Roll No:

The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences M.Sc. Mathematics, 1st Semester Semester End Examination, January 2023 Course Title : Real Analysis Course Code : MAT014C103

Time: 3 Hours

Maximum Marks: 70

Note: Attempt all questions as per instructions given. The figures in the right-hand margin indicate marks.

Section – A

1.

Attempt all questions. (Maximum word limit 50)

2 x 8

- a. "If f + g is integrable on [a, b] then f and g both are integrable on [a, b]." Is this statement true? Justify.
- b. Find $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{r}{r^2 + n^2}$.
- c. Test the convergence of $\int_0^1 \frac{\sin^{-1} x}{x} dx$.
- d. Define Riemann Stieltjes upper and lower sum.
- e. Define pointwise convergence of a sequence of functions with an example.
- f. Find the pointwise limit of (f_n) where $f_n(x) = tan^{-1}nx \ \forall x \in [0, \infty)$.
- g. Give the definition of differentiability of a two-variable function with an example.
- h. If $z = e^{xy^2}$, $x = t \cos t$, $y = t \sin t$ then find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$.

Section – B

2. Attempt any two of the following:

- a. Prove that if a bounded function $f : [a, b] \to \mathbb{R}$ is Riemann integrable with Riemann integral A then $A = \int_a^b f(x) dx$.
- b. State and prove the fundamental theorem of calculus.
- c. Find U(f, P) and L(f, P) of the function $f(x) = x^3 \forall x \in [0, 1]$ where P partitions [0,2] into five equal parts.

3. Attempt any two of the following:

- a. Let $f:[a,b] \to \mathbb{R}$ be a monotonic function and α is increasing function which is also continuous on [a,b] then prove that f is Riemann-Stieltjes integrable.
- b. Define absolutely convergent integral. Prove that every absolutely convergent integral is convergent. Is the converse true? Justify.
- c. Show that integral $\int_0^{\pi/2} \frac{\sin^m x}{x^n} dx$ exists if n < m + 1.
- 4. Attempt any two of the following:
 - a. State and prove the Dirichlet's Test for uniform convergence of a sequence of functions.
 - b. Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \in [0, \infty)$. Check the uniform convergence of (f_n) on $[0, \infty)$.

7 x 2

7 x 2

- c. State and prove Weierstrass M-Test. Using this test, show that the series $\sum_{n=1}^{\infty} f_n(x)$ is uniformly convergent, where $f_n(x) = x^n$, $x \in [0, \frac{1}{2}]$.
- 5. Attempt any two of the following:

7 x 2

- a. Find the radius of convergence and interval of convergence of the power series $\sum \frac{(n!)^2 x^{2n}}{(2n)!}$.
- b. Using $\epsilon \delta$ definition, show that $\lim_{(x,y)\to(0,0)} \frac{xy(x^2-y^2)}{x^2+y^2} = 0.$
- c. Check the differentiability of the function $f(x, y) = \begin{cases} \frac{x^3 y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ at the point (0, 0).

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