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Chapter 5

Solution of Non-Newtonian Boundary Layer Flow in a Convergent Channel using Homotopy Perturbation Method

5.1 Introduction

Magnetohydrodynamics electrically conducting non-Newtonian fluid flow in a convergent channel has not only theoretical appeal but also serves as a model of numerous biological and technical problems, including plasma research, industrial metal casting, nuclear reactors, blood flow concerns, etc. Chemical, aerospace,civil, environmental, mechanical, and bio-mechanical engineering are just some of the fields that may benefit from understanding the theory behind such flows.

In 1915, Jeffry [85] developed the mathematical formulations of incompressible viscous fluid flow through a convergent or divergent channel. The work of Jeffery has been extended by Srivastava [86] to an electrically conducting fluid in the presence of a transverse magnetic field. Millsaps and Pohlhausen [87] performed the numerical computations of Jeffery-Hamel flows between nonparallel planar barriers. Rosenhead [88] presented the solution for two-dimensional incompressible laminar flow in a converging channel with an impermeable wall. Terril [89] analysed the sluggish laminar flow in a converging or diverging channel with suction at one wall and injection at the other.

Falkner and Skan [90] were the first to analyse the two-dimensional laminar boundary layer flow of an incompressible, viscous, non-uniform stream past solid obstacles. Phukan [91] examined the convergent channel flow of an electrically conducting Newtonian fluid. Mahapatra *et al.* [92] investigated the hydromagnetic laminar flow of a viscous fluid in a converging or diverging channel with suction at one wall and equal pressure at the other wall. Sanyal and Adhikari [93] studied the two-dimensional laminar MHD boundary layer flow past a wedge with slip velocity. Alam and Khan [94] have conducted a comprehensive analysis of MHD flow in convergent-divergent channels.

The MHD convergent channel flow of a viscoelastic electrically conducting fluid with slip velocity has been studied by Choudhury and Dey [95]. Hosseini et al. [96] investigated the hydromagnetic flow of an incompressible viscous fluid through a convergent or divergent channel in the presence of a strong magnetic field. Alam and Khan [97] investigated the hydromagnetic effects on mixed convection flow through a diverging channel with a circular obstacle. He [98-100] investigated the preliminary work in the Homotopy perturbation method (HPM), which inspired a large number of researchers, including Ariel *et al.* [101], Belendez *et al.* [102], Ganji and Rajabi [103], Siddiqui *et al.* [104], and many others, to use this method to solve nonlinear equations.

Using the Homotopy Perturbation method, this study aims to examine the two-dimensional boundary layer flow of a non-Newtonian electrically conducting fluid through a convergent channel characterised by Walters liquid (Model $B^{/}$) in the presence of a transverse magnetic field. J.H. and He [105] investigated the Homotopy Perturbation method for bifurcation of a nonlinear problem. With the combination of magnetic and other flow parameters involved in the solution, the non-Newtonian effects across the boundary layer on the dimensionless velocity component and skin friction coefficient were illustrated graphically.

5.2 Mathematical Formulation

The fundamental equations for the steady flow of Walters liquid (Model $B^{/}$) in a two-dimensional boundary layer in the presence of a transverse magnetic field B(x) are provided by.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5.2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{k_0}{\rho} + \sigma B^2(x)\frac{(U-u)}{\rho} \tag{5.2.2}$$

subject to the boundary conditions

$$y = 0:\ u = 0\ ,\ \ v = 0;\ \ y \to \infty\ \ :\ \ u = U(x) \tag{5.2.3}$$

where x-axis coincides with the wall of the convergent channel and y-axis is perpendicular to it. U(x) is the main stream velocity, u and v are the flow velocities in the direction of x and y respectively, ρ is the fluid density, ν is the kinematic viscosity, σ is the electrical conductivity of the fluid and k_0 is the visco-elastic parameter.

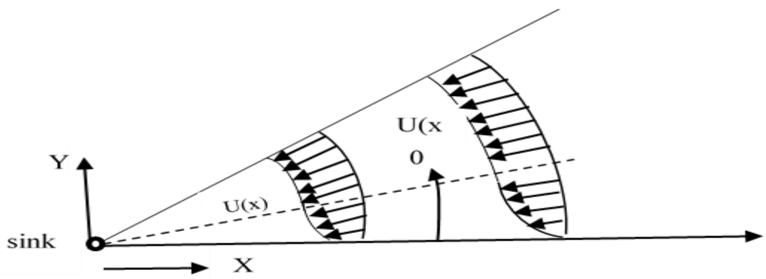

Fig. 5.1 Physical sketch of the flow in a convergent channel

It is assumed that the induced magnetic field is negligible compared to imposed one, the electric field is zero and the electric field due to polarization of charges is also negligible. The velocity of the potential flow along the wall of a convergent channel is given by Schlichting [106]:

$$U(x) = -\frac{u_1}{x} \tag{5.2.4}$$

with $u_1 > 0$ represents two-dimensional motion in a convergent channel with wall (sink) and it leads to similarity solution.

