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Chapter 5

Reactive Mass Diffusion in Viscoelastic Fluid Past a Stretchable Exponential Sheet Due to Variation in Wall Concentration

5.1 Introduction

The viscous layer flow that extends to the expandable surface is an important problem arising from fluid mechanics as a result of its extensive applications in industrial manufacturing, such as polymer sheet extraction, paper production, fiber glass manufacturing, metal processing, wire drawing, etc. Crane [66] firstly studied the exact solution in the closed-form of a steady flow boundary of the membrane of viscous fluid over a simple stretchable plate. Many scientists later built on Crane's work, including Pavlov [68], Gupta and Gupta [83], Chen and Char [84], who studied thermal and mass transport mechanisms with magnetic field influence of different physical phenomena. Andersson [85] observed the hydromagnetic viscoelastic movement of fluid flow past an expandable sheet. The hydromagnetic heat transport flow past a stretchable layer was explained by Char [86]. El-Aziz [87] investigated the heat transition and mass transport of temperature dependent viscous fluid along with thermal conductivity as it passed through a continuously stretching sheet with Ohmic heating.

The mass diffusion caused by chemical reactions in the boundary layer has numerous uses in chemical engineering, fibrous insulation, and atmospheric flows. The reactive diffusion process of species through the boundary layer was defined by Chambre and Young [88]. Stan [89] studied boundary layer flow and chemical surface interactions together. In addition, the impact of chemical reaction on fluid passing over a flat surface was also examined in many articles [90-94].

The flow through a stretchable surface is induced by the flat surface's linear stretching velocity. However, despite being a very critical and constructive flow that occurs often in many manufacturing industries, such flow is rarely studied. Magyari and Keller [95] examined the heat transport through the boundary layer of an expandable sheet when the wall temperature varied exponentially. Khan and Sanjayanand [96] explored viscoelastic flow patterns and heat transmission along with a stretchable layer with viscous dissipation. Elbashbeshy [97] studied numerically the fluid motion and heat transmission of stretchable exponential sheets considering wall suction. Banerjee *et al.* [98] examined mass dissipation in the boundary layer with chemical reaction across an expandable exponential sheet with variable wall concentration.

Nayak *et al.* [99] demonstrated the transition of thermal energy and and mass transport process for chemically reactive hydromagnetic viscoelastic fluid through boundary layer with source/sink. According to Singh and Kumar [100], the heat transition and mass transport process is affected by chemical reactions in a micropolar fluid past a porous channel with thermal radiation and heat generation. Mjankwi *et al.* [101] studied the impact of varying fluid characteristics on heat flux and mass absorption coefficient. Misra and Govardhan [102] investigated the impact of chemical reactions on the boundary layer flow of nanofluids, as well as fluctuations in heat transition and mass transfer.

The mass diffusion in visco-elastic fluid through boundary layer past a stretchable exponential sheet with first order chemical reaction modeled by Walter Liquid (Model B/) is investigated in this paper. The species reaction rate and the concentration variation at the wall are taken as exponential forms. Employing similarity variables, equations guided fluid motion are reduced to self-similar type and then evaluated by solver ‘bvp4c’ of MATLAB. The numerically obtained results are graphically presented. The influence of flow feature factors on the concentration profiles is analyzed from graphs from a physical standpoint.

5.2 Mathematical Formulation

The steady two-dimensional elastico-viscous boundary layer fluid flow past a flat moving plate. The mass diffusion due to chemical reaction in non-Newtonian viscoelastic fluid through boundary layer past a stretchable exponential sheet with deviation in wall concentration is considered. The fluid motion, taking appropriate approximation of boundary layer, is guided by the below mentioned equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5.2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{k_0}{\rho}\left[u\frac{\partial^3 u}{\partial x\,\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\,\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right] \tag{5.2.2}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty) \tag{5.2.3}$$

The corresponding boundary conditions are:

$$u=U_w(x),\ v=0\ \text{ at }\ y=0;\quad u\to 0 \text{ as } y\to\infty \tag{5.2.4}$$

$$C=C_w=C_\infty+C_0 e^{\left(\frac{\lambda x}{2L}\right)} \text{at } y=0\ ;\ C\to C_\infty \text{ as } y\to\infty \tag{5.2.5}$$

The velocity expansion of the sheet $U_w(x)$ is given by

$$U_w(x)=ae^{\left(\frac{x}{L}\right)} \tag{5.2.6}$$

To obtain self-similar forms of above equation, the following similarity transformation are introduced:

$$\Psi=\sqrt{2\nu La}\,f(\eta)e^{\frac{x}{2L}},\quad C=C_\infty+(C_w-C_\infty)\phi(\eta) \tag{5.2.7}$$

where $\eta=y\sqrt{\frac{a}{2\nu L}}\,e^{\frac{x}{2L}}$ denotes similarity variable.

