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First page of paper 1

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UNISERIAL AND BEZOUT CHARACTER OF DISTRIBUTIVE N-GROUPS

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Abstract : Distributive modules have been studied by several researchers like V. Erdogdu (1987), Victor Camillo (1975), A. V. Mikhalev (1999), Askar Tuganbaev (1990). Of these A. V. Mikhalev and Askar Tuganbaev have studied the Uniserial and Bezout Modules over a ring. K C Choudhury and Helen K Saikia have studied the concepts of distributive near-ring groups in (Chowdhury and Saikia, 1995). The idea of Uniserial and Bezout conditions of modules has been expanded to near-ring groups. Characterization of the Distributive near ring group relies greatly on the Uniserial and Bezout features of near ring groups.

Keywords : Near rings, N-groups, DN-groups, Uniserial N-groups, Bezout N-groups.
Mathematics Subject Classification : 16Y30

1. Introduction. In this study, left N-group E is regarded as unitary and N as a right near ring with zero symmetric. This paper's foundational ideas are all referenced in (Pilz, 1983). Here, we defined the fundamental definition and outcomes required for this paper. The symbols $\leq_N, \trianglelefteq_N$ and $\lhd$ are used to mean N-subgroup, normal N-group and ideal respectively. The letters "nr" is frequently used to shorten the words "near ring". Most of the definitions are structured from (Pilz, 1983).

Methods :

Source of Data

Primary source of Data:-Online Questionnaires

Secondary source of data:-Books, Journals, Previous researches, Internet

The research will be done through observing the Journals, Books, online Questionnaires, Previous Research works on near ring, N -group and Distributive modules.

First page of paper 2

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INTUITIONISTIC FUZZY ASPECTS OF MULTIPLICATION N-GROUPS

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Abstract: Intuitionistic fuzzy (IF) sets were first put forward by K. Atanassov as a generalised notation for fuzzy sets in 1983. The concepts of intuitionistic fuzzy near rings, intuitionistic fuzzy N-groups, intuitionistic fuzzy N-subgroups, intuitionistic fuzzy ideals of N-group are described by P. Saikia, L. K. Barthakur and H. K. Saikia in [10, 11]. Fuzzy distributive modules are studied by Sh. B. Semeein and I. M. A. Hadi in [4]. Intuitionistic fuzzy multiplication modules are studied by P. K. Sharma in [13]. We extend the notion of intuitionistic fuzzy multiplication modules to intuitionistic fuzzy multiplication N-groups. Here, we define intuitionistic fuzzy multiplication N-group and some basic definitions that are needed in this sequel. The relations ship of multiplication N-groups, intuitionistic fuzzy multiplication N-groups, DN-groups and intuitionistic fuzzy DN-groups are also studied.

Keywords and Phrases: Near rings, N- groups, DN-groups, multiplication DN-

First page of paper 3

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Original Article

Distributive Character of Multiplication N-groups

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Abstract *- In this paper, Uniserial N-groups and Bezout N-groups are defined. Defining localized near ring, localized N-groups, localized N-subgroups and localized ideals of N-groups the related results are discussed. It is observed that the various characteristics of DN-groups, multiplication N-groups, Uniserial N-groups and Bezout N-groups are also investigated. It is also seen that in arithmetical near rings, uniserial N-groups and DN-groups lead to multiplication N-groups.*

Keywords *- Near rings, Localized N-groups, Multiplication N-groups, Distributive N-groups.*

AMS Subject Classification Codes*: 16Y30.*

1. Introduction

In this study, the concepts of multiplication N-groups and cyclic N-groups in the near rings are defined by Elaheh Khodadaapour and Tahereh Roodbarilor. They discussed the relationship between multiplication N-groups and cyclic N-groups. The left N-group E is regarded as unitary and N as a commutative near ring with zero symmetric. This paper's foundational ideas are all referenced in [6, 9]. Here, we defined the fundamental definition and outcomes required for this paper. The symbols $\leq_N, \trianglelefteq_N$ and $\lhd$ are used to mean N-subgroup, normal N-group and ideal respectively. Max(N) represents the collection of all maximal ideals of N. Most of the definitions have been extracted from [9].

Definition 1.1 If the following standards are satisfied, a nonempty set N combined with the binary operations " + " and "." is referred to as right near ring.
i. $(N, +)$ is a group(not necessarily abelian).
ii. $(N, .)$ is a semi group.
iii. $(p + b)c = pc + bc), \forall p, b, c \in N$.

Definition 1.2 An additive group $(E, +)$ is referred to be a left N-group, if $\exists$ a map $N \times E \to E, (n, u) \to nu$ in which the following standards are satisfied-
i. $(m + n)u = mu + nu$.
ii. $(mn)u = m(nu)$.
It is to be noted that N is itself an N-group over itself. If for $1 \in N$ such that $1.u = u \ \forall \ u \in E$, then E is called an unitary N-group.

In the event that A is a subgroup of $(E, +)$ and $NA \subseteq A$ for any $A \subseteq E$, then E is referred to as an N-subgroup.
If F is a normal subgroup of $(E, +)$ with $na \in F, \ \forall \ n \in N, a \in F$, then F is referred to be a normal N-subgroup of E.
If D is a normal subgroup of $(E, +)$ such that $n(a + e) - ne \in D, \ \forall \ n \in N, a \in D, e \in E$, then D is referred to as an ideal of E.
Let $A \lhd E$. Then the set $\frac{E}{A} = \{a + A: a \in E\}$ forms an N-group under the operations $(k + A) + (s + A) = (k + s) + A$ and $m(s + A) = ms + A, \forall s, k \in E, m \in N$, called quotient N-group.
E is called cyclic if $E = nl$ for some $n \in N, \ l \in E$.
When $x \in E$, Nx is referred to as the principal N-subgroup of E.
E is known as the principal N-group (PNG) if each $A \leq_N E$ is principal.