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Chapter 1

General Introduction

1.1 Introduction

The discipline of physics concerned with the investigation of fluids, whether they are stationary or moving is known as Fluid mechanics. It focuses on the statics, kinematics, and dynamics of fluids. Fluid statics involves analyzing the behavior of a fluid when it is not in motion, while fluid kinematics focuses on understanding the movement of a fluid without the influence of pressure forces. Fluid dynamics, on the other hand, concentrates on studying the motion of a fluid when pressure forces are present. Hydraulics, which arose from experimental investigations, and hydrodynamics, which arose from theoretical studies, are two branches of flow science. However, in recent years, both have been integrated into a single field known as fluid mechanics.

Fluid which includes Liquids and gases share certain common qualities, but they also have their unique set of characteristics. A gas can be compressed easily, whereas liquids cannot. Regardless of the container's size or shape, a given mass of liquid occupies a set volume. Until it is confined by a restricting vessel, a gas will keep expanding since it has no fixed volume. Furthermore, compressibility and viscosity are two important properties of a fluid from the standpoint of fluid mechanics. A solid's elasticity is visible in tension, compression, and shearing stress, but a fluid's elasticity is seen primarily in compression.

In other words, when a fluid is compressed, it increases its pressure in an attempt to maintain its original volume. Compressibility is the term for this property. Furthermore, when two layers of a fluid move over each other, the fluid exhibits resistance. This property is known as viscosity. Mathematically, a fluid is a material that undergoes continuous deformation when subjected to even the slightest amount of shear stress.

Fluid mechanics finds practical use in various fields, including the development of aircraft wings and turbines, as well as the examination of blood circulation within the human body. It has become an essential field of study for engineers and scientists who work in industries such as aerospace, automotive, and energy.

1.2 Classification of Fluids

Some of the most common types of fluid flow issues include:

1.2.1 Viscous and Inviscid flow

When the frictional force significantly affects the fluid flow, the flow is known as viscous flow. Viscosity quantifies the frictional force between two fluid layers as they move. Viscous flow is exemplified by boundary layer flow. In viscous flow, there is a shearing stress or tangential force between two contiguous layers, as well as a pressure or normal stress. If the effects of viscosity are disregarded in the fluid guided equation, the flow can be classified as inviscid. Only pressure or normal stress exists in inviscid flow between two neighbouring layers.

1.2.2 Internal and External flow

External flow denotes the movement of an unconfined fluid through a surface, whereas internal flow regarded as fluid flow that is totally bounded by the surface. In contrast to internal flow, which occurs in a pipe or duct, external flow occurs when fluid moves over a flat plate. When there is a free surface present and the duct is only partially filled, the flow is categorized as open channel flow. In internal flows, viscosity has a notable impact, but in external flows, it is limited to boundary layers formed at the surface of solids.

1.2.3 Compressible and Incompressible flow

The flow of fluids where there are significant changes in density due to changes in pressure and temperature is called compressible flow. In this type of flow, the compressibility of the fluid must be taken into account. In contrast, incompressible flow is the flow of fluids whose density remains constant despite variations in pressure and temperature. In this formof transport, the fluid's compressibility is negligible and can be disregarded practically.

1.2.4 Natural and Forced flow

Natural flow, also known as natural convection, refers to the movement of fluid resulting from density differences caused by temperature variations. In natural flow, the fluid moves spontaneously due to buoyancy forces without any external influence or mechanical means. Forced flow, on the other hand, refers to the movement of fluid that is driven by external forces or mechanical means such as pumps, fans, or blowers. In forced flow, the fluid is forced to move through a system or conduit, regardless of the density differences or temperature variations.

1.3 Non-Newtonian Fluids

The usual linear connection between shear stress and shear rate, as seen in Newtonian fluids, fails down in non-Newtonian fluids. Instead, their flow behavior is more complex and cannot be characterized by a straightforward linear relationship between these two parameters. Non-Newtonian fluids have a viscosity that changes with the magnitude of the shear stress being applied to them. Examples, include ketchup, silly putty, toothpaste, cornstarch and water mixture, blood, multigrade engine oil and many other complex fluids. Non-Newtonian fluids have the characteristic of exhibiting a reversible transition between the states of liquid and solid, which is contingent upon the magnitude of force applied. For example, a mixture of cornstarch and water can behave like a solid when pressure is applied, but will flow like a liquid when the pressure is released.

