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The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

M.Sc. Mathematics, 4th Semester

Semester End Examination, June 2023

Course Title: Fuzzy Set Theory

Course Code: MAT014D402

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. **Answer all questions (Maximum word limit 50):** **[2×8 = 16]**

a. Suppose membership function for the linguistic variable "heavy" and "light" are defined as follows

$$\text{"}Heavy\text{"} = \left\{\frac{0.2}{5}, \frac{0.4}{8}, \frac{0.6}{12}, \frac{0.8}{20}, \frac{1}{80}\right\}$$

Develop membership functions for the following linguistic phrases

(i)Very Heavy

(ii)Fairly Heavy $\left(= [Heavy]^{\frac{2}{3}}\right)$

b. What do you mean by α –cuts of fuzzy set? Find the α –cuts of the following Triangular Fuzzy Numbers

$A = [-1 \quad 1 \quad 3]$

c. Use max-min composition rule to find the composition between the relation

$$\mu_{R_1} = \begin{bmatrix} 0.2 & 0.3 & 0.8 \\ 0 & 0 & 1 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}, \mu_{R_2} = \begin{bmatrix} 0 & 0.2 \\ 1 & 0.9 \\ 0 & 0.5 \end{bmatrix}$$

d. When does a fuzzy relation said to be a compatibility relation?

e. Show that for every $A \in P(X)$, any necessity measure Nec on $P(X)$ and associated possibility measure, Pos satisfy $Nec(A) > 0 \Rightarrow Pos(A) = 1$

f. What is meant by semi continuous Fuzzy Measure?

g. Derive, $\sim A$ for a three valued logic where truth values are 0 , 1/2 and 1.

h. What are the different classifications of fuzzy proposition?

Section – B

2. **Answer any two of the following questions:** **[7× 2=14]**

a. Given fuzzy sets $A_1 = \{(-1,0.5), (0,1), (1,0.5)\}$ on the universal set

a. $X = \{-1, 0, 1,\}$. Let a mapping $f: X \to Y$ be defined by $y = x^2 + 2$. Use extension principle to obtain a fuzzy set B on Y.

b. Define disjoint sum and bounded sum. If A and B are two fuzzy sets defined as

$$A = \{(-1, 0.6), (0, 0.7), (1, 0.4), (2, 0.8)\}$$

$$B = \{(-1, 0.4), (0, 0.6), (1, 0.6), (2, 0.2)\}$$

Find disjoint sum and bounded sum.

c. Find the solution of the fuzzy equation $A + X = B$ where, A and B are two fuzzy numbers given by $A = [3 \quad 4 \quad 5]$ and $B = [12 \quad 20 \quad 32]$

3. **Answer any two of the following questions:** **[7× 2= 14]**

a. Given a fuzzy relation

$$R(x,y) = \begin{bmatrix} 1.0 & 0.0 & 0.8 & 0.0 & 0.6 & 0.8 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.6 & 0.0 & 0.5 & 0.0 \\ 0.8 & 0.0 & 1.0 & 0.8 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.6 & 0.8 & 1.0 & 0.0 & 0.0 & 0.8 \\ 0.6 & 0.0 & 0.0 & 0.0 & 1.0 & 0.6 & 0.0 \\ 0.8 & 0.5 & 0.0 & 0.0 & 0.6 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Is $R(x,y)$ an equivalence relation ? Justify your answer.

b. Prove that Max-min composition is associative i.e. $R_1 o (R_2 o R_3) = (R_1 o R_2) o R_3$ using the following three fuzzy relations

$$R_1 = \begin{bmatrix} 0.2 & 0.9 & 0.6 & 1.0 \\ 0.0 & 0.5 & 0.8 & 0.3 \\ 0.1 & 0.0 & 0.4 & 0.6 \end{bmatrix} \quad R_2 = \begin{bmatrix} 1.0 & 0.2 & 0.4 & 0.0 \\ 0.0 & 0.9 & 1.0 & 0.8 \\ 0.0 & 0.6 & 0.0 & 0.4 \\ 0.3 & 0.5 & 0.0 & 0.2 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.2 & 0.2 & 0.6 \\ 0.0 & 0.3 & 0.4 \\ 0.7 & 0.4 & 1.0 \\ 1.0 & 0.8 & 0.4 \end{bmatrix}$$

c. Consider the fuzzy equation

$$p\ o \begin{bmatrix} 0.8 & 0.5 & 0.9 \\ 0.7 & 0.7 & 0.4 \\ 0.5 & 0.4 & 0.5 \end{bmatrix} = [0.5 \quad 0.5 \quad 0.4]$$, verify the existence and find the maximal solution.

4. **Answer any two of the following questions:** **[7× 2= 14]**

a. A criminal under trial is facing trials. The prosecutors presented three evidences E_1,E_2 and E_3 before two experts. After careful examination of these three evidences, the experts provided the basic probability assignments **m1** and **m2** for different focal points are specified in the following table:

Focal Elements	Expert 1	Expert 2
	m1	**m2**
E_1	0.05	0.15
E_2	0.05	0.05
E_3	0.05	0.05
$E_1 \cup E_2$	0.1	0.05
$E_2 \cup E_3$	0.2	0.1
$E_2 \cup E_3$	0.05	0.1
$E_1 \cup E_2 \cup E_3$	0.5	0.5

(i) Find the Belief measures for each of the focal elements.

(ii) Find combined probability assignment **m1, 2**(E_1) using Dempster's rule of combination.

b. Write short notes on Necessity and Possibility measures of Possibility theory.

c. Describe Plausibility Measure. Prove that,

$$Pl(A) + Pl(A^c) \geq 1$$

5. Answer any one of the following questions: **[12× 1= 12]**

a. (i) Find the truth values of $A <=> B$ proposed by Lukasiewicz.
(ii) Describe Unconditional qualifies and unqualificd proposition of fuzzy logic.

b. (i) Describe Zadeh implication relation with example.
(ii) Use Zadeh implication relation and Fuzzy max-min composite relation, to infer
S' = Speed *is* ABOVE NORMAL where the given rule base is as follows:

Rule 1: IF height is SIIORT, THEN speed is LOW
Rule 2: IF height is MEDIUM, THEN speed is MODERATE

where fuzzy set for height (in feet) and speed (m/s) is given below:

$H_1(\text{Height}) = \text{SHORT} = \left\{\frac{0.6}{5}, \frac{0.3}{6}, \frac{0.1}{7}\right\}$ $H_2(\text{Height}) = \text{MEDIUM HEIGHT} = \left\{\frac{0.8}{5}, \frac{0.4}{6}, \frac{.3}{7}\right\}$

$S_1(\text{Speed}) = \text{LOW} = \left\{\frac{0.9}{5}, \frac{0.2}{7}, \frac{0.1}{9}\right\}$ $S_2(Speed) = \text{MODERATE} = \left\{\frac{0.6}{5}, \frac{.8}{7}, \frac{.7}{9}\right\}$

Given $H'(\text{Height})$ = height *is* ABOVE AVERAGE $= \left\{\frac{0.5}{5}, \frac{0.9}{6}, \frac{.8}{7}\right\}$.

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