# 1

# Introduction

## 1.1 Introduction to Fluid Mechanics

The study of fluids, which include both liquids and gases, is the subfield of physics known as fluid mechanics. It is a cornerstone of many other fields of study, including engineering, physics, and environmental science. Fluid mechanics encompasses the study of fluid behaviour, flow patterns, and the forces acting on fluids. The concepts of fluid mechanics are grounded in the laws of physics, and they dictate how fluids behave. These principles help us understand how fluids flow, how they interact with their surroundings, and how they respond to external forces.

One of the fundamental concepts in fluid mechanics is viscosity, which refers to a fluid's resistance to flow. Viscosity is a property that differs among fluids and is affected by elements such as temperature and pressure. Substances with high viscosity, like honey or molasses, have a sluggish flow, whereas substances with low viscosity, like water or air, have a smoother flow. Another important concept in fluid mechanics is fluid pressure. Pressure is the measurement of the force applied by a fluid on a specific surface divided by the area of that surface. It is responsible for the upward force that allows objects to float in liquids and for the lift generated by wings in aerodynamics.

Fluid flow is an important concept in the field of fluid mechanics and can be categorized into two primary types: laminar flow and turbulent flow. Laminar flow describes a regular, organized pattern of fluid movement with layers flowing parallel to each other. In contrast, turbulent flow is characterized by disorderly, unpredictable motion with swirling and eddying patterns. The shift from laminar to turbulent flow is influenced by variables such as fluid velocity, viscosity, and the roughness of the surfaces the fluid interacts with.

Fluid dynamics, a subset of fluid mechanics, is dedicated to comprehending and forecasting fluid flow patterns. It involves employing mathematical equations, like the Navier-Stokes equations, to depict the movement of fluids. These equations provide a holistic view of fluid behaviour by taking into account factors including velocity, pressure, density, and viscosity.

Fluid mechanics has numerous applications in various fields. It plays a vital role in engineering, where it is necessary for designing and evaluating fluid systems, including pipelines, pumps, and turbines. It is also crucial in the study of aerodynamics, which focuses on the movement of air around various objects like aircraft and automobiles. Moreover, fluid mechanics is valuable in environmental science as it enables us to comprehend natural occurrences like ocean currents and weather formations.

## 1.2 Continuum Hypothesis

At the macroscopic level, the movement of a fluid is typically studied by considering the behaviour of individual molecules. However, in situations where the flow has a significantly larger characteristic length compared to molecular distances, it is more convenient to work with a small but still relatively large lump of fluid. This lump contains a large number of molecules and allows for the analysis of average statistical properties. In this case, the specific molecular structure is not important and is instead replaced by a continuous model of matter that exhibits the appropriate continuum properties. This ensures that the behaviour of the model on a macroscopic scale closely resembles that of a real fluid. On the other hand, when the characteristic length in the flow is not significantly larger than molecular distances, the continuum model is not applicable and the analysis must be done on a molecular scale.

A fluid particle is the tiniest amount of fluid material that has enough molecules to be considered a continuous substance. This field of study focuses on fluids that follow the continuum hypothesis, which assumes that the fluid properties are consistent throughout the entire fluid and in all directions from any given point. These conditions make the fluid uniform and symmetrical.

## 1.3 Non-Newtonian Fluids

The shear stress in a fluid is proportional to the shear strain rate, however in non-Newtonian fluids, this relationship breaks down. To put it another way, the shear stress exerted on a Newtonian fluid has no effect on the fluid's viscosity. On the other hand, non-Newtonian fluids exhibit a more complex bonding between shear stress and shear rate and does not follow a linear flow curve, meaning its apparent viscosity is not constant under different flow conditions.  The viscosity of these fluids can vary depending on factors such as flow geometry, shear rate, temperature, and pressure. This means that their flow behaviour can be different from that of Newtonian fluids.

These types of materials can be conveniently categorized into three general classes:

- **Time independent**: Shear rate at a given point in a time-independent fluid is dictated entirely by the shear stress at that point and such fluids are also referred to as "inelastic," "purely viscous," or "generalized Newtonian fluids."
- **Time dependent**: Time dependent fluids are fluids that exhibit a complex behaviour where the relationship between shear stress and shear rate depends not only on the duration of shearing but also on the fluid's previous movement patterns.
- **Viscoelastic**: Viscoelastic fluids are substances that display properties of both ideal fluids and elastic solids. They have the ability to partially recover their original shape after deformation.