Complex mathematical models experimental techniques are needed to explain the behaviour of non-Newtonian fluids than those used for Newtonian fluids. Therefore, research in this area is ongoing, and new discoveries are constantly being made. Numerous processing industries regularly confront non-Newtonian fluids, such as food processing, oil recovery, inkjet printing, and blood flow in the human body. Understanding the rheological behaviour of non-Newtonian fluids is crucial for designing and optimizing these processes.

1.4 Application of Non-Newtonian Fluids

Non-Newtonian fluids have significant role in fluid dynamics because they are used in a variety of fields, including friction reduction, oil pipeline friction reduction, surface-active agent application in heating and cooling systems and flow tracers. A highly effective flow tracer has been constructed by combining non-Newtonian features into a dye-streak. Moreover, in contemporary technical challenges and industrial processes, there is a need to compute the frictional drag of objects in order to reduce turbulent flow in polymer solutions. Developed countries such as Spain, Villanueva de Tapia, Israel, and Germany have implemented a non-Newtonian fluid speed bump in their roads. Because, when subjected to intense pressure at high velocities, non-Newtonian fluids shift phase from liquid to solid. The Spanish company devised this speed bump that will not hamper slow drivers but will nevertheless halt vehicles travelling too fast. A non-Newtonian fluid is used in the next generation of fireproof lithium batteries. Smartphones, computers, tablets, cameras, strobe heads, and other electronic gadgets are examples of fireproof lithium batteries.

Later, Oak Ridge National Laboratory developed an impact-resistant electrolyte to avoid utilizing a normal additive electrolyte. This modified electrolyte freezes, preventing the electrolyte from coming into contact with each other. As a result, the risks of producing fire following damage are decreased or eliminated. Oak Ridge National Laboratory researchers introduced a novel chemical to batteries that makes them armored on the inside. Powdered silica spheres are used as a component. The electrolyte in the entire battery will stiffen like cornstarch before returning to a liquid state. As a result, non-Newtonian fluids act on the principle of shear thickening, and as shear force is applied, suspension becomes harder. When hit, collided, or struck, the batteries turn into a rock-like hard substance, then return to being batteries when it is safe.

Consequently, numerous applications rely on the concept of non-Newtonian fluids owing to their viscosity properties, which reduces friction in many circumstances and thus reduces the risk of an accident, which is a major breakthrough in modern science and technology.

1.5 Classification of non-Newtonian Fluids

The broad classification of non-Newtonian fluids is based on their flow behaviour. Here are the main categories:

- Shear-thinning fluids: These fluids become less viscous when sheared at higher rates. The viscosity diminishes as the shear rate rises. Examples include ketchup, mayonnaise, and blood.

- Shear-thickening fluids: These fluids become more viscous when sheared at higher rates. The viscosity increases as the shear rate increases. Examples include cornstarch and water mixture, and some industrial slurries.
- Yield stress fluids: These fluids require a certain amount of force (i.e. yield stress) to begin flowing. Examples include toothpaste, shaving cream, and mud.
- Viscoelastic fluids: These fluids exhibit both viscous and elastic behaviour, meaning they can store and release energy. Examples include polymer solutions and gels.
- Thixotropic fluids: These fluids become less viscous over time when left undisturbed. Examples include some types of paint and ink.
- Rheopectic fluids: These fluids become more viscous over time when left undisturbed. Examples include some types of drilling fluids and lubricants.

It's crucial to classify non-Newtonian fluids for understanding their properties and behaviour, as well as for choosing the appropriate mathematical model to describe their flow behaviour.

1.6 Visco-elastic Fluids

A visco-elastic fluid is a kind of non-Newtonian fluid that combines the properties of a viscous fluid with those of an elastic fluid. These fluids are distinguished by their capacity for energy storage and recovery under deformation, making them useful in many industrial and engineering applications. At low shear rates, elastic properties of elastico-viscous fluids predominate, while at high shear rates, viscous properties predominate. This behaviour arises because the fluid contains long-chain polymers that can distort and then return to their original shape. Elastico-viscous fluids possess distinctive flow characteristics that render them valuable in a wide range of applications. As lubricants, they lessen machine wear and friction; as suspensions, they prevent solid particles from settling out of solution. They are also used in the food industry to enhance the consistency and flavour of condiments like sauces and dressings. One important consideration when working with elastico-viscous fluids is their sensitivity to changes in temperature, pressure, and other environmental factors. These fluids can undergo significant changes in viscosity and flow behavior depending on the conditions they are exposed to, which can affect their performance in industrial and engineering applications.