The stream function Ψ connected with velocity components as $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial x}$.

Using relation (5.2.7) in (5.2.2) and (5.2.3), the following set of self-similar equations are obtained:

$$f'''(\eta)+f(\eta)f''(\eta)-2\big(f'(\eta)\big)^2-k_1\left[2f'(\eta)f'''(\eta)-\tfrac{1}{3}f(\eta)f^{\prime v}(\eta)-\big(f''(\eta)\big)^2\right]=0 \tag{5.2.8}$$

$$\phi''(\eta) + S_c\big(f(\eta)\phi'(\eta) - \lambda f'(\eta)\phi(\eta) - \beta\phi(\eta)\big) = 0 \tag{5.2.9}$$

where, $k_1 = \frac{3k_0\, a}{2\mu L} e^{\left(\frac{x}{L}\right)}$ is the modified visco-elastic parameter, $S_c = \frac{\nu}{D}$ is the Schmidt number, $\beta = \frac{2LR_0}{a}$ is the reaction rate parameter.

Transformed condition at the boundary are:

$$f(\eta) = 0,\ f'(\eta) = 1\ at\ \eta = 0\ ; \qquad f'(\eta) = 0,\ f''(\eta) = 0\ as\ \eta \to \infty \tag{5.2.10}$$

$$\phi(\eta) = 1\ at\ \eta = 0\ ;\ \phi(\eta) = 0\ as\ \eta \to \infty \tag{5.2.11}$$

5.3 Method of Solution

The numerical method 'bvp4c' of Matlab is a collocation method used to solve differential equation of the form $\frac{dy}{dx} = \mathrm{g(x, y, q)},\ \mathrm{x} \in [a, b]$ with non-linear boundary conditions $\mathrm{h(y(a), y(b), q)} = 0$, where vector q is a unknown parameter. This method is an effective solver different from the shooting method and it is based on an algorithm. It can compute inexpensively the approximate value of $y(x)$ for any x in $[a, b]$ taking boundary conditions at every step. In this method, the infinity conditions at the boundary are replaced with a finite point which reasonably satisfies the given problem.

The self-similar differential equations (5.2.8) and (5.2.9) are transformed to first order differential equations and then solved by using 'bvp4c'MATLAB solver as follows:

$$f = f_1, f' = f_2,\ f'' = f_3, f''' = f_4\ , \phi = f_5, \phi' = f_6 \tag{5.3.1}$$

From (5.3.1), we can write

$$f_1' = f_2, f_2' = f_3, f_3' = f_4, f_5' = f_6 \tag{5.3.2}$$

Making use of (5.3.1) and (5.3.2), the equations (5.2.8) and (5.2.9) can be written as:

$$f_4' = \frac{3}{f_1}\left[2f_2f_4 - (f_3)^2 - \left(\frac{1}{k_1}\right)\{f_4 + f_1f_3 - 2(f_2)^2\}\right] \tag{5.3.3}$$

$$f_6' = -S_c(f_1f_6 - \lambda f_2f_5 - \beta f_5) \tag{5.3.4}$$

and the applicable boundary conditions (6.2.10) and (6.2.11) reduces as follows:

$$f_1(0) = 0, f_2(0) = 1, (0)\ and\ \ f_2(\infty) = 0, f_3(\infty) = 0 \tag{5.3.5}$$

$$f_5(0) = 1\ \ and\ \ f_5(\infty) = 0 \tag{5.3.6}$$

5.4 Results and Discussion

To find the impact of flow feature factors viz., the viscoelastic parameter k_1, the Schmidt number S_c, the reaction rate parameter β and the parameter λ, computation is carried out numerically using 'bvp4c' solver. To understand the behaviour of diffusion owing to chemical reaction on the viscoelastic fluid through boundary layer past a stretchable exponential sheet, the computed results are plotted in Fig. 5.1–5.8.

To assess the precision of the numerically acquired findings produced by 'bvp4c' and to validate the present work, the skin friction coefficients $f''(0)$ and $f(\infty)$ are computed without taking into account viscoelastic characteristics. The present work yields $f''(0)$= 1.28180838 and $f(\infty)$ =0.90564328, which are well accord with the results found by Magyari and Keller (1999) ($f''(0)$ = 1.281808 and $f(\infty)$= 0.905639) and Banerjee et al. (2018) ($f''(0)$ = 1.281833 and $f(\infty)$= 0.90564328).