Non-Newtonian fluids are commonly found in various industrial and natural processes, such as in the food industry (e.g., ketchup, mayonnaise), oil drilling, polymer processing, and even in biological systems (e.g., blood, mucus). Understanding and characterizing the flow behaviour of non-Newtonian fluids is crucial in many engineering and scientific applications.

## 1.4 Application of non-Newtonian Fluids

The viscosity variation in non-Newtonian fluids depend on the amount of stress or strain rate applied to them. This unique characteristic makes non-Newtonian fluids useful in various applications. Here are some examples:

- Food Industry: Non-Newtonian fluids, such as ketchup, mayonnaise, and chocolate, are commonly used in the food industry. These fluids have shear-thinning behavior, meaning their viscosity decreases when shear stress is applied. This property allows for easy pouring, spreading, and mixing of these food products.
- Cosmetics: Many cosmetic products, like lotions, creams, and shampoos, are non-Newtonian fluids. Their viscosity can be adjusted to provide desired flow properties and enhance user experience. For example, a shear-thinning lotion can be easily applied and spread on the skin, but it thickens again to maintain its protective barrier.
- Paints and Coatings: Non-Newtonian fluids are also used in the manufacturing of paints and coatings. Depending on the intended application, these fluids may exhibit appropriate behaviour. Shear-thinning paints can be easily applied with a brush or roller, while shear-thickening coatings can provide increased impact resistance.

  Depending on the intended application, these fluids may exhibit appropriate behaviour.
- Medical Applications: Non-Newtonian fluids have found applications in the medical field as well. For example, some wound dressings and surgical gels are non-Newtonian fluids. These fluids can conform to irregular surfaces and provide better contact and coverage.
- Oil and Gas Industry: Non-Newtonian drilling fluids are used to transport drill cuttings to the surface, as well as to lubricate and cool drilling equipment. The viscosity of these fluids can be adjusted to optimize drilling efficiency and maintain well stability.
- Personal Protective Equipment: Non-Newtonian fluids are also used in the development of impact-resistant materials for personal protective equipment (PPE). These fluids can be embedded in fabrics or polymers to create flexible and lightweight materials that harden upon impact, providing enhanced protection against shocks or impacts.

These are just a few examples of the many applications of non-Newtonian fluids. The unique flow properties of these fluids make them versatile and valuable in various industries and everyday products.

## 1.5 Visco-elastic fluids and it's applications

Viscoelastic fluids are a type of material that exhibits both viscous and elastic behaviour. These fluids have properties of both liquids and solids, meaning they can flow like a liquid but also possess some degree of elasticity like a solid. The viscosity of a viscoelastic fluid refers to its resistance to flow. Unlike a purely viscous fluid, which flows easily, viscoelastic fluids have a time-dependent viscosity. This means that their viscosity can change depending on the rate at which they are deformed or stressed. The elasticity of a viscoelastic fluid is its capacity to deform and then regain its original shape. This is similar to the behaviour of a solid, but unlike a purely elastic material, viscoelastic fluids can also flow and deform over time.

Here are some applications of viscoelastic fluids:

a) Polymer Solutions: Viscoelastic fluids are commonly used in the production of polymer solutions. These solutions can be used for various applications such as adhesives, coatings, and sealants. The viscoelastic properties of these fluids allow them to adhere well to surfaces, fill gaps, and provide a durable and flexible bond.

b) Personal Care Products: Viscoelastic fluids are used in many personal care products such as hair gels, styling creams, and lotions. These fluids provide the necessary viscosity and elasticity to hold hairstyles in place, enhance shine, and provide a smooth and soft texture to the hair or skin.

c) Biomedical Applications: Viscoelastic fluids find applications in the field of biomedicine. For example, hyaluronic acid-based viscoelastic fluids are used in ophthalmic surgeries, such as cataract surgery, to maintain the shape of the eye during the procedure. These fluids also act as lubricants and protect the delicate tissues of the eye.

d) Enhanced Oil Recovery: Viscoelastic fluids are used in the oil and gas industry for enhanced oil recovery (EOR) techniques. These fluids can be injected into oil reservoirs to improve the flow of oil by reducing its viscosity and increasing its mobility. This allows for the recovery of a larger percentage of oil from the reservoir.

e) Food Processing: Viscoelastic fluids are utilized in the food industry for various purposes. For example, in food manufacturing, these fluids can be used to control the texture, stability, and mouthfeel of products like sauces, dressings, and desserts. They can also be used as thickening agents in food formulations.

f) Consumer Products: Viscoelastic fluids are used in the production of consumer products such as liquid detergents, fabric softeners, and dishwashing liquids. These fluids provide the desired flow properties, stability, and enhanced cleaning performance.

g) Industrial Applications: Viscoelastic fluids find applications in industries such as paints, inks, and coatings. They can be used to improve the flow behaviour, stability, and adhesion of these products, resulting in better performance and durability.