1.7 Boundary layer

A boundary layer is a thin fluid layer that directly interacts with a solid surface. At boundary layer velocity ranges from zero at the surface where fluid particle sticks to u_∞, at the boundary, with V = 0.99u, u = free stream velocity, v = velocity within boundary layer and δ = boundary layer thickness. The fluid behavior inside boundary layer is distinct from those at outside of it, due to the presence of frictional forces and shear stresses at the solid surface. The boundary layer is typically classified as laminar or turbulent. For a laminar boundary layer, the fluid flows smoothly and in parallel layers, with minimal interaction between these layers.

The chaotic nature of fluid flows is noticed in the turbulent boundary layer where eddies and vortices help to mix the fluid to improve heat and mass transmission. The fluid's velocity, its viscosity, and the roughness of the solid surface are used to create an estimate of the boundary layer's thickness. It enhances with reducing velocity of fluid and growing viscosity. When a rough surface is present, the boundary layer's thickness can also increase as per need for the fluid to navigate around the surface irregularities.

The boundary layer serves a vital purpose in fluid dynamics, as it is responsible for initiating the overall dynamics of the system through its interaction with the boundary surface. Boundary layer flow is frequently used in modern science and technology, as well as modern engineering and industrial processes, to calculate the friction drag of bodies and their varied activities in this field. The boundary layer concept can help engineers and scientists optimize the design and performance of systems that involve fluid flow over solid surfaces.

1.8 Governing equations of fluid motion

a) The continuity equation asserts that in every steady state operation, fluid mass enters and departs a system at the same rate. Physically this means that energy can neither be created nor destroyed. In differential form, the equation of continuity is expressible as under:

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad (1.8.1)$$

where, $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$

Here, (u, v, w): velocity component, (x, y, z): axis coordinate, t: time, ρ: density.

This equation must hold at every point of fluid domain.

Here $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$ measures the rate at which the volume of a fluid element at any point (x, y, z) expands. $\frac{D}{Dt}$ is called the material or particle or substantial derivative or differentiation tracing fluid's motion. Also, $\frac{\partial}{\partial t}$ represents local derivative which is connected with time variation at fixed location. Further, $u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$ is termed as convective derivative which is related to the variation of a physical quantity.

In vector form, the continuity equation is expressible as $\frac{Dp}{Dt}+\rho(\vec{\nabla}.\vec{q})=0$ (1.8.2)

Here $\vec{q}=\hat{\imath}u+\hat{\jmath}v+\hat{k}w$ is the fluid velocity vector and $\vec{\nabla}=\ \hat{\imath}\frac{\partial}{\partial x}+\hat{\jmath}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$ is the nabla or, del operator. For incompressible fluid, density ρ is constant and thus, $\vec{\nabla}.\vec{q}=0$ and in cartesian coordinates, it reduces as $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ (1.8.3)

b) Navier-Stokes equations of motion

In vector form, the Navier-Stokes equation for an incompressible and viscous flow is as follows:

$$\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\vec{\nabla})\vec{q}=\ \vec{F}-\frac{1}{\rho}\vec{\nabla}P+v\vec{\nabla}^2\vec{q} \qquad (1.8.4)$$

where $\vec{q}$: fluid velocity vector and $\vec{F}$: total body force per unit mass.

The corresponding set of equations in Cartesian forms may be obtained by substituting $\vec{q}=\hat{\imath}u+\hat{\jmath}v+\hat{k}w$ and $\vec{F}=\hat{\imath}F_x+\hat{\imath}F_y+\hat{\imath}F_z$, where (u, v, w) and $(F_x, F_y,\ F_z)$ denote velocity and force components along coordinate axes. LHS of the equation represents inertia force terms and the RHS contain body force and pressure force per unit mass and viscous force respectively. The body forces may include not only gravity but also electromagnetic and centrifugal forces as well. The Navier-Stokes equations jointly with the continuity equation form the basis of viscous fluids mechanics.

1.9 Walters Liquid (Model B')

The Walters liquid, also known as the B-prime model or Model B′, is a non-Newtonian fluid model that is commonly used to describe the rheological behavior of polymer solutions and melts. This model was developed by Walter in the 1970s and is based on the concept of reptation, which describes the movement of long-chain polymers through a viscous medium. This model assumes that the polymer chains are confined to a tube-like region in the fluid, and that the movement of the chains is restricted by the frictional forces between the chains and the walls of the tube. The model also incorporates the effects of shear rate and temperature on the fluid's behaviour. This model is characterized by two parameters: the relaxation time and the characteristic time. The period of time needed for polymer chains to regain their original shape after being deformed is referred to as the relaxation period. The characteristic time describes the time scale of the flow behavior of the fluid.

