7

Conclusion and Future Scope of Research

7.1 Introduction

The study of different aspects of groups and rings in classical set theory as well as fuzzy set theory has been a topic of high research interest. The research in these areas is constantly developing due to their wide application in the field of science and engineering. Many researchers have given their tremendous contributions in developing theories on near-rings using operations and laws. The work in the thesis is the extension of the distributive and multiplication modules. Also, intuitionistic fuzzy distributive and multiplication N-groups and DN-groups have been considered as main focus areas of

the research.

7.2 Summary and original Contribution of the thesis

The original contributions of the thesis may be summarized as:

7.2.1 Relation between DN-group and uniserial N-group

Near-ring theory is a domain of algebra with many applications. Distributive N-groups have a wide range to study. By these works, this structure will become familiar in near-ring theory. This study describes uniserial N-groups with examples. Several properties of DN-groups are elaborately described using the local condition of near-ring. Different results of the sub factor of an N-group are studied. Defining the left annihilator of an N-groups, many results are established. e.g. " If E is a DN-group with $Nh \cap Ns = 0$, for any non zero $h, s \in E$, then the direct sum of Nh and Ns is cyclic and $l(h+s) = l(h) \cap l(s)$". The relationship between the DN-group and uniserial N-group is derived using the conditions PNG of the DN-group and local near-ring.

7.2.2 Relation between Multiplication N-group and DN-Group

The notions of a multiplication closed subset of near-ring, localized near-ring and localized ideal near-ring are initiated and some of their properties are discussed. Also, localized N-group, localized N-subgroup and localization of N-group at a prime ideal are defined. Various important results, like " For an N-group $E, E = 0$ if and only if $E_P = 0, \forall P \in Max(N)$." are derived. Results between localized DN-group and DNgroup are also studied. The finitely generated condition of localized N-group and cyclic condition of localized N-group are also established.

Arithmetical near-ring, multiplication N-subgroup and multiplication N-groups are initiated with examples. Numerous results on multiplication N-groups, like "If $K \lhd N$ such that $K \subseteq J(N)$ and E is a multiplication N-group, then $KE = 0$ implies $E = 0$." are here observed. Defining annihilator of localized N-groups and multiplica-

tion condition of localized N-group, various results, including "Every cyclic localized N-group is a localized multiplication N-group" are developed. The relationship between localized multiplication N-group and multiplication is established by showing that localized multiplication N-group is also multiplication N-group. Coining the term convey near ring, numerous important results on multiplication N-group related to DN-groups and Bezout N-group, like " If E is an ideal DN-group that generated finitely over convey near ring with inverse property and every ideal DN-group over a strongly regular near-ring is Bezout, then E is a multiplication N-group," are also investigated.

7.2.3 Relation between Intuitionistic Fuzzy DN-group and Weak DN-group

The work summarizes the concepts of intuitionistic fuzzy DN-groups and intuitionistic fuzzy weak DN-groups. By using these concepts, the algebraic properties of intuitionistic fuzzy DN-group and intuitionistic fuzzy weak DN-group structures are studied. This work also focused on the concepts of intuitionistic fuzzy uniserial N-groups, annihilators of N-groups and weak DN-groups and their associations. The outcome "An IF uniserial N-group is an IF DN-group" is an important result of the work. The relationship between (γ,λ)-cut of DN-group and IF weak DN-group is also established by proving that " If E is an IF weak DN-group, then $(\gamma,\lambda)E$ is also a weak DN-group with $\gamma,\lambda \in (0,1], \gamma+\lambda \leq 1$".

7.2.4 Relation between Intuitionistic Fuzzy Multiplication N-group and DN Group

Finally, introducing intuitionistic fuzzy points and characteristic functions of a non-empty subset of a set, some properties associated with them are discussed. Here also Intuitionistic fuzzy multiplication N-group is defined and some results like " E be an IF multiplication N-group if and only if for every $A \leq_{IFN} E, A = (A :_{\chi} E)_{\chi}E$." are derived. It is also established that intuitionistic fuzzy multiplication N-groups has relations with

intuitionistic fuzzy DN-groups.

Although these works are restricted from studying their direct application and societal benefit, there is a great scope that other science communities may use these structures for their different works.

7.3 Future Scope of the Work

There are many scopes to extend the work described in this thesis. Some of them are summarized below:-

1. The uniserial and Bezout N-groups and their relationship can be extended to the concept of fuzzy set theory.

2. There is a potential to broaden the developed relations of the research work in fuzzy soft set theory and intuitionistic fuzzy soft set theory.

3. The study of uniserial and Bezout structures of regular and pure fuzzy N-groups will be a new dimension of further scope of the research.

4. The research work done here may be extended to semi-distributive near-ring groups.

5. There is scope for the introduction of fuzzy semi-distributive near-ring groups and intuitionistic fuzzy semi-distributive near-ring groups to extend the research work.