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Chapter 3

Hydromagnetic Visco-Elastic Boundary Layer Flow Past an Exponentially Stretching Sheet with Suction or Blowing

3.1 Introduction

In fluid mechanics, the study of the boundary layer flow over an extending sheet in a viscous fluid is significant due to its numerous applications in technological processes. This type of flow is frequently observed in many industrial manufacturing processes, such as aerodynamic extrusion of plastic sheets, rolling of synthetic fibres, drawing of copper wires, extraction of polymer sheet, glass blowing, paper production, drawing plastic films, metal spinning, etc.

Crane [52] studied the flow caused by the stretching of a sheet, and his works have since been extended under various physical conditions by a large number of new researchers. In the study of boundary layer flow past a stretching sheet, many researchers assume that the velocity of the stretching sheet is linearly proportional to the distance from the fixed point. According to Gupta and Gupta [53], the stretching of plastic sheets is not necessarily linear in practise. In addition, the study of flow and heat transmission past an exponentially expanding sheet has significant industrial applications. Magyari and Keller [54] focused on heat and mass transfer on boundary layer flow caused by an exponentially continuous stretching sheet, whereas Elbashbeshy [55] investigated the flow past an exponentially stretching surface. Bidin and Nazar [56] investigated the effect of thermal radiation on the constant laminar two-dimensional boundary layer flow and heat transmission over an exponentially stretching sheet. El-Aziz [57] and Ishak [58] examined the flow and heat transmission past an exponentially stretching sheet in the future.

The previous research has been expanded by presuming no slip boundary conditions. The condition in which a liquid adheres to a solid boundary is a central principle of the Navier–Stokes theory. However, this is not true in every circumstance. There is evidence that partial velocity can be observed when the boundary is stretched in fluids that contain particles, such as emulsions, suspensions, foams, and polymer solutions. Velocity slip, which refers to the non-adhesion of a fluid to a solid boundary, has been frequently observed in a variety of situations. As this type of flow has important technological applications in the refining of artificial heart valves and internal cavities, a number of researchers [59-64] have recently examined flow problems with slip flow at the boundary.

The flow field can be substantially affected by suction or blowing of a fluid through a boundary surface. In general, suction increases skin friction, whereas blowing has the opposite effect. Suction and blowing play a key role in the design of thrust bearings, radial diffusers, and thermal oil recovery. Utilising sucyion to remove reactants from chemical processes. The purpose of the blowing is to add reactants, reduce friction, cool the surface, and prevent corrosion.

In this study, the hydromagnetic steady boundary layer flow of viscoelastic electrically conducting fluid characterised by Walters liquid (Model B') over an exponentially stretching sheet was investigated. The effects of the viscoelastic, slip velocity, magnetic, and suction or injection parameters across the boundary layer on the velocity component have been graphically represented using the MATLAB 'bvp4c' solver in conjunction with the other flow parameters involved in the solution. This study also depicts the skin friction coefficient, which is of the uttermost importance from an industrial application standpoint, for a range of visco-elastic parameters.

3.2 Mathematical Formulation

The flow of a two-dimensional incompressible viscous-elastic electrically conducting fluid characterised by Walters liquid (Model $B^{\prime}$)) past a flat sheet intersecting the y = 0 plane is considered. The x-axis is directed along the sheet and points in the direction of fluid motion, while the y-axis is perpendicular to the sheet. Using two equal and opposing forces along the x-axis, the wall is stretched while the origin remains fixed. The fluid motion is generated by applying a substantial force to an elastic boundary sheet stretched from a perforation. In the flow direction coordinate x, the velocity of the boundary sheet has an exponential form. Flow occurs in the upper portion of the y > 0 plane.