The B-prime model has been used to describe the rheological behavior of a wide range of polymer solutions and melts, including polyethylene, polypropylene, and polystyrene. It has also been used to analyze the movement of a wide variety of complex fluids, including emulsions and suspensions. Overall, the B-prime model is a useful tool for understanding the rheological behavior of non-Newtonian fluids, particularly those that involve long-chain polymers. Its ability to accurately describe the flow behavior of these fluids has made it an important model in the field of polymer science and engineering.

For liquids with short memory, as Walters (1962) shown,

$$\tau^{ik} = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\delta}{\delta t} e^{(1)ik} \tag{1.9.1}$$

where, $\eta_0 = \int_0^\infty N(\lambda) d\tau$ and $k_0 = \int_0^\infty \lambda N(\lambda) d\tau$.

Convective derivative for contrvariant tensor is

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - \frac{\partial v^k}{\partial x^m} b^{im} - \frac{\partial v^i}{\partial x^m} b^{mk} \tag{1.9.2}$$

where v_i denotes velocity vector. Taking into consideration very short memories, Walters liquid (Model B') is close approximation to it so that the terms involving

$$\int_0^\infty \lambda^n N(\lambda)\, d\lambda, \lambda \geq 2$$

have been neglected. As this simplified model accounts for extremely short memories, it provides a close enough approximation of Walters liquid (Model B').

1.10 Homotopy Perturbation method (HPM)

HPM is a robust analytical technique for finding solution of differential equation which is non-linear in nature and are difficult to solve using traditional methods. The method was first introduced by J.H. He in 1999 and has since been widely used in various fields of engineering and science.

HPM involves the construction of a homotopy function, which is a continuous function that connects the initial conditions of the differential equation to its exact solution. The homotopy function is then perturbed by a small parameter, which is used to decompose a differential equation into a set of linear and nonlinear equations.

The following steps are involved in this method:

- Create the homotopy function that links the differential equation's starting conditions to its precise solution.
- Perturb the homotopy function by introducing a small parameter, which help to transform the differential equation into linear and nonlinear systems.
- Solving series of linear and nonlinear equations using iterative techniques.
- Approximate solution for differential equation is obtained by taking the limit of the perturbation parameter to zero.

The HPM is superior to other numerical and analytical techniques in a number of ways. It is an easy and effective technique that doesn't need the use of sophisticated numerical algorithms or high-performance computing tools. It's also a versatile approach that may be used for a broad variety of nonlinear differential equations in engineering, science, and other disciplines.

However, the HPM also has some limitations. It may not be suitable for solving differential equations with singularities or for systems of differential equations. It also requires the selection of appropriate initial conditions and perturbation parameters, which can be

1.11 Magneto-hydrodynamic (MHD)

Magneto-hydrodynamics, also known as hydro-magnetic or magneto-fluid dynamics, is a branch of physics that deals with the movement of a fluid that conducts electricity when magnetic field is present in the electro-fluid-mechanical energy dialogue. The term "magneto-hydrodynamics" is a combination of "magneto" (related to magnetic fields), "hydro" (related to liquids), and "dynamics" (related to movement).

MHD is based on the fact that when a fluid is electrically conductive, it can interact with magnetic fields. When a magnetic field is applied to an electrically conducting fluid, it causes the fluid to develop an electric field, which consequently produces a magnetic field. The interplay between the magnetic field and the electrically conducting fluid can give rise to a diverse array of fascinating phenomena, such as the generation of electric currents, the production of magnetic fields, and the generation of heat and energy.

MHD has many practical applications in engineering and science. For example, MHD is used in power generation to transform the kinetic energy of a fluid into an electrical output. MHD is also used in the design of propulsion systems for spacecraft and submarines. MHD is also used in the study of astrophysical phenomena, such as the behavior of plasmas in the Sun and other stars. MHD is particularly important for understanding the dynamics of the Sun's corona, which is a highly electrically conductive region that is strongly influenced by magnetic fields.

1.12 Heat Transfer

Heat transfer is the transmission of thermal energy from a higher-temperature region to a lower-temperature region. The thermal energy transfer continues until the object and its surroundings reaches thermal equilibrium. Heat flow cannot be directly measured in terms of energy transfer. Many disciplines of science and engineering are interested in assessing temperature distribution and heat flow, including chemical process industry, automotive engineering, thermal insulation, thermal processing, power plant engineering, bio-hear transmission, and aerospace technology etc. The different modes of heat transfer are described below:

- Conduction is the process of heat transfer within a substance through the collision of molecules. This type of heat transfer involves the movement of heat from an area of greater temperature to one of lower temperature. Metals are good conductors of heat, while non-metals are generally poor conductors.