At first, our focus is concentrated on the impact of viscoelastic factor k_1 on chemically reactive mass diffusion. The reactive concentration curves $\phi(\eta)$ for variation of k_1 are presented in Fig. 5.1 and Fig. 5.2 for $\lambda = 1$ and $\lambda = -1$. From the figures, it is observed that for both direct and inverse variations of exponential surface concentration, the dimensionless concentration at a location and the thickness of solute boundary layer drop gradually with growing η but slightly enhancing pattern is observed for rising values of k_1.

Secondly, we look into the effect of Schmidt number S_c on chemically reactive mass diffusion. The reactive concentration curves $\phi(\eta)$ for variation of S_c are presented in Fig. 5.3 and Fig. 5.4 for $\lambda = 1$ and $\lambda = -1$. It is noticed that for both direct and inverse variations of exponential surface concentration, the concentration of the fluid reduces rapidly at every point and the boundary layer thickness diminishes as S_c enhances. The growth of Schmidt number diminishes the diffusion coefficient and thus enhances the mass transfer rate. As a result, the thickness of the species boundary layer is reduced.

Next, the changes in concentration curves $\phi(\eta)$ for variation of reaction rate parameter β are discussed. The concentration curves $\phi(\eta)$ for different values of β are exhibited in Fig. 5.5 and Fig. 5.6 for first order chemical reaction for direct and inverse variations of exponential wall concentration distribution. In both situations, the concentration at a point diminishes with the growth of reaction rate. Also, the rising value of β, reduces the thickness of species boundary layer.

The impact of the parameter λ which is related to wall concentration distribution is very significant in controlling the reactive mass diffusion. In Fig. 5.7 and Fig. 5.8, the reactive species profiles $\phi(\eta)$ are depicted for several values of λ. Out of which, in Fig. 5.7 all values of λ are non-negative, whereas in Fig. 5.8 all are non-positive. From Fig. 5.7, it is noticed that the concentration slightly enhancing as λ enhances and the thickness species boundary layer slightly increased with λ. But the concentration of the fluid diminishing with increasing η. On the other hand, for negative variation of λ, the concentration at fixed point diminishes for higher negative values of λ. Hence, it can be understood that below a certain value of $\lambda < 0$, absorption of mass occurs at the stretchable sheet and it increases with the growing magnitude of $\lambda < 0$. As a consequence, the thickness of species boundary layer enhances with the rising magnitude of λ <0.

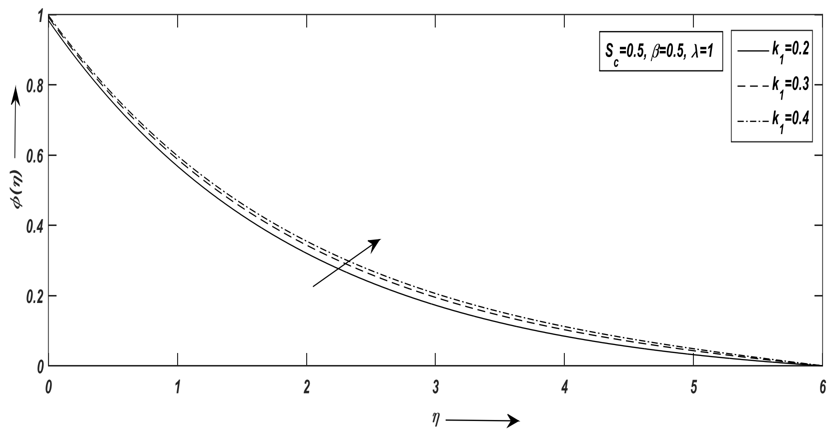

Fig. 5.1 Concentration curves $\phi(\eta)$ for variation of $k_1$with $\lambda > 0$

