n+2}(x-1)^n\right\}$

f) Prove that $x = 0$ is a singular point of $2x^2y'' + 7x(x+1)y' - 3y = 0$.

g) Show that $y'' + \lambda y = 0,\ y(0) = 0,\ y(\pi) = 0$ is a Sturm Liouville problem.

h) Transform $x^2y'' - 2xy' + 2y = 0$ into an self- adjoint equation.

Section – B

2. Attempt any two of the following questions: **6 x 2=12**

a) Verify Existence and Uniqueness theorem for the initial value problem

$y' = 1 + y^2, y(0) = 0.$

Also, find its unique solution.

P.T.O.

b) Show that linearly independent solutions of $y'' - 2y' + 2y = 0$ are $e^x \sin x$ and $e^x \cos x$ .What is the general solution? Find the solution $y(x)$ with the property $y(0) = 2, y'(0) = -3$.

c) Test for exactness and solve $(1 + x^2)y'' + 4xy' + 2y = \sec^2 x$ given that $y(0) = 0, y'(0) = 1$.

3. Attempt any two of the following questions: **7 x 2=14**

a) Determine the nature of the critical point (0,0) of the system

$$\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy$$

and discuss the stability of the system for the roots of the characteristic equation.

b) Put the initial-value problem $\ddot{x} + 2\dot{x} - 8x = e^t; x(0) = 0, \dot{x}(0) = -4$ into the form of matrix system and hence solve it.

c) What do you mean by critical point for autonomous system? Discuss in brief about various types of critical points with suitable diagrams.

4. Attempt any two of the following questions: **7 x 2=14**

a) Obtain the power series solution of $y'' + (x - 3)y' + y = 0$ in powers of $(x - 2)$.

b) Find the power series solution in powers of $(x - 1)$ of the initial value problem

$$xy'' + y' + 2y = 0, \ y(1) = 1, y'(1) = 2.$$

c) Solve the Bessel equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ in series, taking $2n$ as non- integral.

5. Attempt any two of the following questions: **7 x 2=14**

a) For the initial value problem

$y'' - 4\lambda y' + 4\lambda^2 y = 0, \ y'(1) = 0, \ y(2) + y'(2) = 0,$ find the eigenvalues and eigenfunctions.

b) Obtain the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$y'' + \lambda y = 0, \ y(0) = 0, \ y(1) = 0$.

c) For the boundary value problem

$u'' + \lambda u = x, \ u(0) = u\left(\frac{\pi}{2}\right) = 0,$ construct Green's function.

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