- Convection refers to the overall process of heat transfer that takes place when a fluid moves over a solid body or when there is difference intemperature between the fluid and solid surface. This heat transfer is caused by the relative motion of the fluid and the surface.

  Forced convection occurs when fluid motion is purposefully created by pump, blower, wind, etc. The heat transmission is considered to be free connection if the fluid motions are set up by the buoyancy effect generated by density differences caused by temperature differences in the fluid. Both mechanisms are equally significant in a mixed connection.

- Radiation is the process of transferring thermal energy through empty space using electromagnetic waves. Unlike conduction or convection, radiation does not require any existence of material medium. All solids, solid bodies, liquids, and gases release energy as radiation at normal or very high or low temperatures. Many heating and cooling activities and equipment, such as fossil fuel burning, furnace operation, thermal crawling, and petroleum refinery tube stills, rely heavily on heat transfer by radiation.

1.13 Mass Transfer

Mass transfer refers to the process of transferring mass from one location to another. It is an important field of study in chemical engineering, materials science, and other related fields. Mass transfer can occur in three main ways: diffusion, convection, and mass transfer by chemical reaction.

- Diffusion is the process through which molecules go from a high concentration to a low concentration region. Diffusion occurs due to the random motion of molecules and is driven by a concentration gradient.
- Convection is the transfer of mass due to the movement of a fluid. In convective mass transfer, the fluid carries the mass from one point to another. Convection can occur naturally (natural convection) or artificially (forced convection).
- Mass transfer by chemical reaction occurs when a chemical reaction causes a transfer of mass from one point to another. This type of mass transfer is important in many industrial operations like chemical synthesis and wastewater treatment.

Mass transfer concept is significant in various fields of engineering and science. It’s role is significant in the design and optimization of many chemical and industrial processes, such as distillation, absorption, and extraction. Understanding the principles of mass transfer is essential for the efficient and effective design of these processes. Overall, mass transfer is a complex and important field of study that has many practical applications in various industries. Its study has led to many important advances in chemical engineering, materials science, and other related fields.

1.14 Porous Media

A porous media is a material that contains interconnected voids or pores. The pores within a porous medium can be occupied by a fluid, such as air, water, or oil A porous medium's characteristics are determined by the size, shape, and distribution of its pores and also by the features of the fluid that fills them. Fluids act differently in porous medium compared to those in non-porous media. Pores shape, connectivity and surface properties of the solid matrix influence fluid dynamics in porous media. Permeability of a porous material is referred to as its fluid conductivity. The permeability value is dictated by the porous material's structure. Porous material permeability is a macroscopic characteristic. It's only useful for samples large enough to include a lot of pores. Compaction reduces the permeability of a porous substance in the same way that it reduces porosity. Darcy's law is employed to explain the movement of fluids through porous medium. This law establishes the relationship between the rate of fluid flow and the pressure gradient across the porous medium.

Porous media are found in many natural and engineered systems, such as soil, rock, and biological tissues. The study of porous media is important in many fields, including geology, hydrology, petroleum engineering, and materials science. Porous materials also have considerable impact on mass transportation. The transport of dissolved or suspended particles in porous media is affected by the properties of the fluid and the porous medium, as well as the properties of the particles themselves. Overall, the study of porous media is important in understanding a wide range of natural and engineered systems. The behavior of fluids and particles in porous media is complex and depends on many factors like fluid properties, properties of porous medium, geometry and connectivity of pores.

1.15 No-slip and Slip Boundary Conditions

The well-known ‘no-slip' velocity criterion is commonly acknowledged and empirically confirmed to be correct for several bulk flows of viscous Newtonian fluids over a surface at the macroscopic level. According to this assumption, fluids adhere to the contact surface, implying that there is no relative movement between the surface and the nearby layer of fluid. Slip boundary condition, on the other hand, is a condition that describes the behaviour of fluids in contact with a surface that allows some degree of slip. Slip occurs when the fluid velocity at the surface is not zero, but rather has a non-zero value relative to the surface. Slip can occur due to the presence of a thin layer of gas or liquid at the surface, or due to the surface roughness. The slip boundary condition is important in understanding the behaviour of fluids at the nanoscale, where the no-slip condition may not be valid. In these cases, the slip boundary condition can be used to describe the behaviour of fluids in contact with surfaces that allow some degree of slip. Overall, the choice of boundary condition depends on the specific problem being studied. In most practical applications, the no-slip boundary condition is valid and can be used to model the behaviour of fluids in contact with solid surfaces. However, in some cases, such as at the nanoscale or in the presence of surface coatings, the slip boundary condition may be more appropriate.

