Behavioural Study of Visco-Elastic Fluid Flow Characterized by Walters Liquid (Model B')

A THESIS SUBMITTED AS PARTIAL FULFILLMENT

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

MATHEMATICS

To

By

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May 2024

DECLARATION

I hereby declare that the content embodied in the PhD thesis entitled "**Behavioural Study of Visco-Elastic Fluid Flow Characterized by Walters Liquid (Model B')**" is the result of research work carried out by me in the Department of Mathematics, The Assam Royal Global University, Guwahati, India, under the supervision of Dr. Kamal Debnath.

In keeping with the general practice of reporting research observations, due acknowledgments have been made wherever the work described is based on the findings of other researchers.

Further, I declare that this thesis as a whole or any part thereof has not been submitted to any university (or institute) for the award of any degree/ diploma.

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ACKNOWLEDGEMENTS

I am extremely grateful and indebted to my Supervisor Dr. Kamal Debnath, Associate Professor & Head, Department of Mathematics, Royal School of Applied and Pure Sciences, The Assam Royal Global University, for his valuable advice, resourceful guidance and constant encouragement in compilation of this thesis.

I shall remain highly grateful towards Prof. (Dr.) Anuradha Devi, Dean, Royal School of Applied & Pure Sciences, The Assam Royal Global University, for believing me and encouraging me for this entire research work.

I would also like to thanks all faculty members of Department of Mathematics, Royal School of Applied & Pure Sciences, The Assam Royal Global University, for their encouragement, love and valuable support during the progress of my research work.

I am always thankful to The Assam Royal Global University, the pioneer of higher education in north-east India for providing me all the necessary facilities to carry out my research work.

Lastly, I convey my gratitude towards my family, friends, without their support and encouragement it would have been certainly much harder to write this doctoral thesis.

Thanks all of you

Signature

Bikash Koli Saha

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Abbreviations

U(x)	Main stream velocity
u	Velocity in the x-direction
v	Velocity in the y-direction
ρ	Fluid density
μ	Coefficient of fluid viscosity
ν	Kinematic viscosity
σ	Electrical conductivity of the fluid
k_0	Visco-elastic parameter
B(x)	Magnetic field
B_0	Constant Magnetic field
k_1	Non-Newtonian parameter
M	Magnetic parameter
η	Similarity variable
U	Stretching velocity
U_0	Reference velocity
N	Velocity slip factor
N_1	Initial value of velocity slip factor
v_w	Suction or blowing parameter
$V(x)$	Velocity of suction or blowing
V_0	Initial strength of suction or blowing
λ	velocity slip parameter
S	Modified Suction or blowing parameter
τ	skin friction coefficient
K	Permeability parameter
k^*	Modifed Permeability parameter
δ	Velocity slip parameter
β	Thermal slip parameter
Pr	Prandtl parameter
C	Concentration
C_W	Plate concentration
C_∞	Free stream concentration
n	Power-law exponent
D	Diffusion coefficient
R_0	Constant

$R(x)$	Variable reaction rate
L	Reference length
Re_x	Local Reynolds number
Da_x	Local Darcy number
S_c	Schmidt number
β	Thermal slip factor/ Reaction rate parameter
C_p	Specific heat
T	Temperature
K	Thermal conductivity of fluid
T_w	Plate temperature
T_∞	Free stream temperature
U_w	Plate velocithy/stretching parameter
U_∞	Free stream velocity
A	Velocity ratio parameter
A	Unsteadiness parameter
A	Slip Length
Ψ	Stream function and
T_0	Rate of surface temperature
c	Stretching parameter
a	Straining parameter/velocity slip parameter
G_1	Velocity slip factor
G_0	Initial value of velocity slip factor
H_1	Thermal slip factor
H_0	Initial value of thermal slip factor

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