These are just a few examples of the many applications of viscoelastic fluids. Their unique combination of viscosity and elasticity makes them valuable in numerous industries, from personal care to oil extraction and beyond.

## 1.6 Walters Liquid (Model $B'$)

Walters liquid Model $B'$ is a fluid model used to explain the rheological properties of polymer solutions and melts. It was developed by Walter in the 1970s and is based on the idea of reptation, which refers to the movement of long polymer chains through a viscous medium. According to this model, the polymer chains are confined to tube-like regions within the fluid and their movement is hindered by the frictional forces between the chains and the tube walls. The model also takes into account the influence of shear rate and temperature on the fluid's behaviour. The model is characterized by two parameters: the relaxation time, which represents the time taken for polymer chains to return to their original shape after deformation, and the characteristic time, which describes the time scale of the fluid's flow behaviour.

This model is commonly used to explain how different polymer solutions and melts, such as polyethylene, polypropylene, and polystyrene, behave under different conditions. It is also applied to study the flow of complex fluids like emulsions and suspensions. This model is particularly valuable in understanding the flow properties of non-Newtonian fluids that involve long-chain polymers. Due to its accuracy in describing the flow behaviour of these fluids, the B-prime model is highly regarded in the field of polymer science and engineering.

For liquids with short memory, as Walters (1962) shown,

$$\tau^{ik} = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\delta}{\delta t} e^{(1)ik} \tag{1.5.1}$$

where, $\eta_0 = \int_0^\infty N(\lambda)d\tau$ and $k_0 = \int_0^\infty \lambda N(\lambda)d\tau$.

Convective derivative for contrvariant tensor is

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - \frac{\partial v^k}{\partial x^m} b^{im} - \frac{\partial v^i}{\partial x^m} b^{mk} \tag{1.5.2}$$

where $v_i$ denotes velocity vector. Taking into consideration very short memories, this hypothetical model is a close approximation of Walters liquid (Model $B'$) so that the terms involving

$$\int_0^\infty \lambda^n N(\lambda)\, d\lambda, \lambda \geq 2$$

have been neglected. As this simplified model accounts for extremely short memories, it provides a close enough approximation of Walters liquid (Model B').

## 1.7 Fluid Flow Governing Equations

The equations that govern fluid dynamics are a collection of mathematical equations that explain how fluids behave. These equations are derived from fundamental principles such as conservation of mass, momentum, and energy.

- **Conservation of Mass (Continuity equation)**: This equation states that mass cannot be created or destroyed, and it governs the conservation of mass in a fluid. It is expressed as:

  $$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{q}) = 0 \tag{1.7.1}$$

  where, ρ: fluid density, t: time, $\vec{q}$: velocity vector, and $\nabla.(\rho\vec{q})$: divergence of mass flow rate.

- Conservation of Momentum (Navier-Stokes equations):

  These equations describe the conservation of momentum in a fluid and are derived from Newton's second law. They are expressed as:

  $$\frac{\partial(\rho\vec{q})}{\partial t} + \nabla.(\rho\vec{q} \otimes \vec{q}) = \nabla \mathrm{p} + \nabla.\tau + \rho \mathrm{g} \tag{1.7.2}$$

  where p: pressure, stress tensor, g: acceleration due to gravity, and ⊗: tensor product.

- Conservation of Energy (Energy equation):

  This equation governs the conservation of energy in a fluid. It is expressed as:

$$\frac{\partial(\rho e)}{\partial t} + \nabla.(\rho e\vec{q}) = -\nabla.(\mathrm{p}\vec{q}) + \nabla.(k\nabla T) + \rho\mathrm{g}.\vec{q} \qquad (1.7.3)$$

  where e: internal energy per unit mass, k: thermal conductivity, T: temperature.

These equations, along with appropriate boundary conditions, form a system of partial differential equations that can be solved to determine the flow behaviour of fluids in various scenarios.

## 1.8 Boundary Layer and it's classification

The boundary layer refers to a thin layer of fluid that develops along the surface of an object while it moves through another fluid medium like air or water. It is identified by a gradual transition in velocity and other flow characteristics from the surface of the object to the surrounding fluid. The boundary layer is of great significance in determining the drag force encountered by the object, as well as heat transfer and other phenomena.