1.16 Self-similar solution

Self-similar solution refers to a solution of differential equation that exhibits a scaling property. Specifically, a self-similar solution is a solution that can be transformed into itself by a scaling transformation. This means that the solution is invariant under a rescaling of the independent and dependent variables. Self-similar solutions are often encountered in physical problems that exhibit a certain degree of symmetry or scaling behavior. They are particularly useful in problems where the solution cannot be obtained analytically, but can be approximated using numerical methods. By using self-similarity, number of variables in the problem can be reduced and thus numerical solutions can be obtained more efficiently.

1.17 Chemical reaction

Chemical reactions in fluids are an important area of study in chemistry and chemical engineering. Fluids can include gases, liquids, and even plasmas. The behavior of chemical reactions in fluids is influenced by factors such as temperature, pressure, concentration, and the presence of catalysts or inhibitors. In liquids, chemical reactions occur when molecules or ions come into contact with each other. Reaction rates are affected by a variety of parameters. These include reactant concentration, temperature, and the presence of catalysts or inhibitors. In some cases, reactions can occur at the interface between two immiscible liquids, such as in the case of emulsions or suspensions. Understanding the behavior of chemical reactions in fluids is important in many fields, such as chemical engineering, environmental science, and materials science. By studying the mechanisms of chemical reactions in fluids, scientists and engineers can develop new materials, processes, and technologies that have practical applications in industry and everyday life.

1.18 Non-dimensional parameters

The fluid flow problem involves the utilization of certain non-dimensional or dimensionless parameters, some of which are listed below:

1. **Reynolds Number:** “Reynolds number $\mathrm{R_e}$ is a dimensionless value that quantifies the relationship between the forces of inertia and viscosity.” When the Reynolds number $\mathrm{R_e}$ is very tiny, it indicates that viscous forces are more dominating than inertia forces. Such forces are generally referred to as "creeping or viscous flow," however if the Reynolds number $\mathrm{R_e}$ is very big in comparison to inertial effects, the flow problem is referred to as "inviscid analysis”. $\mathrm{R_e}$ is also employed to examine how laminar to turbulent flow regimes change. Heat transfer problems frequently involve the Reynolds number.

Mathematically,

$$Re = \frac{\rho VL}{\mu} = \frac{VL}{\delta}$$, Where

v = Characteristics velocity of the fluid

L = Characteristics length

ρ = density

μ = dynamic viscosity

δ = Kinematic viscosity

2. **Prandtl Number:** “Prandtl number P_r is a dimensionless number that represents the ratio between the diffusivity of momentum (kinematic viscosity) and the diffusivity of heat.” The Prandtl number is a constant of the material and does not depend on the property of the flow. For gas P_r= 1, for air P_r = 0·7, for water at 60^0 F, P_r = 7 whereas for glycerin P_r = 7250. Its importance found in the German physicist Ludwig Prndtl.

Mathematically,

$P_r = \frac{\delta}{\alpha} = \frac{\mu}{k}\ c_p$,where

δ = kinematic viscosity

$\alpha =$ thermal diffusivity

c_p = specific heat

μ = viscosity
k = thermal conductivity

3. **Nusselt Number: "**Nusselt number $\mathrm{N_u}$ measures the efficiency of convective heat transfer relative to that of conductive heat transfer." It's purpose is to examine the heat transfer properties of the fluid flow. A large $\mathrm{N_u}$ causes turbulent flow, whereas a small $\mathrm{N_u}$ near to 1 causes laminar flow.

Mathematically,

$\mathrm{N_u} = \frac{hL}{K},$ where

L = Characteristic length

h = Convection heat transfer co-efficient

k = thermal conductivity

4. **Eckert Number:** The fluid's kinetic energy divided by its enthalpy gives the dimensionless Eckert number E_c. It's used to figure out how much energy is lost in a fluid flow. For air E_c = 1 when the velocity of the sound and the temperature difference is of order 100° C.. Ernst R.G. Eckert was the inspiration for the name.