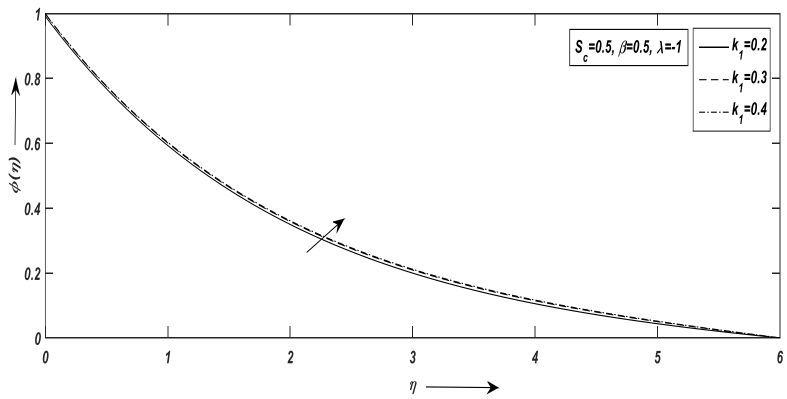

Fig. 5.2 Concentration curves $\phi(\eta)$ for variation of $k_1$with $\lambda < 0$

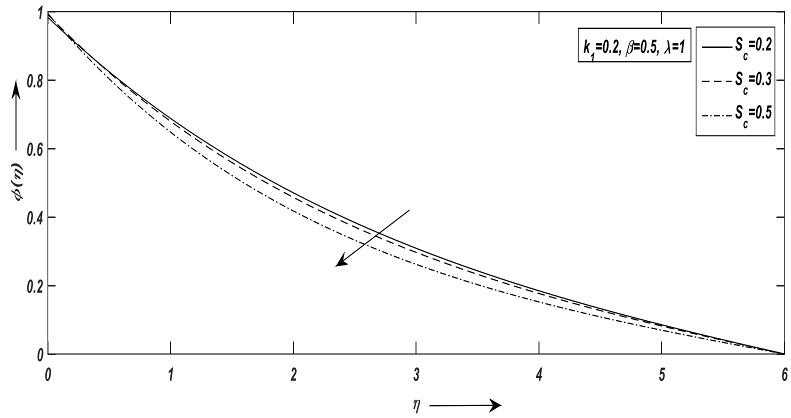

Fig. 5.3 Concentration curves $\phi(\eta)$ for variation of S_c with $\lambda > 0$

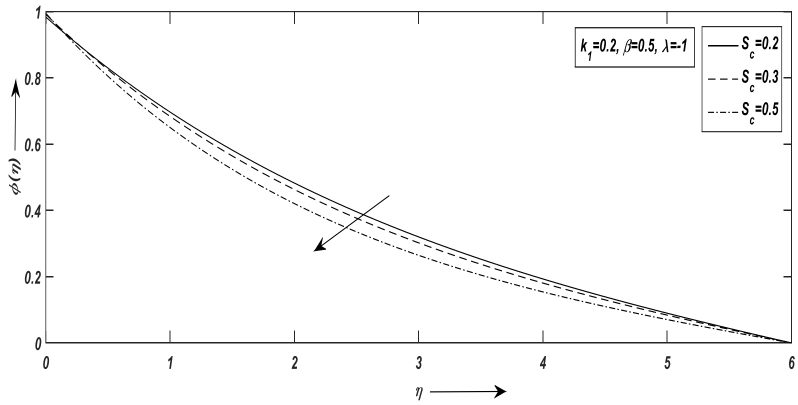

Fig. 5.4 Concentration curves $\phi(\eta)$ for variation of S_c with $\lambda < 0$