In fluid dynamics, boundary layer can be classified as follows:

- **Velocity Boundary Layer**

The velocity boundary layer refers to a thin layer of fluid in close proximity to a solid surface where the speed of flow experiences a rapid change. It is formed due to the viscous effects of the fluid, which cause the fluid particles to slow down as they come into contact with the solid surface. This layer is important in understanding the behaviour of fluid flow near solid boundaries. In this layer, the speed of the flow gradually increases from being still at the surface to reaching the speed of the surrounding fluid. The thickness of this layer of changing velocity is determined by the fluid's viscosity and the rate at which the velocity changes near the surface.

- **Thermal Boundary Layer**

The thermal boundary layer refers to the region near a solid surface where heat transfer occurs mainly through conduction. It is created due to a temperature disparity between the solid surface and the fluid in its vicinity. This temperature difference leads to the formation of a thin layer of fluid. Within this layer, there is a substantial variation in temperature, and heat is transferred from the solid surface to the fluid through interactions between molecules. The thickness of this layer determined by several factors like flow conditions, thermal conductivity, the temperature disparity between the solid surface and the fluid

- **Concentration Boundary Layer**

The concentration boundary layer is a thin layer of fluid that is found close to a solid surface where the concentration of a particular substance is significantly different from the bulk fluid. This can occur in various situations, such as in the atmosphere, in water bodies, or in industrial processes. The concentration boundary layer can occur near the surface or bottom due to factors like temperature gradients, turbulence, or biological activity. This layer can impact the distribution of dissolved gases, nutrients, or pollutants in the water. This layer can affect mass transfer rates and efficiency of the process.

The boundary layer plays a crucial role in various engineering applications. It affects the drag force experienced by objects moving through a fluid, the heat transfer from a heated surface, and the mass transfer in chemical reactions. Understanding and analysing the boundary layer behaviour is essential for designing efficient and streamlined objects, such as aircraft wings, car bodies, and heat exchangers.

## 1.9 Self-similar solution

Self-similar solutions are commonly used in fluid mechanics to analyse physical characteristics of flow, heat, and mass transfer. This concept is a generalization of similarity in geometry, where two objects have the same shape and can be obtained from each other through uniform scaling. In fluid flow, self-similarity is based on the idea that temperature and velocity distributions will collapse if plotted in dimensionless form using an appropriate similarity variable. The partial differential equations are reduced to ordinary differential equations, facilitating analysis. Self-similarity is important in understanding boundary layer flow with heat and mass transfer. In the study of partial differential equations, self-similar solutions arise when there is no characteristic length or time scale present.

## 1.10 Slip and no-slip boundary conditions

In fluid dynamics, the slip and no-slip conditions refer to the behaviour of fluid flow at a solid boundary. The no-slip condition means that when a fluid comes into contact with a solid surface, it sticks to the surface and doesn't move relative to it. The fluid molecules directly touching the boundary are at rest, and the fluid and solid surface are not moving.

This condition is commonly observed in most practical situations, where the fluid sticks to the solid surface due to molecular forces.

On the other hand, the slip condition arises when there is movement between the fluid and the solid surface. In this case, the velocity of the fluid molecules touching the barrier is not zero with respect to the surface. Slip conditions are typically observed in situations involving very thin fluid layers, such as at the nanoscale or when dealing with certain types of liquids or surfaces with low friction.

When a fluid meets a solid barrier, the slip or no-slip condition depends on the fluid and the surface. It can have a significant impact on the overall flow behaviour, especially in microfluidic systems or when studying fluid flow in confined spaces. It is important to note that slip conditions are more commonly observed at small scales, while the no-slip condition is generally applicable at larger scales and is often assumed in fluid flow calculations and simulations.

## 1.11 Magnetohydrodynamics

Magnetohydrodynamics (MHD), is a subfield of physics concerned with the behaviour of electrically conductive fluids in the presence of magnetic fields. It combines principles from both magnetism and fluid dynamics to understand and analyze the interactions between magnetic fields and flowing fluids.

MHD has applications in various fields, including astrophysics, plasma physics, engineering, and geophysics. It helps explain phenomena such as the behaviour of plasma in fusion reactors, the dynamics of flow of conducting fluids in industrial processes. In MHD, the way the fluid behaves is affected by the Lorentz force, which occurs when the magnetic field interacts with the charged particles in the fluid. This force can accelerate or decelerate the fluid, cause it to rotate, or generate electric currents within the fluid.

