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The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

M.Sc. Mathematics, 3rd Semester

Semester End Examination, January 2023

Course Title : Number Theory-I

Course Code : MAT014D301

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt **all questions**. (Maximum word limit 50) **2 x 8**

a. For any integer n, prove that $3 \mid n(2n^2 + 7)$.

b. If $ax + by = \gcd(a, b)$ for some integers x and y then show that $\gcd(x, y) = 1$.

c. If $\gcd(a, 35) = 1$ then show that $a^{12} \equiv 1 (mod\ 35)$.

d. For any odd integer a, show that $a^2 \equiv 1 (mod\ 8)$.

e. If $n = 2^{k-1}$ then show that $\sigma(n) = 2n - 1$ for $k \geq 2$.

f. Define Mobius function (μ). Also, find $\mu(30)$.

g. Define Legendre's symbol. Also, find $\left(\frac{2}{13}\right)$.

h. Define quadratic residue with an example.

Section – B

2. Attempt **any two** of the following: **6 x 2**

a. For positive integers a and b, prove that $\gcd(a, b) \times lcm(a, b) = ab$.

b. Given any two integers a and b with $b > 0$, prove that there exist unique integers q and r such that $a = bq + r$ where $0 \leq r < b$.

c. Find the values of x and y such that $\gcd(312, 2054) = 312x + 2054y$.

3. Attempt **any two** of the following: **7 x 2**

a. Find the remainders of 2^{50} and $\sum_{n=1}^{100} n!$ when divided by 12.

b. Show that the system of linear congruences $ax + by \equiv r (mod\ n)$ and $cx + dy \equiv s (mod\ n)$ has a unique solution if $\gcd(ad - bc, n) = 1$.

c. State and prove the converse of Wilson's theorem. Also, find the remainder of 15! when divided by 17.

4. Attempt **any two** of the following: **7 x 2**

a. If n is a positive integer and p is a prime then prove that the highest exponent e such that $p^e \mid n!$ is $\sum_{k=1}^{\infty} \left[\frac{n}{p^k}\right]$.

b. Define τ –function. Show that it is multiplicative. Also, show that $\prod_{d \mid n} d = n^{\tau(n)/2}$ where $n > 1$ is a positive integer.

P.T.O.

c. Define σ −function and find $\sigma(1246)$. Also, for $k \geq 2$, if $2^k - 1$ is prime and $n = 2^{k-1}(2^k - 1)$ then show that $\sigma(n) = 2n$.

5. Attempt **any two** of the following: **7 x 2**

a. State and prove Lagrange's theorem.

b. Prove that if p is a prime and $d|p-1$ then there are exactly $\phi(d)$ incongruent integers having order d modulo p.

c. Prove that if p is an odd prime and $\gcd(a,p) = 1$ then the congruence $x^2 \equiv a(mod\ p^n)$, $n \geq 1$ has a solution if and only if $\left(\frac{a}{p}\right) = 1$.

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