To obtain similarity solutions we introduce the following change of variables:

$$\eta(x,y) = y\sqrt{-\frac{U(x)}{x\nu}} = y\sqrt{\frac{u_1}{x^2\nu}} = \frac{y}{x}\sqrt{\frac{u_1}{\nu}} \tag{5.2.5}$$

and the stream function

$$\psi(x,y) = -\sqrt{\nu u_1}\, F(\eta) \tag{5.2.6}$$

Then, we obtain the velocity components as

$$u = \frac{\partial\psi}{\partial y} = U(x)F'(\eta) \tag{5.2.7}$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\eta}{x}\sqrt{u_1 \nu}\, F'(\eta) \tag{5.2.8}$$

The equation of continuity (5.2.1) is identically satisfied for the velocity components.

Similarity solution exists if the magnetic field B(x) has the special form Chiam [107]

$$B(x) = \frac{B_0}{x} \tag{5.2.9}$$

Using the equations (5.1.4) to (5.1.9), the equation (5.2.2) takes the form as follows:

$$F''' - F'^2 - k_1\left[4F'F''' - 2F''^2\right] + M(1 - F') + 1 = 0 \tag{5.2.10}$$

where prime indicated the differentiation with respect to η. k_1 and M denote the modified non-Newtonian parameter and magnetic parameter respectively.

The corresponding boundary conditions are

$$F'(0) = 0, \qquad F'(\infty) = 1 \quad \& \quad F''(\infty) = 0 \tag{5.2.11}$$

5.3 Method of Solution

Now we consider

$$z = \sqrt{M}\eta \quad \& \quad f(z) = \sqrt{M}F(\eta) \tag{5.3.1}$$

Then

$$f'(z) = F'(\eta), f''(z) = \frac{1}{\sqrt{M}}F''(\eta), \qquad f'''(z) = \frac{1}{M}F'''(\eta) \tag{5.3.2}$$

Using the equations (5.3.2) in the equation (5.2.10), we get the following differential equation

$$f''' - k_1\left[4f'f''' - 2f''^2\right] + (1 - f') = \varepsilon\left(f'^2 - 1\right) \tag{5.3.3}$$

where $\varepsilon = \frac{1}{M}$.

The modified boundary conditions are

$$f'(0) = 0, \qquad f'(\infty) = 1 \quad \& \quad f''(\infty) = 0 \tag{5.3.4}$$

Using Homotopy perturbation method, equation (2.3.3) is constructed as follows:

$$(1 - p)(f''' - f') + p\left[(f''' - f') - k_1\left(4f'f''' - 2f''^2\right) - \varepsilon f'^2 + \lambda\right] \tag{5.3.5}$$

where $\lambda = 1 + \varepsilon$

We consider $f = f_0 + pf_1 + p^2 f_2 + \cdots \ldots \ldots \ldots \ldots$, and thus equation (5.3.5) becomes

$$\begin{aligned}(1-p)[f_0''' + pf_1''' + p^2 f_2''' + \cdots \ldots \ldots) - (f_0' + pf_1' + p^2 f_2' + \cdots \ldots..)] \\ + p[(f_0''' + pf_1''' + p^2 f_2''' + \cdots \ldots \ldots) - (f_0' + pf_1' + p^2 f_2' \\ + \cdots \ldots..) - k_1\{4(f_0' + pf_1' + p^2 f_2' + \cdots \ldots..)(f_0''' + pf_1''' + p^2 f_2''' \\ + \cdots \ldots \ldots) - 2(f_0'' + pf_1'' + p^2 f_2'' + \cdots \ldots \ldots)^2\Big\} \\ - \varepsilon(f_0' + pf_1' + p^2 f_2' \ldots \ldots..)^2 + \lambda\Big] \\ = 0 \qquad (5.3.6)\end{aligned}$$

Terms independent of p gives,

$$f_0''' - f_0' = 0 \qquad (5.3.7)$$

The boundary conditions are,

$$f_0'(0) = 0, \;\; f_0'(\infty) = 1, \;\; f_0''(\infty) = 0 \qquad (5.3.8)$$

Term containing only p gives,

$$f_1''' - f_1' - 4k_1 f_0' f_0''' - 2{f_0''}^2 - \varepsilon {f_0'}^2 + \lambda = 0 \qquad (5.3.9)$$