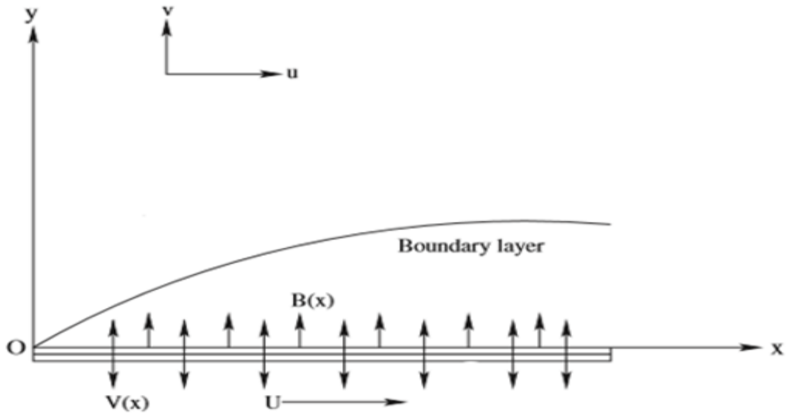

Fig. 3.1 Geometrical model of the flow problem

The basic governing equations for steady two-dimensional boundary layer flow of Walters liquid (Model B/) in the presence of transverse variable magnetic field B(x) applied normal to the sheet are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (3.2.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{k_0}{\rho}\left[u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}\right]-\frac{\sigma B^2}{\rho}u \qquad (3.2.2)$$

where u and v denote the velocity component in the x- and y- directions, $\nu=\frac{\mu}{\rho}$ is the kinematic fluid viscosity, μ is the coefficient of fluid viscosity, ρ is the fluid density, σ is the electrical conductivity of the fluid and k_0 is the visco-elastic parameter.

The appropriate boundary conditions are given by

$u = U + N\nu \frac{\partial u}{\partial y}, \qquad v = -V(x) \qquad \text{at} \qquad y = 0; \qquad u = 0 \qquad \text{as} \qquad y \to \infty$

(3.2.3)

Here $U = U_0 e^{\frac{x}{L}}$ is the stretching velocity, U_0 is the reference velocity, $N = N_1 e^{\frac{x}{2L}}$ is the slip factor which changes with x, $N_1$is the initial value of velocity slip factor. The no-slip case is obtained for $\mathrm{N} = 0$. $V(x) > 0$ represents velocity of suction where as $V(x) < 0$ denotes velocity of blowing. A special type of velocity $V(x) = V_0 e^{\frac{x}{L}}$ at the wall is considered, where V_0 is the initial strength of suction.

Little inspection provides us a set of similarity solution of the above system of equations as

$$u = U_0 e^{\frac{x}{L}} f^{/}(\eta),\ v = -\sqrt{\frac{\nu U_0}{2L}}\ e^{\frac{x}{2L}}\{f(\eta) + \eta f^{/}(\eta)\} \tag{3.2.4}$$

where $\eta = \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y$

The expressions of u and v satisfy the continuity equation (3.2.1) identically.

Substituting values of u and v in equation (3.2.2), we obtained the self-similar equation as follows:

$$f^{///}(\eta) + f(\eta)f^{//}(\eta) - 2\{f^{/}(\eta)\}^2 - M^2 f^{/}(\eta) - K_1[6f^{/}(\eta)f^{///}(\eta) - f(\eta)f^{IV}(\eta) - 3\{f^{//}(\eta)\}^2] = 0 \tag{3.2.5}$$

and the relevant boundary conditions are

$$f(\eta) = S, f^{/}(\eta) = 1 + \lambda f^{//}(\eta) \text{ at } \eta = 0;\ f^{/}(\eta) = 0, f^{//}(\eta) = 0 \text{ as } \eta \to \infty \tag{3.2.6}$$

where the prime indicates differentiation with respect to η. $M = \sqrt{\frac{2\sigma B_0^2 L}{\rho U_0}}$ is the magnetic parameter, $k_1 = \frac{\mathrm{k_0 U_0}}{2\rho\nu\mathrm{L}} e^{\frac{\mathrm{x}}{\mathrm{L}}}$ is the visco-elastic parameter, $\lambda = N_1\sqrt{\frac{U_0\nu}{2L}}$ is the velocity slip parameter, $S = {v_0}\Big/{\sqrt{\frac{U_0\nu}{2L}}} > 0$ (or < 0) is the suction (or blowing) parameter.

