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Conclusion and Scope for Future Work

8.1 Introduction

The purpose of this thesis is to investigate different aspects of mathematical modelling in order to properly portray eco-epidemic scenarios. The models under consideration focus specifically on epidemics that impact plant populations. There are three distinct compartments that are used to classify plant density: susceptible, infective, and recovered. The application of pesticides is considered as a control measure. The eco-epidemic models being studied are highly nonlinear and intricate. The fundamental purpose of this work is to construct mathematical models and then analyse those models in order to validate their behaviour. The proposed models yield several significant conclusions.

8.2 Summary and original Contribution of the thesis

The original contributions of the thesis may be summarized as:

8.2.1 Developing a basic eco-epidemic model of spread of diseases between susceptible and infected plants, utilising pesticides as a control measure

The study has provided significant insights into the functioning of eco-epidemiological systems and disease management in agricultural ecosystems. The analysis and development of the mathematical model yielded crucial findings on plant population treatments, disease patterns, and pesticide application. The examination and description of equilibrium points have enhanced the development of efficient disease control measures by clarifying the system's stability and behaviour. This model holds ecological importance due to its boundedness rate and positivity, mirroring real-world results. The integration of mathematical models with ecological and epidemiological data in this study highlighted the importance of employing diverse approaches to address complex environmental challenges.

8.2.2 Stability and control of a plant epidemic model with pesticide intervention

Through the development of a new plant model that takes into account the application of pesticides, this research has shed light on efficient strategies for the management of agricultural diseases. The detailed experiments, which included assessments of pesticides and diseases, enabled the mathematical analysis to uncover critical parameters that influence the efficacy of pesticides and the dynamics of their application. According to the model, the fundamental reproduction rate, denoted by R_0, is an essential component in deciding whether or not plant populations will be able to prevent the spread of disease or whether they will become extinct.

8.2.3 Analysis of Stability, Sensitivity Index and Hopf Bifurcation of Eco-Epidemiological SIR Model under Pesticide Application

In this work, a compartmental plant-pesticide model represented by a system of ordinary differential equations (ODEs) was proposed and analysed. The plant populations were divided into three compartments: susceptible, infected, and recovered populations. Pesticides were applied to all plants as a control measure to reduce disease transmission from infected to susceptible plants, impacting both susceptible and infected populations. The necessary mathematical analysis for the biological validity of the proposed model was first presented. The boundedness theorem indicates that each plant population is bounded above as $t \to \infty$. The total plant population $N(t)$ is also bounded above as $t \to \infty$, meaning the system will not collapse due to population explosion. The uniqueness and existence of solutions are crucial aspects of mathematical modelling, and in this model, unique solutions exist. Without unique solutions, there could be two different equilibria, such as two different disease endemic equilibria, leading to different equilibrium states from different initial populations. The study also determined a domain in which the system's solutions exist. The proposed system has three feasible equilibrium points. The first is the trivial equilibrium point T_0 , which always exists and is stable if $r < \mu$. If $r > \mu$, T_0 becomes unstable, resulting in the appearance of the disease-free equilibrium (DFE) point T_1 and the endemic equilibrium point T_2. The basic reproduction number R_0 of the infection was determined using the next-generation matrix method. Sensitivity analysis was conducted to understand the relationship between R_0 and the associated parameters. Finally, a biologically plausible set of parameters was employed to conduct numerical simulations, with the goal of comparing the analytical findings. Furthermore, numerical simulations were used to produce Hopf bifurcation curves across different parameter spaces.

8.2.4 Optimizing Plant Epidemic Control: A Mathematical Model Integrating Susceptible and Infectives Plants, and Herbivores with Pesticide Intervention