The boundary conditions are,

$$f_1'(0) = 0, \;\; f_1'(\infty) = 1, \;\; f_1''(\infty) = 0 \qquad (5.3.10)$$

Terms containing only p^2 gives,

$$f_2''' - f_2' - 4k_1(f_0' f_1''' + f_1' f_0''') - 4f_0'' f_1'' - 2\varepsilon f_0' f_1' = 0 \qquad (5.3.11)$$

The boundary conditions are,

$$f_2'(0) = 0, \; f_2'(\infty) = 1, \; f_2''(\infty) = 0 \qquad (5.3.12)$$

Terms containing only p^3 gives,

$$f_3''' - f_3' - 4k_1(f_0' f_2''' + f_1' f_1''' + f_2' f_0''') - 2\left({f_0''}^2 + 2f_2'' f_0''\right) f_0'' f_1'' - \varepsilon\left({f_1'}^2 + 2f_2' f_0'\right) \\ = 0 \qquad (5.3.13)$$

The boundary conditions are,

$$f_3'(0) = 0,\ f_3'(\infty) = 1,\ f_3''(\infty) = 0 \tag{5.3.14}$$

Solving equations (5.3.7), (5.3.9), (5.3.11) *and* (5.3.13) with the help of boundary condition, we get (5.3.8), (5.3.10), (5.3.12), (5.3.14)

$$f(z) = \lambda(z + e^{-z} - 1) + p^3(D + Ee^{-z} + Fe^{-2z} + Gze^{-z} + Hz) + \ldots\ldots\ldots\ldots\ldots \tag{5.3.15}$$

Differentiating equation (5.3.15) with respect to z, we obtain

$$f'(z) = \lambda(1 - e^{-z}) + p^3[-Ee^{-z} - 2Fe^{-2z} + G(e^{-z} - ze^{-z}) + H] + \ldots\ldots\ldots\ldots\ldots \tag{5.3.16}$$

From equation (5.3.16) we can easily find the dimensionless velocity $F'(\eta)$ across the boundary layer. The constants of the solution of the differential equations are not presented here for the sake of brevity.

5.4 Results and Discussions

The skin friction coefficient at the wall of the convergent channel is given by

$$\tau = f''(0) - k_1\left(\frac{3}{\nu}\right) f'(0)f''(0) \tag{5.4.1}$$

where,

$$f'(0) = -p^3(E + 2F - G - H) \text{ and } f''(0) = \lambda + p^3(E + 4F - 2G)$$

This work analyses the impact of the visco-elastic parameter on the two-dimensional laminar MHD boundary layer flow via a converging channel. The Homotopy perturbation approach is used to provide an analytical solution to this issue. Using Matlab software, numerical estimates of the velocity and skin friction at the wall have been performed for a range of values of the flow parameters. The non-dimensional parameter k_1 displays the non-Newtonian impact. Setting $k_1 = 0$ yields all of the Newtonian fluid's equivalent findings.

For various values of visco-elastic, magnetic, and other flow characteristics, the fluctuations in dimensionless velocity $F'(\eta)$ versus the variable η over the boundary layer are shown in Figs. 5.2 to 5.4. The velocity has been seen to grow with increasing values of η in both Newtonian and non-Newtonian cases. It shows that when the parameter η increases, the thickness of the boundary layer that forms close to the convergent channel decreases. However, for certain fixed values of the magnetic parameter, the velocity decreases with rising values of the visco-elastic parameter in compared to the Newtonian fluid. Additionally, it can be shown in Figs. 5.2 to 5.4 that $F'(\eta)$ diminishes as the magnetic parameter's values rise. It demonstrates that when the magnetic parameter grows, the boundary layer thickness increases.

The dimensionless shearing stress at the convergent channel wall measured in accordance with the magnetic parameter M for different visco-elastic and other flow parameter values is shown in Fig. 5.5. The shearing stress is shown to decrease in both Newtonian and non-Newtonian scenarios when the visco-elastic parameter's values increase. Also noted is that although the magnetic parameter M first causes the shearing stress to rise, as M's value rises, the shearing stress finally decreases. The fact that the boundary layer thickness decreases as the magnetic parameter increases is consistent with this.

