

Mathematical Modelling and Integrated Management of Eco-Epidemiological System under the Application of Pesticides

A THESIS SUBMITTED IN PARTIAL FULFILLMENT

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

MATHEMATICS

To



THE ASSAM

ROYAL GLOBAL UNIVERSITY

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October 2024

Dedicated To My Family,
My Teachers & My Friends

DECLARATION

I hereby declare that the content embodied in the PhD thesis entitled "**Mathematical Modelling and Integrated Management of Eco-Epidemiological System under the Application of Pesticides**" is the result of research work carried out by me in the Department of Mathematics, **The Assam Royal Global University, Guwahati, India**, under the supervision of Prof. (Dr.) Anuradha Devi and co-supervision Dr. Aditya Ghosh.

In keeping with the general practice of reporting research observations, due acknowledgments have been made wherever the work described is based on the findings of other researchers.

Further, I declare that this thesis as a whole or any part thereof has not been submitted to any university (or institute) for the award of any degree/ diploma.

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Abbreviations

DE	Differential Equation
ODE	Ordinary Differential Equations
PDE	Partial Differential Equations
DDE	Delay Differential Equations
SIR	Susceptible-Infectious-Recovered
LTI	Linear time-invariant
IFF	if and only if
PMP	Pontryagin's Maximum Principle
MATLAB	Matrix Laboratory
R_0	Basic reproductive number
DFE	Disease-free equilibrium
EE	Endemic equilibrium
GH	Generalised-Hopf
BT	Bogdanov-Takens
VOCs	Volatile organic compounds
IPM	Integrated Pest Management
FAO	Food and Agriculture Organization

List of Symbols

$=$	Equal to
\neq	Not equal to
$<$	Less than
$>$	Greater than
\leq	Less than or equal to
\geq	Greater than or equal to
\approx	Approximately equal to
\Rightarrow	Implies
\rightarrow	Convergence
$/$	Division
\det	Determinant
tr	Trace
π	Pi
\in	belongs to
\exists	there exists
$\frac{d}{dx}$	Derivative
\int	Integral
$\frac{\partial}{\partial x}$	Partial derivative
α	Alpha

β	Beta
γ	Gamma
δ	delta
Δ	Delta
ε	Epsilon
θ	Theta
ω	omega
φ	Phi
η	Eta
λ	Lambda
μ	Mu
ν	Nu
ξ	Xi
ρ	Rho
τ	Tau
χ	Chi
ψ	psi
Ψ	Psi
ζ	Zeta
σ	Sigma
Ω	Omega
\propto	is proportional to

\Leftrightarrow	if and only if
∞	infinity
\therefore	Therefore
\because	because / since
e	Exponential
$\ \ $	norm of
\max	Maximum
\min	Minimum
\lim	Limit
\inf	Infimum
\sup	Supremum

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