MHD has practical applications in various technologies. For example, it is used in the design and optimization of magnetohydrodynamic generators, which employ the kinetic energy of a conducting fluid to generate electricity without the need for mechanical devices. MHD is also utilized in the development of magnetohydrodynamic pumps and propulsion systems. It finds utility in various scientific and technological domains, aiding in the comprehension of natural phenomena and facilitating the creation of inventive engineering remedies.

## 1.12 Heat Transfer in Fluid Flow

Heat transfer in fluid flow refers to the process of transferring thermal energy between a fluid and its surroundings or between different parts of the fluid itself. It plays a crucial role in various engineering applications, including cooling systems, heating processes, and energy conversion systems.

There are three primary modes of heat transfer in fluid flow:

- **Conduction**: Conduction occurs when heat is transferred through direct molecular interactions. In fluid flow, conduction typically takes place near solid boundaries where the fluid is in contact with a heated or cooled surface. The heat is conducted through the fluid by molecular collisions, leading to a temperature gradient within the fluid.
- **Convection**: Convection involves the transfer of heat through the movement of a fluid. This movement can happen naturally due to temperature differences or can be forced using external devices like pumps or fans. In most situations involving fluid flow, convection is the primary method of heat transfer.
- **Radiation**: The transmission of heat through electromagnetic waves is called radiation. In fluid flow, radiation can occur between the fluid and its surroundings, as well as between different parts of the fluid itself. The extent of radiation heat transfer depends on factors such as the temperature of the fluid and its surroundings, as well as the emissivity and absorptivity of the surfaces involved.

To analyse and predict heat transfer in fluid flow, engineers use various mathematical models and empirical correlations. The fluid properties, flow velocity, temperature gradients, and the geometry of the system are all taken into consideration by these models. Flow simulations tools are often employed to study complex fluid flow and heat transfer phenomena. Understanding heat transfer in fluid flow is essential for designing efficient heat exchangers, cooling systems, and thermal management devices. By optimizing the flow patterns, surface geometries, and operating conditions, engineers can enhance heat transfer rates, improve energy efficiency, and ensure the safe operation of various systems.

## 1.13 Mass Transfer in Fluid Flow

Mass transfer in fluid flow refers to the process of transferring substances (such as gases, liquids, or particles) from one location to another within a flowing fluid. It involves the movement of these substances due to concentration gradients or pressure differences.

There are various mechanisms of mass transfer in fluid flow, including:

i. **Diffusion:** Diffusion refers to the motion of molecules or particles as they move from an area of greater concentration to an area of lesser concentration. In fluid flow, diffusion plays a significant role in mass transfer, especially when there are concentration gradients across the fluid.
ii. **Advection:** Advection refers to the bulk movement of a substance with the fluid flow. It occurs when the fluid carries the substance along with it. Advection can be caused by factors such as fluid velocity, pressure gradients, or external forces.
iii. **Convection:** Convection involves the combined effect of advection and diffusion. It occurs when the fluid flow and concentration gradients work together to transport substances. Convection is particularly important in scenarios where the fluid flow is turbulent or when there are significant variations in concentration or temperature.

Mass transfer in fluid flow is crucial in various engineering applications. For example:

- In chemical reactors, mass transfer is essential for the efficient conversion of reactants to products.
- In wastewater treatment, mass transfer helps in the removal of pollutants from water.
- In heat exchangers, mass transfer allows for efficient transfer of heat between two fluids.
- In natural processes like evaporation and condensation, mass transfer plays a vital role.

Mass transfer in fluid flow is crucial for designing efficient separation processes, optimizing chemical reactions, and improving the performance of various systems. By controlling mass transfer, engineers can enhance the efficiency, selectivity, and overall performance of processes involving fluid flow.

## 1.14 Porous Media

Porous media refers to materials that have interconnected void spaces or pores. These void spaces allow for the flow of fluids or gases through the material. Examples of porous media include soil, rocks, sponges, and filters. The study of porous media is important in various fields, including geology, hydrology, environmental science, and engineering. Understanding how fluids and gases move through porous media is crucial for applications such as groundwater flow, oil and gas recovery, and water filtration. The way fluids behave in porous materials is influenced by several factors, such as the dimensions and structure of the pores, the characteristics of the fluid, the properties of the material, and the pressure and temperature conditions. Different types of porous media have different permeability, which is a measure of how easily fluids can flow through them.

Porous media can exhibit different flow regimes, such as Darcy flow, where the rate at which fluid flows is directly proportional to the gradient of pressure, and non-Darcy flow, where additional factors such as viscous forces and inertia become significant. Researchers and engineers use various techniques and models to study and predict fluid flow in porous media. These include laboratory experiments, numerical simulations, and analytical solutions based on mathematical equations.