Mathematically,

$E_c = \frac{v^2}{C_p \Delta t}$, Where,

v = Characteristic velocity

C_p = Special heat

ΔT = characteristic temperature differences

5. **Grashof Number:** "Grashof Number Gr_L is a non-dimensional number that represents the approximate ratio of buoyancy to viscous force in a fluid." It is commonly used to investigate natural convection situations in both heat transfer and mass transfer mechanisms. The flow is laminar when the Grashof number is small, and turbulent when the number is large. Franz Grashof, a German engineer, was the inspiration for the name.

Mathematically,

$$Gr_L = \frac{g\beta\,(\Delta T)\,L^3}{\mu^2} = \frac{g\beta\,(\Delta T)\,L^3}{\delta^2}, as\ \mu = \delta\rho \text{ (For Vertical Plate)}$$

$$Gr_D = \frac{g\beta\,(\Delta T) D^3}{\delta^2}, (for\ pipes), \text{ where}$$

g = acceleration due to gravity

β = Volumetric thermal expansion Co – efficient

L = Characteristics length, D = diameter.

ρ = density, μ = viscosity, δ = Kimatic Viscosity

ΔT = temperature difference between surface and the free stream

6. **Schmidt Number:** "Schmidt number S_c is a non-dimensional number that represents the ratio of momentum (viscosity) to mass diffusivity in fluid flows." The number is named after Ernest Heinrich Wilhelm Schmidt, a German engineer.

Mathematically,

$$S_c = \frac{\delta}{D} = \frac{\mu}{\rho D}, \text{ where}$$

δ = kinematic viscosity

D = mass diffusivity

μ = dynamic viscosity

ρ = fluid density

1.19 MATLAB Code ‘bvp4C’

MATLAB inbuilt code is an algorithm has developed to solve the mathematical problem in a very short duration of time. MATLAB code ‘bvp4c’ is a function in MATLAB. It is used to address boundary value issues. It does this by discretizing the ODE using a finite difference approach and solve for the unknown function at a set of discrete points. The function takes the following inputs:

- A function handle to the ODE of the form $\frac{dy}{dx} = \mathrm{g(x, y, q)},\ \mathrm{x} \in [a, b]$. Here, y is an unknown function, x is an independent variable, and p is a vector of parameters that can be used in the ODE.
- Boundary conditions are controlled by a function of the form $\mathrm{g(y(a), y(b), q) = 0}$, where $\mathrm{y(a)}$, and $\mathrm{y(b)}$ are the values of the unknown function at the left and right boundaries, respectively.
- An initial guess for the solution.
- A vector of nodes at which to solve the ODE.
- A vector of parameters to be used in the ODE and boundary conditions.

The function returns a structure containing the solution to the ODE, including the values of the unknown function at the nodes, as well as information about the accuracy of the solution. The obtained solutions are easily compared with their physical phenomenon in the various application of modern science and technology.

1.20 Motivation of the study

Visco-elastic fluids possess both viscous and elastic properties and have ability to store and recover energy under deformation, making them useful in many industrial and engineering applications. Walters Liquid (Model B') accurately describe the flow behaviour of a wide range of polymer solutions and melts, including polyethylene, polypropylene, polystyrene and complex fluids, has made it an important model for polymer science and technology. The behavioural study of the boundary layer can help engineers and scientists optimize the design and performance of systems that involve fluid flow over solid surfaces. Magnetohydrodynamic is an important field of study that has many practical applications in astrophysics, geophysics and engineering and science. The study of porous media is important in many fields, including geology, hydrology, petroleum engineering, materials science and mass transport processes. The temperature distribution and heat flow determination are essential for chemical process industry, automotive engineering, thermal insulation, thermal processing, power plant engineering, bio-heat transmission, and aerospace technology, etc. The process of transferring mass from one point to another is an important field of study in chemical engineering, materials science, and other related fields. This prompted the author to examine the flow characteristics of viscoelastic fluids by Walters Liquid (Model B') with different geometry and flow feature parameters.

1.21 Objectives of the study

The objectives of this study are as follows:

- Investigate the flow behaviour of visco-elastic fluid characterized by Walters Liquid (Model B') with different geometry and flow feature parameters.
- Examine the effects of various physical flow parameters in combination with visco-elastic parameter on the velocity, temperature and other related fields.
- Compare the computed results with other theoretical results.
- Interpret results from graphs and tables to get physical insight of the solved problems.

1.22 Literature Survey

A literature review involves a thorough examination and evaluation of previously published works on a particular subject matter. It is an essential component of any research work, and its importance cannot be overstated. It helps to identify research gaps, develop a research methodology, establish the context of the research, avoid duplication of research, identify key concepts and theories, and provide evidence for research outcomes.

