Roll No: [ ][ ][ ][ ][ ][ ][ ][ ][ ]

The Assam Royal Global University, Guwahati
Royal School of Applied & Pure Sciences
UG GE-2/GE-6
Semester End Examination, January 2023
Course Title: Fundamentals of Mathematics
Course Code: MAT012G102

Time: 3 Hours   Maximum Marks: 70

Note: Attempt all questions as per instructions given.
The figures in the right-hand margin indicate marks.

SECTION – A

1. **Answer the following questions:** $[2 \times 8 = 16]$
   a. Distinguish between scalar matrix and square matrix.
   b. Show that the matrix $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is singular.
   c. Find f_x and f_y for $f(x,y) = 5x^2 + 2xy + 6y^2$
   d. Integrate $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ with respect to x, where x varies from 0 to 4.
   e. If by rotation of the rectangular axes the equation $17x^2 - 18xy - 7y^2 = 1$ reduces to the form $ax^2 + by^2 = 1$, find the angle through which the axes are rotated.
   f. What is the condition that the line $lx + my + n - 0$ is a tangent to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$?
   g. Find $L^{-1}\left\{\frac{s-2}{(s-2)^2+9}\right\}$.
   h. Evaluate $L\{e^{-2t}\cos 2t\}$.

SECTION – B

2. **Answer any two of the following questions:** $[6 \times 2 - 12]$

   a. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$.

   b. Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 3 & 2 & -4 & 1 \\ 5 & -3 & 2 & 6 \end{bmatrix}$ reducing it to echelon form.

   c. Find the characteristic roots of the matrix:

   $$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$ ***P.T.O***

3. **Answer any two of the following questions:** $[7 \times 2 = 14]$

    a. Verify Rolle's theorem for the functions: (i) $f(x) = x^3 - 6x^2 + 11x - 6$ in the interval $[1,3]$ (ii) $f(x) = x^2 - 1$ in the interval $[1,2]$

    b. Evaluate $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$, if (i) $u = \log(x^2 + y^2)$ (ii) $u = 2x^2 + 4xy + 5y^2$

    c. Find the reduction formula for $\int \sin^n x \, dx$ and evaluate $\int_0^{\frac{\pi}{2}} \sin^n x \, dx$.

4. **Answer any two of the following questions:** $[7 \times 2 = 14]$

    a. General equation of second degree represents a parabola under certain condition- Establish it.

    b. Reduce the equation $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ to the standard form.

    c. Find the equation of the chord of contact of tangents from a given point (x_1, y_1) to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

5. **Answer any two of the following questions:** $[7 \times 2 = 14]$

    a. Find the Laplace transform of the following:
       (i) $t\sin^2 3t$ (ii) $(t + 2)^2 e^{2t}$

    b. Obtain inverse Laplace transform of the following:

       (i) $\frac{s+2}{s^2 - 4s + 13}$ (ii) $\frac{1}{s(s+1)^2}$

    c. Solve $y''(t) + y(t) = t$, $y'(0) = 1$, $y(\pi) = 0$.

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