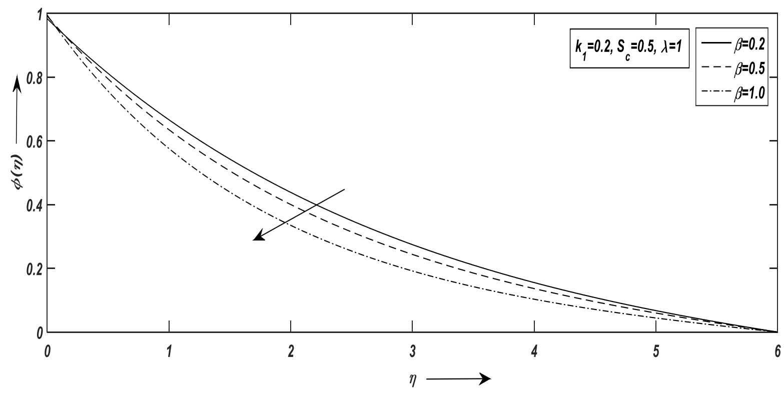

Fig. 5.5 Concentration curves $\phi(\eta)$ for variation of β with $\lambda > 0$

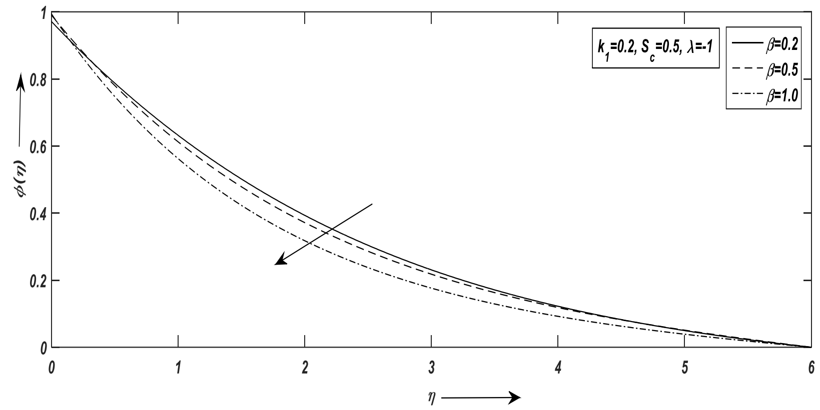

Fig. 5.6 Concentration curves $\phi(\eta)$ for variation of β with $\lambda < 0$

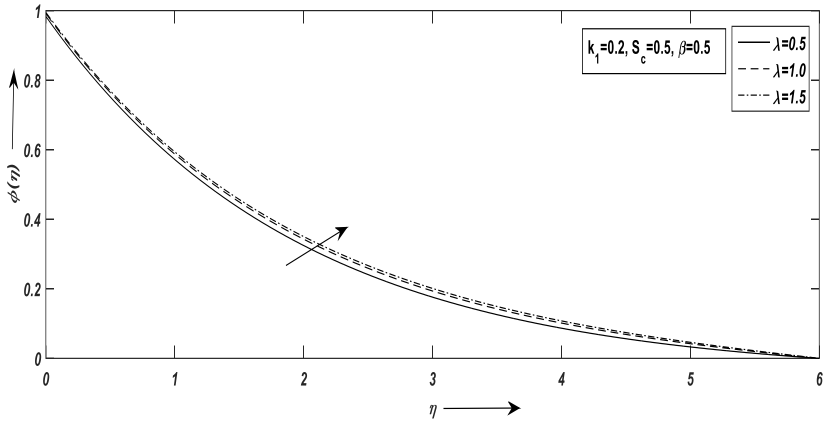

Fig. 5.7 Concentration curves $\phi(\eta)$ for variation of $\lambda > 0$

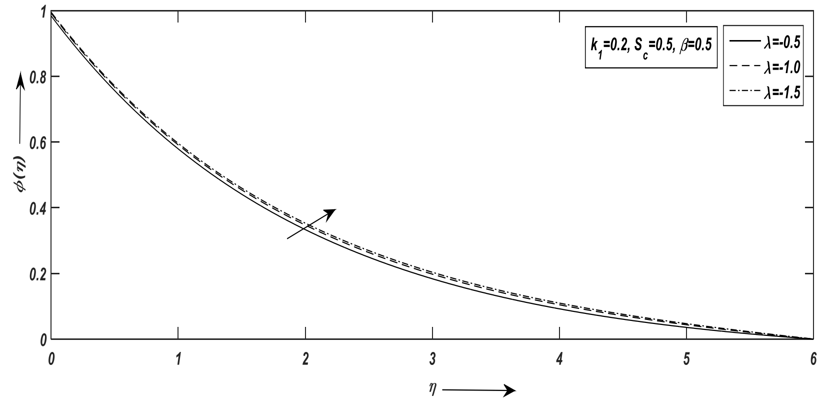

Fig. 5.8 Concentration curves $\phi(\eta)$ for variation of $\lambda < 0$

5.5 Conclusion

Mass diffusion with chemical reaction in viscoelastic fluid flow through boundary layer induced by a stretchable exponential sheet is investigated. The chemical reaction rate and distribution of wall concentration for the species are taken as variables. Utilizing similarity variables, the fluid guided equations are reduced into nonlinear self-similar forms and then evaluated by employing MATLAB solver 'bvp4c'. The present investigation brings out the fact that growing visco-elastic parameter enhances the fluid concentration. Also, as the Schmidt number rises, the fluid concentration and the thickness of species boundary layer significantly reduce and the reaction rate factor has a comparable effect to the Schmidt number. The variable wall concentration, characterized by the parameter λ, controls the mass transfer. Most importantly, below a certain value of λ, mass absorption occurs at the sheet. This model study is intended to serve as inspiration for future experimental studies. Fluid simulation can assist in the visualization of a flow problem.