3.3 Method of Solution

The self-similar nonlinear differential equations (3.2.5) is transformed to first order differential equations as follows:

$$f = y_1, f' = y_2,\ f'' = y_3, f''' = y_4\ , \theta = y_5, \theta' = y_6 \tag{3.3.1}$$

From (3.3.1), we can write as

$$y_1' = y_2, y_2' = y_3, y_3' = y_4, y_5' = y_6 \tag{3.3.2}$$

Making use of (3.3.1) and (3.3.2), the equations (3.2.5) can be written as

$$y_4' = \frac{1}{k_1 y_1}(6k_1 y_2 y_4 - 3k_1 y_3^2 + y_4 + y_1 y_3 - 2y_2^2 - M^2 y_2) \tag{3.3.3}$$

and the relevant boundary conditions (3.2.7) reduces as follows:

$$y_1(0) = s,\ y_2(0) = 1 + \lambda y_3(0) \tag{3.3.4}$$

$$y_1(\infty) = 0,\ y_2(\infty) = 0 \tag{3.3.5}$$

The above equations (3.3.3) along with the boundary conditions (3.3.4) and (3.4.5) are solved using MATLAB bvp4c solver for different values of flow parameters involved in the solution.

3.4 Results and Discussion

Coefficient of skin friction is approximately expressed as

$$\tau = f''(0) + k_1 f(0) f'''(0) \tag{3.4.1}$$

For different values of flow parameters such as the visco-elastic parameter k_1, magnetic parameter M, velocity slip parameter λ and suction/blowing parameter S, numerical calculations for the velocity and skin friction at the stretched sheet have been performed using the Matlab solver 'bvp4c'.

In Fig. 3.2, velocity profiles are shown for a range of viscoelastic parameter values $k_1(= 0.3, 0.4, 0.5)$ with constant values for $S = 0.1,\ M = 0.1$ and $\lambda = 0.1$. According to the velocity curves, the rate of transport reduces when k_1 is increased. In every situation, the velocity vanishes at a certain distance from the sheet $\eta = 3$. The nature of velocity is shown in Fig. 3.3 for different values of the magnetic parameter $M(= 0.1, 0.5, 1.0)$ with constant values of $S = 0.1,\ \lambda = 0.1$ and $k_1 = 0.4$. It is observed that the fluid velocity decreases as the magnetic parameter increases. Because the transverse magnetic field opposes the fluid motion, the rate of transfer is greatly decreased. This is due to the Lorentz force, which grows as M rises and creates additional flow resistance. The thermal boundary layer's thickness increases as M grows.

In Fig. 3.4, velocity profiles are shown for a range of slip parameter values $\lambda\,(= 0.1, 0.3, 0.5)$ with constant values of $S = 0.1,\ \mathrm{M} = 0.1$ and $\mathrm{k}_1 = 0.4$. Velocity initially reduced as slip parameters increased, but the rate of transport enhanced as the distance $(\,\eta)$ from the sheet increased, and it eventually vanished at a certain distance from the sheet. As slip develops, the flow velocity next to the sheet is no longer equal to the sheet's stretching velocity. With an increase in λ , the slip velocity rises; as a result, the fluid velocity falls. It is noted that the dragging of the stretched sheet can only be partially communicated to the fluid under the slip situation. It is observed that λ has a significant impact on the solution.

The effects of suction and blowing parameters $S(= 0.1, 0.3, 0.5 \text{ and } -0.1, -0.3, -0.5)$ with constant values of $\lambda = 0.1,\ \mathrm{M} = 0.1$ and $\mathrm{k}_1 = 0.4$ on velocity profiles in the presence of slip at the stretched sheet's boundary are shown in Figs. 3.5 and 3.6. When the suction parameter is increased, it is shown that velocity first diminishes but gradually rises and eventually vanishes at a distance from the sheet, while blowing results in the opposite pattern of flow. The boundary layer thickness and velocity field are seen to grow when the wall suction $(\,S > 0)$ is considered, but blowing results in the opposite behaviour.