The present study utilised advanced mathematical modelling, computer analysis, and optimisation tools to investigate the dynamics of infectious diseases in ecological systems. Through a thorough analysis of stability characteristics, equilibrium points, and sensitivity to parameter modifications, crucial thresholds determining the persistence and elimination of diseases were discovered. Additionally, the main factors driving transmission and prevalence were identified. By employing Pontryagin's Maximum Principle, optimal control methods were developed to minimise the burden of disease while optimising control expenditures. These strategies offer practical insights into the effectiveness of various intervention measures. The integration of multiple disciplines enhanced the understanding of patterns and changes in disease occurrence, providing essential direction for public health policy based on solid data and targeted intervention tactics. These findings are significant resources in the ongoing fight against infectious diseases, equipping policymakers and public health professionals with the necessary tools and knowledge to protect human health and ecological integrity. The study used rigorous mathematical frameworks and computational approaches to analyse the dynamics of infectious disease transmission within ecological systems and to develop optimal control mechanisms for managing disease spread. The study aimed to understand the complex relationships between key factors influencing disease spread and changes in population size, as well as develop effective methods for reducing disease prevalence.

8.2.5 A Mathematical Analysis of Plant-Pesticide Interaction: Existence, Uniqueness, and Optimal Control

This paper presents a mathematical analysis of plant-pesticide interactions, focusing on developing a model that examines plant population dynamics under pesticide application. It establishes conditions for the existence and uniqueness of solutions to the model of Chapter 4, ensuring its accuracy in predicting the spread of plant diseases. The study also introduces optimal control strategies to minimize pesticide use while effectively controlling plant epidemics, aiming to balance agricultural productivity with environmental and economic considerations. These findings offer valuable guidance for sustainable and efficient pest management in agriculture.

8.3 Future Scope of Work

Mathematical modelling is a rapidly expanding field of research in science and engineering, with particular significance in the biological sciences. Eco-epidemic modeling is a prominent topic in biomathematical research. The work on eco-epidemic modelling can extend into numerous other areas, including the following:

1. The effectiveness of pesticide treatments is critical in agricultural contexts to maintain crop health and production. On the other hand, the dynamics of pesticide-plant interactions are frequently complex and multifaceted. When pesticides are used to treat plant diseases, it is commonly expected that the affected plants will recover immediately. However, this may not always be the case. The results of the pesticide treatment may not always become evident immediately because there is occasionally a delay in the recovery process. This delay could be caused by a number of factors, including the time it takes for the pesticide to take effect, the resistance of the pathogens, or the plants' ability to recover from infection-related damage.
2. To better understand and model this delayed recovery phenomenon, our research proposes extending the existing model to include time delays via delay differential equations (DDEs). DDEs are mathematical tools that allow us to model systems in which the evolution of a variable is determined not only by its current state but also by its previous state. By incorporating time delays into our model, we can better capture the dynamics of plant-pesticide interactions over time, resulting in a more realistic portrayal of the system's behaviour.
3. In addition to considering time delays, one can explore the use of fractional-order derivatives in eco-epidemiological modelling. Traditional differential equations describe how quantities change with respect to time using integer-order derivatives (e.g., first, second, and third derivatives). However, fractional order derivatives take this idea a step further by supporting orders that are not integers. This lets us understand more complex dynamics and long-range dependencies in ecological and epidemiological systems. By incorporating fractional-order derivatives into our model, one can gain a deeper understanding of the underlying mechanisms driving plant-pesticide interactions. Fractional calculus provides a powerful framework for modelling systems with memory effects and

non-local interactions, which are often observed in ecological systems. This extension will allow us to explore new aspects of the system's behaviour and potentially uncover hidden patterns or dynamics that may not be apparent with traditional integer-order models.

4. Furthermore, the thesis suggests that the effectiveness of pesticide treatments can be influenced by the quantity of pesticides applied, which in turn affects the contact rates between plants and pesticides. These contact rates play a crucial role in determining the spread of infections and the efficacy of pest control measures. To model the relationship between pesticide quantity and contact rates, the parameter α is used (5.1). By adjusting the value of α, researchers can simulate different scenarios and investigate how variations in pesticide application impact the dynamics of plant-pest interactions.

In summary, our research adds to existing models of how plants and pesticides interact by using delay differential equations to include time delays, investigating the use of fractional order derivatives in eco-epidemiological modelling, and looking into how pesticide quantity and contact rates are related. By considering these factors, researchers will be able to provide a more comprehensive understanding of the dynamics of plant-pesticide interactions and develop more effective strategies for pest management in agricultural systems.