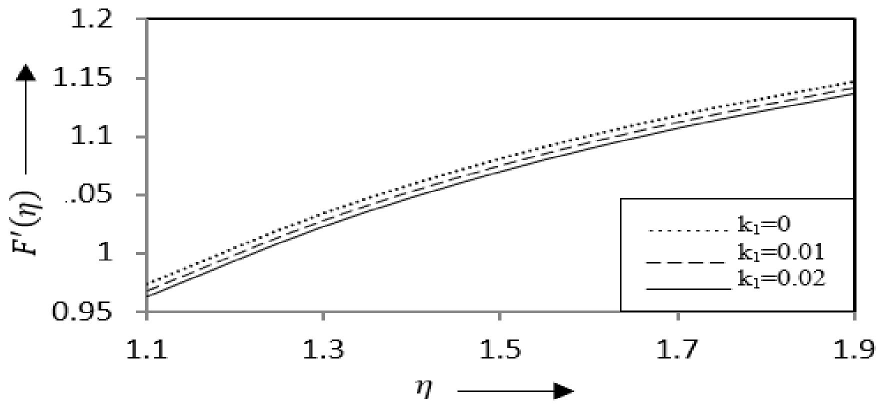

Fig. 5.2 Velocity distribution against η for $M = 8$

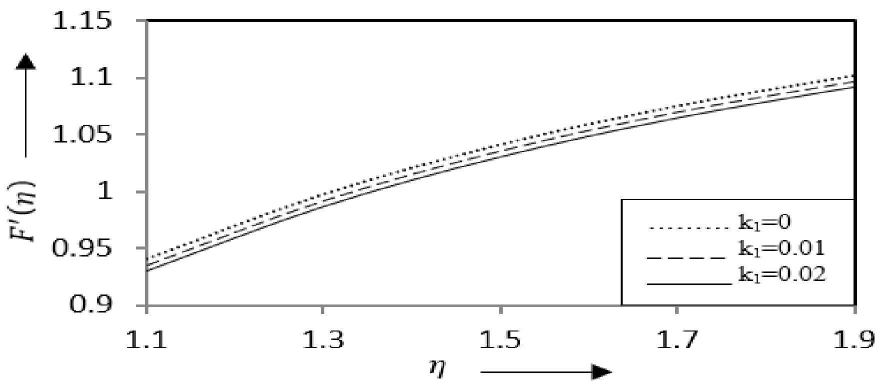

Fig. 5.3 Velocity distribution against η for $M = 10$

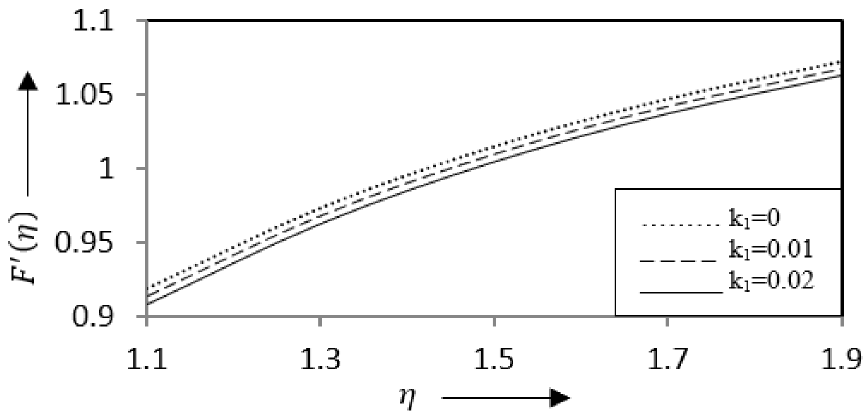

Fig. 5.4 Velocity distribution against η for M = 12

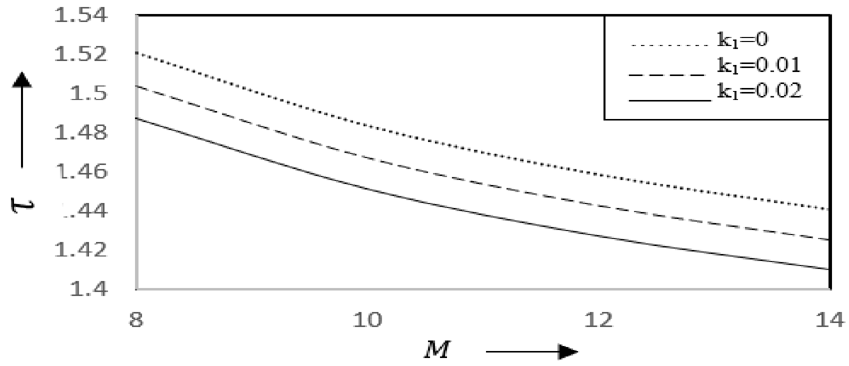

Fig. 5.5 Skin friction coefficient for different values of K_1

5.5 Conclusion

In order to solve nonlinear differential equations, the homotopy perturbation approach is used in this study. This technique improves upon the standard perturbation approach. It has been shown that the visco-elastic and magnetic parameters have a significant impact on the flow field. Research shows that in the special situation of non-Newtonian fluid represented by Walters Liquid(Model B')., boundary layer flow is achievable. For the purpose of contrasting the findings acquired by analytical approach, the identical issue may also be solved using numerical methods. It would be fascinating to see future research examine the subject from the perspective of flow simulation. This opens up a lot of possibilities for the future of the work.