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# The Assam Royal Global University, Guwahati

## Royal School of Applied & Pure Sciences

**M.Sc. Mathematics, 3rd Semester**

**Semester End Examination, January 2023**

**Course Title: Mathematical Logic**

**Course Code: MAT014D303**

**Time: 3 Hours** **Maximum Marks: 70**

**Note: Attempt all questions as per instructions given**

**The figures in the right-hand margin indicate full marks for the questions**

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## SECTION A

1. **Attempt all questions:** **2x8**

a. Verify tautology of the statement $((q \vee r) \rightarrow (\sim r \rightarrow q))$.

b. Show that $(p \rightarrow q)$ is logically equivalent to $(p|(q|q))$.

c. Are $((\sim A \rightarrow B) \rightarrow C)$ and $(\sim A \rightarrow (B \wedge C))$ *wfs.* in the formal system $L$? Justify with reason.

d. A sequence $A_1, A_2, \ldots, A_n$ is a *proof* of $A_n$, if for each $i$ $(1 \leq i \leq n)$, either $A_i$ is _________ or $A_i$ ________.

e. Justify that the formal system $L$ is not complete.

f. Give an example of extension of the formal system $L$?

g. Translate into symbols using universal quantifier "No number is both odd and even".

h. Is $(A_1^1(x_2) \rightarrow A_1^3(x_1, a))$ a well-formed formula? Justify with reason.

## SECTION B

2. **Answer any two of the following:** **6x2**

a. i) Find the statement form in which only the connectives $\sim$ and $\wedge$ occur, which is logically equivalent to $(p \leftrightarrow q)$.

ii) Show that the singleton sets $\{\downarrow\}$ and $\{|\}$ are adequate sets of connectives.

b. Find the statement form in *conjunctive normal form* which are logically equivalent to the following $((p \vee q) \wedge r)$;

***P.T.O.***

c. i) Test the validity of the argument form $(p \rightarrow (q \vee r)), \sim r : \therefore (\sim q \rightarrow \sim p)$

ii) Suppose $\mathcal{A}_1, \dots, \mathcal{A}_n; \therefore \mathcal{A}$ is a valid argument form, prove that $\mathcal{A}_1, \dots, \mathcal{A}_{n-1}; \therefore \mathcal{A}_n \rightarrow \mathcal{A}$ is also a valid argument form.

3. **Answer any two of the following:** **7x2**

a. Without using Deduction theorem for $L$, show that for any $wfs.\ \mathcal{A}, \mathcal{B}, \mathcal{C}$ of $L$, $i)\ \{A, (\mathcal{B} \rightarrow (A \rightarrow C))\} \vdash_L (B \rightarrow C),\ ii) \vdash_L (A \rightarrow A)$ and $\vdash_L (\sim B \rightarrow (B \rightarrow A))$.

b. If $\Gamma \cup \{\mathcal{A}\} \vdash_L \mathcal{B}$, show that $\Gamma \vdash_L (\mathcal{A} \rightarrow \mathcal{B})$ where $\mathcal{A}$ *and* $\mathcal{B}$ are $wfs.$ of $L$ and $\Gamma$ is a (possibly empty) set of $wfs.$ of $L$.

c. Suppose $L'$ is a formal deductive system which differs from the formal system $L$ only in having the axiom 3 as $((\sim A \rightarrow \sim B) \rightarrow ((\sim A \rightarrow B) \rightarrow A))$ for any $wf. A, B$. Show that $i) \vdash_{L'} (\sim(A \rightarrow \sim B) \rightarrow A)\ ii) \sim A \vdash_{L'} (A \rightarrow B)$ and $iii)\ A \vdash_{L'} (\sim A \rightarrow B)$

4. **Answer any two of the following:** **7x2**

a. Suppose $L^*$ be consistent extension of $L$, Show that there is a complete extension $J$ of $L^*$.

b. Suppose $L^*$ be consistent extension of $L$ and $\mathcal{A}$ be a $wf.$ of $L$ which is not a theorem of $L^*$. Show that $L^{**}$ is also consistent, where $L^{**}$ is the extension of $L$ obtained from $L^*$by including $(\sim\mathcal{A})$ as an additional axiom.

c. Show that if $A$ is a theorem of the formal system $L$, then $A$ is a tautology. Using definition of valuation, prove that $((\sim A \rightarrow \sim B) \rightarrow (B \rightarrow A))$ is a tautology for any $wf. A, B$ of $L$.

5. **Answer any two of the following:** **7x2**

a. Show that a $wf.$ of the first order language $\mathcal{L}$ which is a tautology is true in any interpretation of $\mathcal{L}$.

b. i) If $A$ is a $wf.$ of $\mathcal{L}$ and $A$ is a tautology, then show that $A$ is a theorem of $K$.

ii) If $x_i$ does not occur free in $\mathcal{A}$, show that $\vdash_K ((\mathcal{A} \rightarrow (\forall x_i)\mathcal{B}) \rightarrow (\forall x_i)(\mathcal{A} \rightarrow \mathcal{B}))$

c. i) Show that $((\forall x_1)(\exists x_2)A_1^2(x_1, x_2) \rightarrow (\exists x_1)(\forall x_2)A_1^2(x_1.x_2))$ is not logically valid.

ii) Write alphabets of symbols of first order language $\mathcal{L}$. Give an example of atomic formula.

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