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The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences
M.Sc. Mathematics 3rd Semester
Semester End Examination, January, 2023
Course Title: Graph Theory
Course Code: MAT014C302

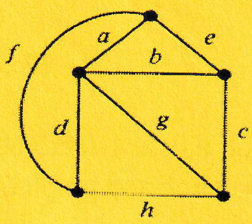
Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.
The figures in the right-hand margin indicate marks.

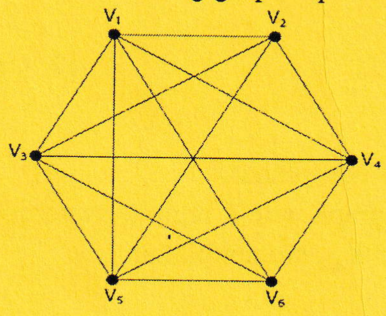
Section – A

Q1. Attempt **all questions.** (Maximum word limit 50) **[2 x 8 = 16]**

(i) Explain the Konigsberg Bridge problem.
(ii) Explain isomorphism of graphs with a proper example.
(iii) Define spanning tree. Find all spanning trees of the following graph



(iv) Justify that the incident matrix of a disconnected graph with two components G_1 and G_2 can be written in block diagonal form.
(v) Draw the spanning cycles of K_5.
(vi) Define point covering and line covering of graph.
(vii) State and justify Euler's polyhedron formula with a proper example.
(viii) Is the following graph a planar graph? Justify.



Section – B

Q2. Answer any two of the following: **[6 × 2 = 12]**

(i) Justify that if a graph G is disconnected then its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other in V_2. Is the converse true? If yes, then prove it.
(ii) What is directed graph? Discuss different types of digraphs.
(iii) Prove that a connected graph with even degree vertices is a Euler graph.

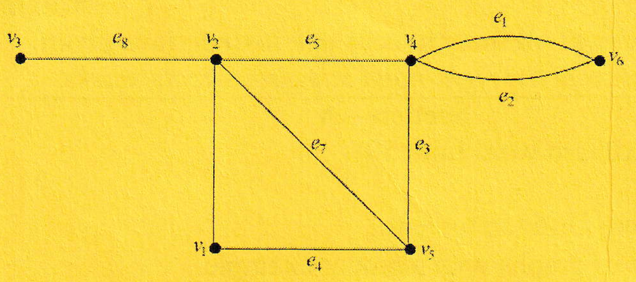
P.T.O.

Q3. Answer any two of the following: **[7 × 2 = 14]**

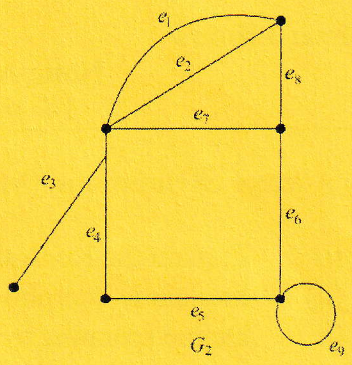
(i) What is tree in a graph? How many edges are there in a tree with n- vertices? Prove it. Also, give a proper example.

(ii) Define block in graph. Suppose G is a connected graph with at least three vertices. Then show that G is a block if every two points of G lie on a common cycle.

(iii) Define the cycle matrix of any graph G. Find the cyclic matrices of the following graphs



G_1



G_2

Q4. Answer any two of the following: **[7 × 2 = 14]**

(i) What is n –factorisation? Justify that the complete graph K_{2n} is 1-factorable.

(ii) Prove that the following statements are equivalent for a connected graph G,
  (a) G is Eulerian.
  (b) Every point of G has even degree.
  (c) The set of lines of G can be partitioned into cycles.

(iii) For any non-trivial connected graph G, justify that
$\alpha_0 + \beta_0 = p = \alpha_1 + \beta_1$, where $\alpha_0, \beta_0, \alpha_1, \beta_1$ have their usual meaning.

Q5. Answer any two of the following: **[7 × 2 = 14]**

(i) Prove that the complete graph of five vertices is non-planar.

(ii) Show that in any simple, connected and planar graph with f regions, n vertices and e edges ($e > 2$), $e \leq 3n - 6$. Justify that K_5 and $K_{3,3}$ are nonplanar.

(iii) State and prove the five-colour theorem.

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