For different values of the visco-elastic parameter $\mathrm{k}_1(= 0.3, 0.4, 0.5)$ with fixed values of $\mathrm{S} = 0.1\ and\ \lambda = 0.1$, the nature of the skin friction coefficient versus magnetic parameter is shown in Fig. 3.7. It is observed that when visco-elastic and magnetic parameter values rise, the skin friction coefficient falls. For various values of the visco-elastic parameter $\mathrm{k}_1(= 0.3, 0.4, 0.5)$with fixed values $\mathrm{M} = 0.1.\ and\ \lambda = 0.1$, the nature of the skin-friction coefficient against suction and blowing parameter is shown in Figs. 3.8 and 3.9. It is noticed that the skin friction coefficient reduces as visco-elastic and suction parameters increase, while the opposite nature is seen when τ is plotted against suction parameter.

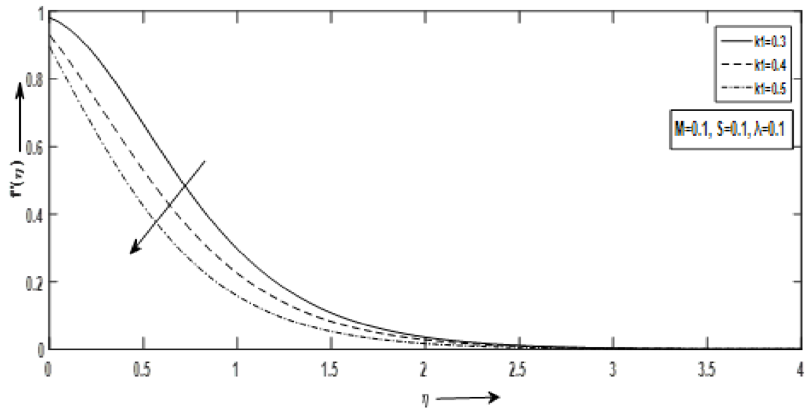

Fig. 3.2 Velocity profile $f'(\eta)$ against η for various values of k_1

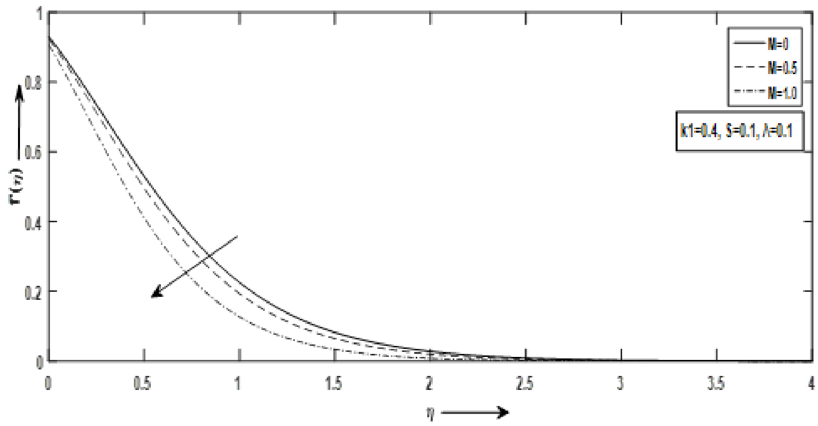

Fig. 3.3 Velocity profile $f'(\eta)$ against η for various values of M

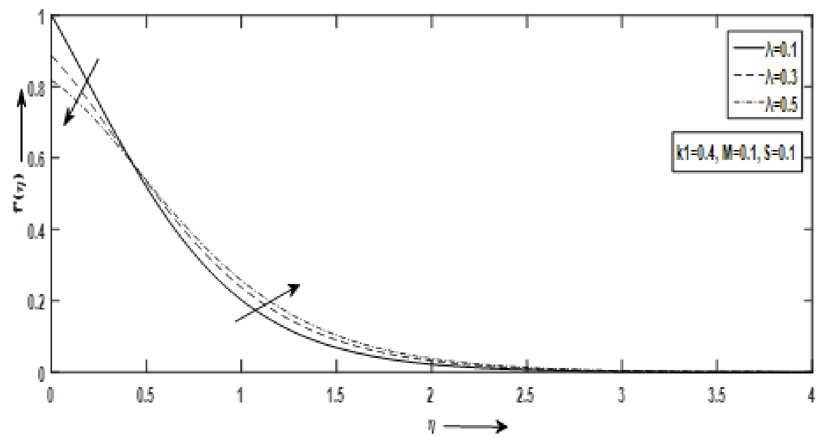

Fig. 3.4 Velocity profile $f'(\eta)$ against η for various values of slip parameter λ