Oldroyd [1] delved into the exploration of equations of state and its rheology, offering valuable insights into their formulation. Oldroyd [2] engaged in a thorough examination of the influence of visco-elastic properties on the sustained movement of certain conceptual Elastico-viscous fluids, initiating a comprehensive discourse on the subject matter. Walters [3] dedicated scholarly attention to the dynamics of visco-elastic fluid confined within co-cylinders having same axis , unraveling the intricacies of this phenomenon. Additionally, an exploration of materials endowed with memories [4] resulted in the formulation of a solution to the corresponding problem, augmenting the understanding of such complex systems. Soundalgekar [5] undertook an insightful examination of the fluid dynamics pertaining to elastico-viscous substances, focusing specifically on the flow characteristics exhibited by such fluids when encountering an oscillating plate. The research conducted by Johri and Sharma [6] centered around the analysis of laminar flow driven by free convection in an incompressible visco-elastic fluid. Jha [7] studied free convection of Walters liquid flow with heat sources. Nanousis [8] conducted a pioneering study that showcased the phenomenon of magnetohydrodynamic flows in rotating elastico-viscous fluids. Ariel [9] presented a comprehensive analysis of the flow characteristics exhibited by visco-elastic fluids as they traverse through a channel permeated by a porous medium. Kim [10] offered a comprehensive explanation of the natural convection phenomena occurring along a vertically undulating plate for Power-law fluid.

Andrienko *et al.* [11] thoroughly investigated the resonance characteristics exhibited by visco-elastic fluids during Poiseuille flow, shedding light on this intriguing phenomenon. Helmy *et al.* [12] analysed the boundary layer problem of power law hydromagnetic fluids using the integral technique. Sajid *et al.* [13] explained non-Newtonian fluid flow confined between two parallel plates due to mixed convection of visco-elastic fluids in porous media. Eldabe *et al.* [14] investigated magnetohydrodynamic flow of elastic-viscous fluids past a porous domain near an accelerating plate. Grosan *et al.* [15] studied the boundary layer in free convection past a vertical cone submerged in Power-law fluid, focusing on internal heat generation. Liu [16] presented the kinetics of a non-Newtonian fluid's unstable, one-way flow through a porous material. Chowdhury [17] explored the influences of magnetic field on natural convection of an elastic-viscous fluid over an infinitely long vertical plate. Veena *et al.* [18] evaluated the heat and mass transmission of an electrically conductive viscoelastic fluid through a stretched sheet in a porous medium. Reza *et al.* [19] explored the behaviour of flow velocity and heat transport of elastic-viscous fluid as it past a porous plate as a result of suction or blowing. Prakash *et al.* [20] examined chemically reactive heat transmission of elastic-viscous dusty fluid flow using Walters liquid model-B.

Chang *et al.* [21] numerically analyzed the mass transport due to natural convection of elastic-viscous fluid flow employing Walters B model. The investigation specifically considered the influence of wall suction on the system's dynamics. The effects of mass transport due to natural convection of Walters liquid subject to magnetohydrodynamics were studied by Reddy *et al.* [22]. Choudhury and Das [23] explored the fascinating realm of viscoelastic hydromagnetic horizontal oscillatory channel flow with heat source. Choudhury and Das [24] investigated the effect of a hall current on the transient flow of an elastico-viscous fluid through a permeable plate with an impulsive start. The reactive solute diffusion and thermal transition of viscoelastic fluid past a porous inclined surface was examined by Choudhury and Das [25].

Choudhury and Dey [26] investigated the influence of viscoelasticity on magnetohydrodynamic natural-convection flow through a vertical sheet utilizing Walters liquid model $B^{/}$. Dey [27] presented magnetohydrodynamic unsteady flow of elastico-viscous fluid in an annulus with thermal radiation, a heat source, and a sink. Chen et al. [28] examined numerically the unsteady viscoelastic magnetohydrodynamic fluid flow through a stretchable s employing Maxwell fractional fluid model. Khan *et al.* [29] examined the impact of multiple slips on an unsteady hydromagnetic elastic-viscous buoyant nanofluid, using the Jeffrey fluid model with Soret and radiation past a stretchable porous sheet. Ibezim *et.al.* [30] conducted experimental research on the flow behavior of flexible polymer solutions through a unique micro-porous structure. The primary emphasis was on studying the interaction between the viscoelastic properties of the fluid and the micro-porous structure. In addition, some non-Newtonian fluid-related books [31-35] pertinent to my investigation are studied.