Roll No: | | | | | | | | | |

The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences
M.Sc. Mathematics, 1st Semester
Semester End Examination, January 2023
Course Title : Real Analysis
Course Code : MAT014C103

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.
The figures in the right-hand margin indicate marks.

Section – A

1. Attempt **all questions.** (Maximum word limit 50) **2 x 8**

a. "If $f + g$ is integrable on $[a, b]$ then f and g both are integrable on $[a, b]$." Is this statement true? Justify.

b. Find $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r}{r^2+n^2}$.

c. Test the convergence of $\int_0^1 \frac{\sin^{-1} x}{x} dx$.

d. Define Riemann Stieltjes upper and lower sum.

e. Define pointwise convergence of a sequence of functions with an example.

f. Find the pointwise limit of (f_n) where $f_n(x) = tan^{-1}nx \ \forall \ x \in [0, \infty)$.

g. Give the definition of differentiability of a two-variable function with an example.

h. If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$ then find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$.

Section – B

2. Attempt **any two** of the following: **6 x 2**

a. Prove that if a bounded function $f : [a, b] \to \mathbb{R}$ is Riemann integrable with Riemann integral A then $A = \int_a^b f(x)dx$.

b. State and prove the fundamental theorem of calculus.

c. Find $U(f, P)$ and $L(f, P)$ of the function $f(x) = x^3 \ \forall x \in [0,1]$ where P partitions $[0,2]$ into five equal parts.

3. Attempt **any two** of the following: **7 x 2**

a. Let $f: [a, b] \to \mathbb{R}$ be a monotonic function and α is increasing function which is also continuous on $[a. b]$ then prove that f is Riemann-Stieltjes integrable.

b. Define absolutely convergent integral. Prove that every absolutely convergent integral is convergent. Is the converse true? Justify.

c. Show that integral $\int_0^{\pi/2} \frac{\sin^m x}{x^n} dx$ exists if $n < m + 1$.

4. Attempt **any two** of the following: **7 x 2**

a. State and prove the Dirichlet's Test for uniform convergence of a sequence of functions.

b. Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \in [0, \infty)$. Check the uniform convergence of (f_n) on $[0, \infty)$.

P.T.O.

c. State and prove Weierstrass M-Test. Using this test, show that the series $\sum_{n=1}^{\infty} f_n(x)$ is uniformly convergent, where $f_n(x) = x^n$, $x \in \left[0, \frac{1}{2}\right]$.

5. Attempt **any two** of the following: **7 x 2**

a. Find the radius of convergence and interval of convergence of the power series $\sum \frac{(n!)^2 x^{2n}}{(2n)!}$.

b. Using $\epsilon - \delta$ definition, show that $\lim_{(x,y)\to(0,0)} \frac{xy(x^2-y^2)}{x^2+y^2} = 0$.

c. Check the differentiability of the function $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at the point (0,0).

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