## 1.15 Chemical reaction in fluid

Chemical reactions in fluid flow refer to the phenomenon where chemical reactions occur in a flowing fluid. This can happen in various scenarios, such as in industrial processes, combustion engines, and chemical reactors. When a fluid flows, it carries different chemical species, such as reactants and products, along with it. As these species come into contact with each other and with catalysts or surfaces, chemical reactions can take place. The rate and extent of these reactions depend on factors like the concentration of reactants, temperature, pressure, and the presence of catalysts.

Chemical reactions in fluid flow are important in a variety of applications. In industrial processes, like petroleum refining and chemical manufacturing, fluid flow reactions are used to produce desired products from raw materials. In combustion engines, fuel is mixed with air, and the resulting chemical reactions generate energy for propulsion. In chemical reactors, reactants are introduced into a flowing fluid to achieve specific chemical transformations. To study and analyse chemical reactions in fluid flow, various mathematical models and computational techniques are employed. These models incorporate the principles of fluid mechanics and chemical kinetics to describe the transport of reactants, heat, and mass within the fluid. They also account for factors like mixing, turbulence, and reaction rates to predict the behaviour of the system.

Understanding chemical reactions in fluid flow is essential for optimizing process design, improving reaction efficiency, and ensuring safety. By studying the fluid dynamics and reaction kinetics, engineers and scientists can make informed decisions about reactor design, reactant feeding, temperature control, and other parameters to achieve desired reaction outcomes.

## 1.16 Homotopy Perturbation Method

Homotopy Perturbation Method (HPM) is a powerful mathematical approach used to solve complicated differential equations that cannot be easily solved using traditional methods. It was developed by J.H. He in 1999 and has gained popularity in engineering and scientific disciplines.

The method involves creating a continuous function called a homotopy function, which connects the initial conditions of the differential equation to its exact solution. This function is then modified with a small parameter to break down the differential equation into a system of linear and nonlinear equations.

The following steps are involved in this method:

- Homotopy function connects the initial conditions of a differential equation to its exact solution.
- Modify the homotopy function by adding a small parameter, which aids in converting the differential equation into systems that are both linear and nonlinear.
- Solving series of linear and nonlinear equations using iterative techniques.
- Approximate solution for a differential equation involves taking the perturbation parameter to zero and finding the limit.

The HPM stands out from other numerical and analytical techniques due to several advantages. It is a straightforward and efficient method that does not require complex numerical algorithms or advanced computing tools. It is a flexible method that works with a variety of nonlinear differential equations in various fields such as engineering, science, and other disciplines.

Despite its strengths, the HPM does have certain limitations. It may not be suitable for tackling differential equations that contain singularities or for solving systems of differential equations. Additionally, it relies on the careful selection of initial conditions and perturbation parameters.

## 1.17 MATLAB Code ‘bvp4C’

The 'bvp4c' function in MATLAB is an inbuilt algorithm that is designed to efficiently solve mathematical problems with boundary value conditions. It utilizes a finite difference method to discretize the ordinary differential equation and find the solution at a series of discrete points.

The function takes the following inputs:

- A function handle to the ODE of the form $\frac{dy}{dx} = \mathrm{g}(\mathrm{x}, \mathrm{y}, \mathrm{q}),\ \mathrm{x} \in [a, b]$, where y is the unknown function, x is the independent variable, and p is a vector of parameters that can be used in the ODE.
- A function handle to the boundary conditions of the form $\mathrm{g}(\mathrm{y}(\mathrm{a}), \mathrm{y}(\mathrm{b}), \mathrm{q}) = 0$, where $\mathrm{y}(\mathrm{a})$, and $\mathrm{y}(\mathrm{b})$ are unknown function values to left and right boundaries.
- An initial guess for the solution.
- A vector of nodes at which to solve the ODE.
- A vector of parameters to be used in the ODE and boundary conditions.

The function provides a structure that contains the solution to the ODE. This includes the values of the unknown function at specific points, as well as details about the accuracy of the solution. The solutions obtained can be easily compared to real-world phenomena in various fields of science and technology.

## 1.18 Dimensionless parameters

To ensure that physical equations are dimensionally correct, dimensionless parameters can be generated by dividing a physical quantity by a characteristic value or reference value of that quantity in the system. The principle of similarity can then be applied to establish non-dimensional parameters that relate a model to a prototype. These parameters are useful as they allow for comparisons and analysis of different systems without the need for specific units. They help in simplifying complex equations, identifying important variables, and understanding the behaviour of a system.

