SOME ASPECTS OF FLUID FLOW PROBLEMS WITH EMPHASIS ON VISCO-ELASTICITY

A THESIS SUBMITTED AS PARTIAL FULFILLMENT
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To

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By

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DECLARATION

I hereby declare that the content embodied in the PhD thesis entitled **"Some Aspects of Fluid Flow Problems with Emphasis on Visco-Elasticity"** is the result of research work carried out by me in the Department of Mathematics, The Assam Royal Global University, Guwahati, India, under the supervision of Dr. Kamal Debnath.

In keeping with the general practice of reporting research observations, due acknowledgments have been made wherever the work described is based on the findings of other researchers.

Further, I declare that this thesis as a whole or any part thereof has not been submitted to any university (or institute) for the award of any degree/ diploma.

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i 1 Introduction 1.1 Introduction to Fluid Mechanics The study of fluids, which include both liquids and gases, is the subfield of physics known as fluid mechanics. It is a cornerstone of many other fields of study, including engineering, physics, and environmental science. Fluid mechanics encompasses the study of fluid behaviour, flow patterns, and the forces acting on fluids. The concepts of fluid mechanics are grounded in the laws of physics, and they dictate how fluids behave. These principles help us understand how fluids flow, how they interact with their surroundings, and how they respond to external forces. One of the fundamental concepts in fluid mechanics is viscosity, which refers to a fluid's resistance to flow. Viscosity is a property that differs among fluids and is affected by elements such as temperature and pressure. Substances with high viscosity, like honey or molasses, have a sluggish flow, whereas substances with low viscosity, like water or air, have a smoother flow. Another important concept in fluid mechanics is fluid pressure. Pressure is the measurement of the force applied by a fluid on a specific surface divided by the area of that surface. It is responsible for the upward force that allows objects to float in liquids and for the lift generated by wings in aerodynamics.

2 Fluid flow is an important concept in the field of fluid mechanics and can be categorized into two primary types: laminar flow and turbulent flow. Laminar flow describes a regular, organized pattern of fluid movement with layers flowing parallel to each other. In contrast, turbulent flow is characterized by disorderly, unpredictable motion with swirling and eddying patterns. The shift from laminar to turbulent flow is influenced by variables such as fluid velocity, viscosity, and the roughness of the surfaces the fluid interacts with. Fluid dynamics, a subset of fluid mechanics, is dedicated to comprehending and forecasting fluid flow patterns. It involves employing mathematical equations, like the Navier-Stokes equations, to depict the movement of fluids. These equations provide a holistic view of fluid behaviour by taking into account factors including velocity, pressure, density, and viscosity. Fluid mechanics has numerous applications in various fields. It plays a vital role in engineering, where it is necessary for designing and evaluating fluid systems, including pipelines, pumps, and turbines. It is also crucial in the study of aerodynamics, which focuses on the movement of air around various objects like aircraft and automobiles. Moreover, fluid mechanics is valuable in environmental science as it enables us to comprehend natural occurrences like ocean currents and weather formations. 1.2 Continuum Hypothesis At the macroscopic level, the movement of a fluid is typically studied by considering the behaviour of individual molecules. However, in situations where the flow has a significantly larger characteristic length compared to molecular distances, it is more convenient to work with a small but still relatively large lump of fluid. This lump contains a large number of molecules and allows for the analysis of average statistical properties. In this case, the specific molecular structure is not important and is instead replaced by a continuous model of matter that exhibits the appropriate continuum properties. This ensures that the behaviour of the model on a macroscopic scale closely resembles that of a real fluid. On the other hand, when the characteristic length in the flow is not significantly larger than molecular distances, the continuum model is not applicable and the analysis must be done on a molecular scale. A fluid particle is the tiniest amount of fluid material that has enough molecules to be considered a continuous substance. This field of study focuses on fluids that follow the continuum hypothesis, which assumes that the fluid properties are consistent throughout

