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The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

M.Sc. Mathematics 1st Semester

Semester End Examination, January, 2023

Course Title: Algebra-I

Course Code: : MAT014C101

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

Q1. Attempt **all questions.** (Maximum word limit 50) **[2 x 8 = 16]**

a. Define solvable group. Show that for any abelian group G is solvable.

b. Show that $< Q, +>$ cannot be isomorphic to $< Q^*, \circ >$ where $Q^* = Q - \{0\}$.

c. Let $R =$ ring of 2×2 matrices over Z, let $A = \left\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : a, b \in Z\right\}$. Show that A is an Ideal of R.

d. Show that in an integral domain D with unity every prime element is irreducible.

e. Define subfield with suitable examples.

f. Show that the intersection of any non-empty family of subfields of a field F is a subfield of F.

g. Define Module in a ring.

h. Define Free module. Give examples.

Section – B

Q2. Answer any two of the following **$[7 \times 2 = 14]$**

a. A group G is solvable iff $G^{(n)} = \{e\}$, for some positive integer n.

b. Any subgroup of H of a solvable group G is solvable.

c. Let $f: G \to G'$ be a homomorphism, let $a \in G$ such that $o(a) = n$ and $o(f(a)) = m$. Show that $o(f(a)) | o(a)$ and f is one-one iff $m = n$.

Q3. Answer any two of the following **$[7 \times 2 = 14]$**

a. Let $B \subseteq A$ be two ideals of a ring R. Then $R/A \cong \frac{R/B}{A/B}$.

b. Let R be a ring and N be an ideal of R then R/N is a ring.

c. In UFD every irreducible element is prime.

Q4. Answer any two of the following **$[7 \times 2 = 14]$**

a. If any two elements a, b of K are algebraic over F then,

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i) $a \pm b, a, b$ are algebraic over F

ii) If $b \neq 0$, then ab^{-1} is algebraic over F.

b. Prove that if $a \in K$ is algebraic over F of an odd degree then $F(a) = F(a^2)$.

c. $a, b \in K$ are algebraic over F of degrees m and n respectively, where m and n are relatively prime. Prove that $F(a, b)$ is of degree mn over F.

Q5. Answer any two of the following **[$6 \times 2 = 12$]**

a. Let R be a ring with unity and M be an R-module. Show that M is cyclic iff $M \cong R/I$, where I is left ideal of R

b. If f is an homomorphism from R-modules M onto N, then $\frac{M}{Kerf} \cong f(M)$.

c. Show that Q (the field of rational numbers) is not a free Z-Module (Z is the ring of integers).

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