Few non-dimensional parameters are listed below:

i. **Reynolds Number:** The Reynolds number $\mathrm{R_e}$ is a dimensionless value that quantifies the relationship between the forces of inertia and viscosity. When the Reynolds number $\mathrm{R_e}$ is very tiny, it indicates that viscous forces are more dominating than inertia forces. Such forces are generally referred to as "creeping or viscous flow," however if the Reynolds number $R_e$ is very big in comparison to inertial effects, the flow problem is referred to as "inviscid analysis”. The Reynolds number $R_e$ is also employed to examine the transition from laminar to turbulent flow regimes. Heat transfer problems frequently involve the Reynolds number. Osborne Reynolds is the name of the number.

Mathematically,

$Re = \frac{\rho VL}{\mu} = \frac{VL}{\delta}$, Where

$v$ = Characteristics velocity of the fluid

L = Characteristics length

$\rho$ = density

$\mu$ = dynamic viscosity

$\delta$ = Kinematic viscosity

**i. Prandtl Number:** The Prandtl number $P_r$ is a dimensionless parameter used to characterize the ratio of the kinematic viscosity and the thermal diffusivity. It is a material property that remains constant and does not depend on the flow characteristics. It indicates the relative magnitude of the thermal boundary layer compared to the viscous boundary layer. For gas $P_r$= 1, for air $P_r$ = 0·7, for water at $60^0$ F, $P_r$ = 7 whereas for glycerin $P_r$ = 7250. Its importance found in the German physicist Ludwig Prndtl.

Mathematically,

$P_r = \frac{\delta}{\alpha} = \frac{\mu}{k}\ c_p$ ,where

δ = kinematic viscosity

$\alpha =$ thermal diffusivity

$c_p$ = specific heat

μ = viscosity

k = thermal conductivity

ii. **Grashof Number:** The Grashof Number $Gr_L$ is a non-dimensional number that represents the approximate ratio of buoyancy to viscous force in a fluid. It is commonly used to investigate natural convection situations in both heat transfer and mass transfer mechanisms. When the Grashof number is low, the flow becomes laminar, and when the Grashof number is high, the flow becomes turbulent. Franz Grashof, a German engineer, was the inspiration for the name.

Mathematically,

$$Gr_L = \frac{g\beta\ (\Delta T)\ L^3}{\mu^2} = \frac{g\beta\ (\Delta T)\ L^3}{\delta^2}, as\ \mu = \ \delta\rho \text{ (For Vertical Plate)}$$

$$Gr_D = \frac{g\beta\ (\Delta T) D^3}{\delta^2}, (for\ pipes), \text{ where}$$

$g$ = acceleration due to gravity

$\beta$ = Volumetric thermal expansion Co – efficient

$L$ = Characteristics length, D = diameter.

$\rho$ = density, $\mu$ = viscosity, $\delta$ = Kimatic Viscosity

$\Delta T$ = temperature difference between surface and the free stream

iii. **Schmidt Number:** The Schmidt number $S_c$ is a non-dimensional number that represents the ratio of momentum (viscosity) to mass diffusivity in fluid flows. The number is named after Ernest Heinrich Wilhelm Schmidt, a German engineer.

Mathematically,

$$S_c = \frac{\delta}{D} = \frac{\mu}{\rho D}, \text{ where}$$

$\delta$ = kinematic viscosity

D = mass diffusivity

$\mu$ = dynamic viscosity

$\rho$ = fluid density

iv. **Soret Number:** The Soret number ($S_r$) is a dimensionless parameter used to quantify the effect of thermal gradients on mass diffusion in a mixture. The Soret number measures the extent to which a thermal gradient influences the migration of different species in a mixture. Mathematically, it is expressed as:

$$S_r = \frac{Dt}{Dm},$$

Where, Dt = thermal diffusion coefficient and Dm = mass diffusion coefficient

## 1.19 Motivation of the study

Viscoelastic fluids are important in various industries and biological systems. Understanding their behaviour can lead to more efficient processes and improved product formulations. Studying these fluids helps advance our understanding of rheology and develop new theories and models. Working with viscoelastic fluids involves solving complex engineering problems, which can be intellectually stimulating. Collaboration between different disciplines is common in this field, leading to exciting discoveries and new collaborations. Research on viscoelastic fluids has the potential to impact industries and improve products and processes in fields like healthcare, consumer products, and manufacturing. Working on viscoelastic fluids offers the opportunity to tackle challenging problems, contribute to scientific knowledge, and make a positive impact on society. This prompted the author to investigate some aspects of fluid flow problems with emphasis on viscoelasticity taking different geometry in combination of important flow feature parameters.