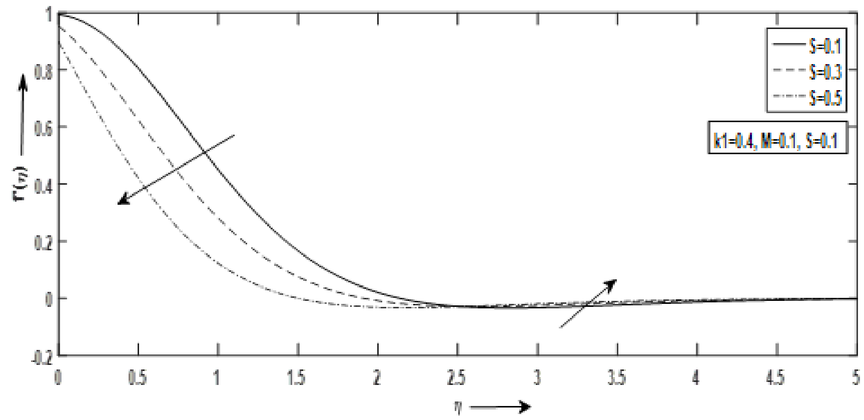

Fig. 3.5 Velocity profile $f'(\eta)$ against η for various values of S (suction)

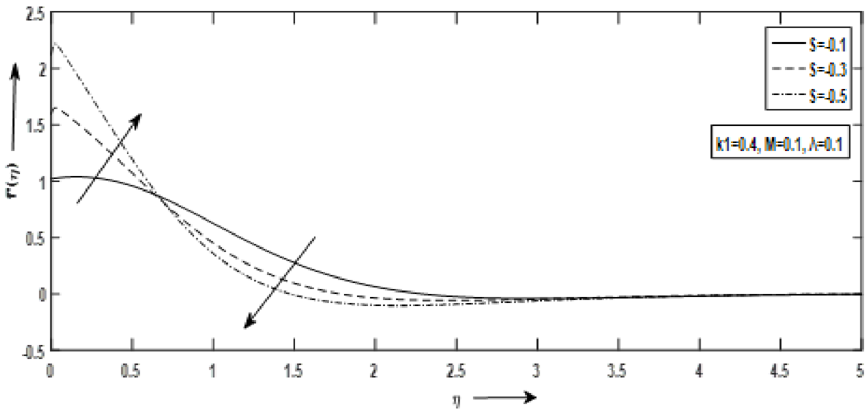

Fig. 3.6 Velocity profile $f'(\eta)$ against η for various values of S (blowing)

