Roll No:

The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

M.Sc. Mathematics 3rd Semester

Semester End Examination, January 2023

Course Title: Mathematical Methods

Course Code: MAT014C301

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt all questions: **2 x 8 = 16**

a) Show that for the variational problems, the functional

$$I[y(x)] = \int_1^2 (y^2 + x^2y'^2)\ dx$$

satisfies the differential equation

$$x^2y'' + 2xy' - y = 0.$$

b) Find the extremal of the functional

$$\int_0^1 [(y')^2 + 12xy]\, dy$$

subject to the conditions $y(0) = 0\ and\ y(1) = 1$.

c) Show that the function $\phi(x) = 1$ is a solution of Fredholm integral equation

$$\phi(x) = \int_0^1 x\,(e^{xt} - 1)\phi(t)dt = e^x - x.$$

d) Find the solution of the integral equation

$$\phi(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 x\, t\phi(t)dt,$$

if the resolvent kernel of this equation is given by $\frac{6}{5}xt$.

e) Find the second approximation of the integral equation

$$\phi(x) = x - \int_0^x (x\ -\ t)\,\phi(t)dt$$

starting with initial approximation $\phi_0(x) = x$.

P.T.O.

f) Find the resolvent kernel of the integral equation
$\phi(x) = f(x) + \int_{log2}^{x} e^{t-x}\,\phi(t)dt.$

g) If $\alpha\{F(t)\} = \frac{S^2-S+1}{(2S+1)^2(S-1)}$, find $\alpha\{F(2t)\}$, applying the change of scale property.

h) Show that the finite Fourier sine transform of the function

$$F(x) = 2x, 0 < x < 4 \; is \; -\frac{32}{n\pi} cosn\pi.$$

Section – B

2. Attempt any two of the following questions: **6 x 2=12**

a) Find the extremal of the functional $I = \int_{2}^{\pi} (y'^2 - y^2)\, dx$ under the conditions $y(0) = 0, y(\pi) = 1$ and subject to the constraint $\int_0^{\pi} y\, dx = 1$.

b) Find the shortest distance between the point (1,0) and the ellipse $4x^2 + 9y^2 = 36$.

c) Show that for reflection of extremals in variational problems, the angle of incidence and the angle of reflection are equal.

3. Attempt any two of the following questions: **7 x 2=14**

a) Using the method of resolvent kernel, solve the integral equation

$$\phi(x) = x + \int_0^{\frac{1}{2}} \phi(t)\, dt.$$

b) Determine the eigenvalues and eigenfunctions for the following integral equation:

$$\phi(x) = \lambda \int_{-1}^{1} (5xt^3 + 4x^2t + 3xt)\, \phi(t)dt$$

c) Solve the integral equation:

$$\phi(x) = sinx - \frac{x}{4} + \int_0^{\frac{\pi}{2}} x\, t\phi(t)dt$$

4. Attempt any two of the following questions: **7 x 2=14**

a) Reduce the following initial value problem

$$\frac{d^2y}{dx^2} - sinx\frac{dy}{dx} + e^x y = 0; y(0) = 1, y'(0) = -1$$

into Volterra integral equation.

P.T.O.

b) With the aid of resolvent kernel, find the solution of the integral equation

$$\phi(x) = 1 + x^2 + \int_0^x \frac{1 + x^2}{1 + t^2}\phi(t)dt.$$

c) Solve:

$$\phi(x) = 1 + x + \int_0^x (x - t)\,\phi(t)dt \text{ with } \phi_0(x) = 1.$$

5. Attempt any two of the following questions: **7 x 2=14**

a) Evaluate

$$\alpha^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$$

by use of the convolution theorem.

b) Solve the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} = -8t,$$

using Laplace transform technique, given $x(0) = x'(0) = 0$.

c) Find the Fourier transform of

$$F(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos \frac{x}{2}\, dx.$$

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