## 1.20 Objectives of the study

The objectives of this study are as follows:

- Examine few aspects of fluid flow issues with emphasis on visco-elasticity.
- Investigate the impact of various flow dominant physical flow parameters involved in the solution in combination with visco-elastic parameter.
- Analyse data from graphs and tables to gain a physical understanding of the issues that were resolved.

## 1.21 Literature Survey

A literature survey is an essential component of any research work. It includes a comprehensive examination and evaluation of existing literature, academic articles, books and other relevant sources related to a particular topic. Literature review is important because it provides the basis for research, helps identify gaps and research problems, evaluates existing research, supports arguments, improves research methods, and generates ideas and inspirations. This is a crucial step in the research process and contributes to the overall quality and credibility of the research work.

To generalize the linear relationship between stress and strain rate tensors, there are two types of thoughts that prevail in literature. The first type of constitutive model must have originated from the concepts proposed by Walters, Oldroyd and his collaborators. The second constitutive model was given by Tredell, Noel, Coleman, Erickson, and his associates Sravastava 1968. In this study, emphasis is placed on the first type of thinking.

Oldroyd [1] proposed the generalisation of a one-dimensional empirical rheological mathematical model of state. Oldroyd [2] examined the effects of visco-elasticity on the constant motion of certain ideal fluids. For the situation of materials having memory, the solution has been shown by imagining the motion of a visco-elastic fluid confined between two co-axial cylinders. It is observed that Walters has addressed a number of flow issues in the published literature. Thomas and Walters [3], Beard and Walters [4], etc. are only a few examples.

The plane Couette flow of the Walters liquid Walters B' as investigated by Soundalgekar [5] under conditions of equal rates of injection at one wall and suction at the other. Mishra and Acharya [6] showed that the Walters liquid model B' may flow between two coaxial porous cylinders. Jhori and Sharma [7] studied the laminar flow of a viscoelastic fluid under free-convection conditions. The free convection flow of a non-Newtonian fluid in the presence of heat sources has been described by Jha [8]. Nanousis [9] performed an analysis of MHD flows in a rotating elastic-viscous fluid. Natural convective flow of a non-Newtonian fluid down a vertical plate has been investigated by Kim [10]. Raptis [11] investigated the effect of radiation on the viscoelastic behaviour of fluids. Ghosh *et al.* [12] showed that a dusty non-Newtonian fluid may flow hydromagnetically.

Eldabe *et al.* [13] examined magnetohydrodynamic flow of elastic-viscous fluids past a porous domain near an accelerating plate. Pillai et al. [14] examined the flow of a visco-elastic boundary layer through a porous media. Liu [15] presented the kinetics of a non-Newtonian fluid's unstable, one-way flow through a porous material. The flow behaviour through a stretched sheet has been investigated in a variety of scenarios by Veena et al. [16]. Reza and Gupta [17] investigated the momentum and heat transmission of a viscoelastic fluid via a porous plate under suction or blowing. Norouzi *et al.* [18] have taken into account the convective heat transfer in a curved duct for an elastico-viscous fluid.

Choudhury and Dey [19] have studied the heat and mass transport of visco-elastic flow characteristics of Walters liquid (Model B′) through porous medium. Chang *et al.* [20] numerically analyzed the mass transport due to natural convection of elastic-viscous fluid flow employing Walters B model.Free convective flow between a long vertical wavy wall and a parallel flat wall of equal transpiration has been described by Choudhury and Das [21], including the effects of visco-elasticity. Visco-elastic MHD oscillatory horizontal channel flow and heat transfer with heat source have been explored by Choudhury and Das [22], and their impacts on MHD unsteady free convective flow have been analysed.

Choudhury and Das [23] have discussed the visco-elastic MHD free convective flow through porous media in radiation and chemical reaction with heat and mass transfer. Choudhury and Das [24] studied the influence of visco-elasticity on MHD heat and mass transfer flow through a porous medium bounded by an inclined surface with chemical reaction. Choudhury and Dey [25] also studied the unsteady thermal radiation effects on MHD convective slip flow of visco-elastic fluid past a porous plate embedded in porous medium. Chen et al. [26] investigated numerically the unsteady viscoelastic MHD fluid flow through a stretchable s employing Maxwell fractional fluid model. A few books [27–30] based on Newtonian and non-Newtonian fluids theory are studied to proceed further in this research work.