3 the entire fluid and in all directions from any given point. These conditions make the fluid uniform and symmetrical. 1.3 Non-Newtonian Fluids The shear stress in a fluid is proportional to the shear strain rate, however in non-Newtonian fluids, this relationship breaks down. To put it another way, the shear stress exerted on a Newtonian fluid has no effect on the fluid's viscosity. On the other hand, non-Newtonian fluids exhibit a more complex bonding between shear stress and shear rate and does not follow a linear flow curve, meaning its apparent viscosity is not constant under different flow conditions. The viscosity of these fluids can vary depending on factors such as flow geometry, shear rate, temperature, and pressure. This means that their flow behaviour can be different from that of Newtonian fluids. These types of materials can be conveniently categorized into three general classes: ▪ Time independent: Shear rate at a given point in a time-independent fluid is dictated entirely by the shear stress at that point and such fluids are also referred to as "inelastic," "purely viscous," or "generalized Newtonian fluids." ▪ Time dependent: Time dependent fluids are fluids that exhibit a complex behaviour where the relationship between shear stress and shear rate depends not only on the duration of shearing but also on the fluid's previous movement patterns. ▪ Viscoelastic: Viscoelastic fluids are substances that display properties of both ideal fluids and elastic solids. They have the ability to partially recover their original shape after deformation. Non-Newtonian fluids are commonly found in various industrial and natural processes, such as in the food industry (e.g., ketchup, mayonnaise), oil drilling, polymer processing, and even in biological systems (e.g., blood, mucus). Understanding and characterizing the flow behaviour of non-Newtonian fluids is crucial in many engineering and scientific applications. 1.4 Application of non-Newtonian Fluids The viscosity variation in non-Newtonian fluids depend on the amount of stress or strain rate applied to them. This unique characteristic makes non-Newtonian fluids useful in various applications. Here are some examples:

4 ▪ Food Industry: Non-Newtonian fluids, such as ketchup, mayonnaise, and chocolate, are commonly used in the food industry. These fluids have shear-thinning behavior, meaning their viscosity decreases when shear stress is applied. This property allows for easy pouring, spreading, and mixing of these food products. ▪ Cosmetics: Many cosmetic products, like lotions, creams, and shampoos, are non- Newtonian fluids. Their viscosity can be adjusted to provide desired flow properties and enhance user experience. For example, a shear-thinning lotion can be easily applied and spread on the skin, but it thickens again to maintain its protective barrier. ▪ Paints and Coatings: Non-Newtonian fluids are also used in the manufacturing of paints and coatings. Depending on the intended application, these fluids may exhibit appropriate behaviour. Shear-thinning paints can be easily applied with a brush or roller, while shear-thickening coatings can provide increased impact resistance. Depending on the intended application, these fluids may exhibit appropriate behaviour. ▪ Medical Applications: Non-Newtonian fluids have found applications in the medical field as well. For example, some wound dressings and surgical gels are non-Newtonian fluids. These fluids can conform to irregular surfaces and provide better contact and coverage. ▪ Oil and Gas Industry: Non-Newtonian drilling fluids are used to transport drill cuttings to the surface, as well as to lubricate and cool drilling equipment. The viscosity of these fluids can be adjusted to optimize drilling efficiency and maintain well stability. ▪ Personal Protective Equipment: Non-Newtonian fluids are also used in the development of impact-resistant materials for personal protective equipment (PPE). These fluids can be embedded in fabrics or polymers to create flexible and lightweight materials that harden upon impact, providing enhanced protection against shocks or impacts. These are just a few examples of the many applications of non-Newtonian fluids. The unique flow properties of these fluids make them versatile and valuable in various industries and everyday products.

List of Figures

Abbreviations

u	Velocity in the x-direction
v	Velocity in the y-direction
ρ	Fluid density
μ	Coefficient of fluid viscosity
ν	Kinematic viscosity
σ	Electrical conductivity of the fluid
k_0	Visco-elastic parameter
B(x)	Magnetic field
B_1	magnetic parameter
k_1	Modified visco-elastic parameter
M	Modified magnetic parameter
η	Similarity variavle
D_M	molecular diffusivity
D_T	thermal diffusivity
F_0	initial velocity slip factor
F_1	velocity slip factor
G_0	initial thermal slip factor
G_1	thermal slip factor
H_0	initial mass slip facto
H_1	mass slip factor
S	Suction/Blowing parameter
τ	skin friction coefficient
k	Permeability parameter
g	Acceleration due to gravity
β^*	volumetric coefficient of thermal expansion
Pr	Prandtl parameter
C	Mass concentration
C_W	Plate mas concentration
C_∞	Free stream mass concentration
S_r	Soret number
D	Diffusion coefficient
R	Variable reaction rate
L	Reference length

v_w	Applied suction or blowing parameter
Re_x	Local Reynolds number
Da_x	Local Darcy number
S_c	Schmidt number
β	Chemical reaction rate parameter
C_p	Specific heat
T	Temperature
K	Thermal conductivity of fluid
T_w	Plate temperature
T_∞	Free stream temperature
U_w	Plate velocithy
U_∞	Free stream velocity
λ	mixed convection parameter
v_w	prescribed velocity of suction/blowing
Ψ	Stream function and
C_0	constant measuring rate of concentration
λ	controlling surface concentration parameter
Δ	Modified velocity slip parameter,
β	Modified thermal slip parameter
γ	Modified mass slip parameter