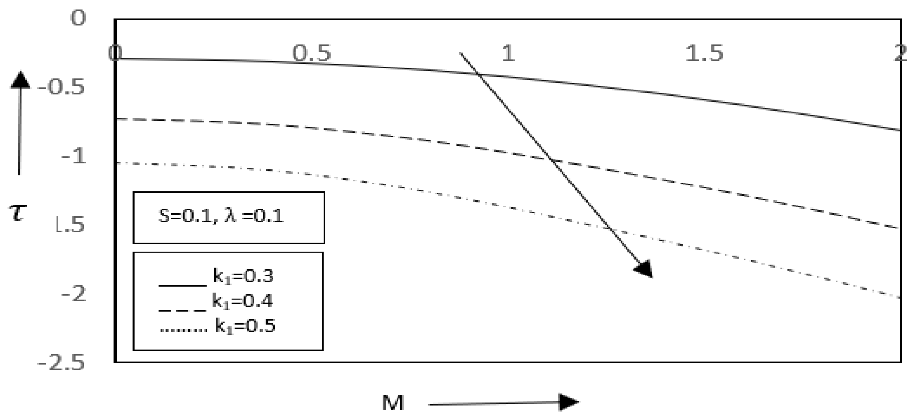

Fig. 3.7 Skin friction τ against M for various values of k_1

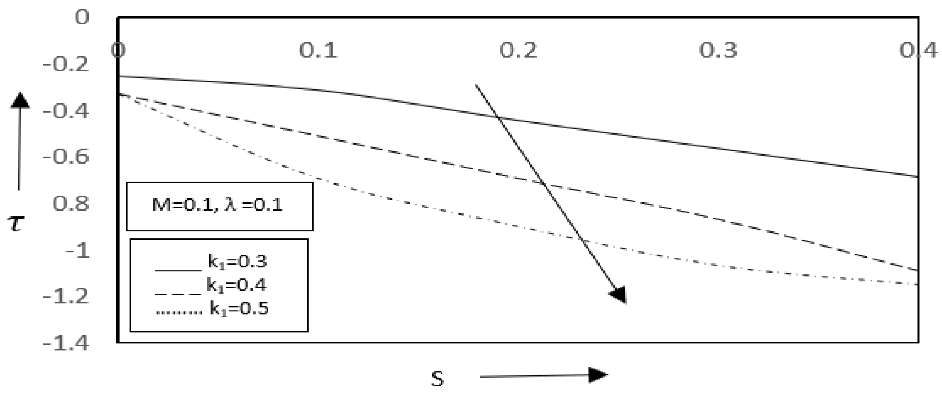

Fig. 3.8 Skin friction τ against S (suction) for various values of k_1

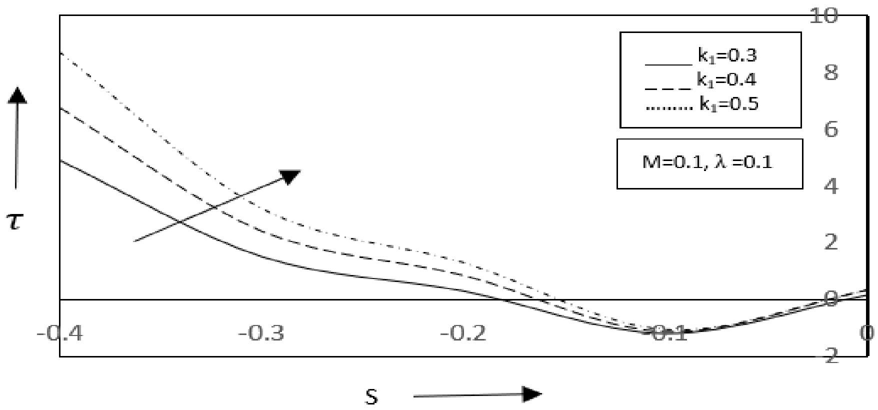

Fig. 3.9 Skin friction τ against S (blowing) for various values of k_1

3.5 Conclusion

Using the MATLAB solver 'bvp4c', the solutions to the highly nonlinear differential equations involved in this investigation are obtained. The impacts of slip, viscoelastic, magnetic, and suction or injection parameters on the velocity and skin-friction coefficients are examined graphically. Observations indicate that the flow field is substantially influenced by the aforementioned flow parameters. A prospective research investigating the problem's flow simulation would be extremely intriguing.

The current study's findings highlight the following key aspects:

- At a distance far from the sheet, the transport rate diminishes and declines with rising values of the visco-elastic parameter.
- It has been found that when the magnetic parameter grows, the fluid velocity reduces.

- As the slip parameter rises, the velocity first decreases but then increases, causing the velocity of the fluid to decline.
- In the case of blowing, the velocity falls at first but increases with increasing values of the visco-elastic and suction parameters, whereas the opposite trend is seen.
- As the values of the viscoelastic and magnetic parameters rise, the skin friction coefficient falls.
- In the presence of suction and an increase in viscoelastic characteristics, the skin friction coefficient decreases, whereas the reverse trend is seen in the presence of blowing.