# 1

# General Introduction

## 1.1 Overview of the study

Mathematical biology is a rapidly growing, well-defined field of research and the most stimulating application of mathematics in modern times [49]. Biology is a science that aids human health in various ways, and the competence of biomathematics is foreseeable. It helps to surge food production, battle against diseases, and safeguard and protect the environment. Many people use mathematical modelling to positively describe the biological system. It also delivers guidelines for policymakers, physicians, and scientists to produce informed decisions. There are various problems in the biological system, particularly epidemics and ecosystems, that still require recognition, and mathematical modelling will be beneficial in resolving those concerns.

Populations at every level are constantly in danger of disease. From an eco-epidemiological and mathematical perspective, the disease's effects and the attack rate of one species on another play a significant role in the ecological system. Investigators are showing more attention to combining these two research areas. Eco-epidemiology is an advanced branch of mathematical biology that deals with both epidemiological and ecological issues at the same time. The efficacious invasion of a parasite into a host population and the resultant host-parasite dynamics can most probably rely on additional members of the host community—predators.

The term "mathematical model" refers to a theoretical and intellectual model that uses mathematical language to describe the behaviour of the system. Typically, mathematical modelling translates the real-world phenomena into mathematical problems. Subsequently, the mathematical model solves these framed problems and deduces their solutions in a language familiar to the real world. There are numerous opportunities to improve the model using real-world parameters, which exist for investigative purposes. As a result, the present study intends to understand plant diseases and infections in the ecosystem using mathematical models. This chapter provides an elaborate introduction to the study.

### 1.1.1 Background of the study

For the past few decades, mathematical modelling has become an integral part of ecological investigation. Mathematical models make one's evaluations and forecasts in ecology more reliable and objective. There are several definitions for mathematical models. Among those definitions, the most commonly accepted one is as follows: The mathematical model of a real object is a totality of the formalised dependences, logical connections, and formulas that facilitate the learning of a real object without its experimental analysis. The ecological research objects are:

- Communities
- Populations
- Ecosystems

Conducting experiments on those objects is unlikely, as it could lead to alterations or even the destruction of ecological entities. In this context, it is obvious that mathematical modelling plays a major role in ecological studies [63]. Hence, the eco-epidemiological model is significant in both theoretical ecology and applied mathematics. Infectious diseases play an important role in eco-epidemiological models. Consequently, various mathematical models have been established. Several existing works develop prey-predator models of disease in prey. Moreover, in recent years, eco-epidemiological systems involving disease in predators have emerged as the most fascinating area of investigation among all existing mathematical models [80].

The dynamic relationship between prey and predator is one of the central themes in mathematical ecology because of its integrity. Experts have extensively examined the traditional predator-prey models over the past few years. Researchers extensively employ mathematical models to define the interaction between pests and plants and to evaluate the effectiveness of control measures. Furthermore, it facilitates the investigator's ability to mediate the dynamic interaction among populations in the model. Researchers have found several pest control models useful for comprehending the mechanism of intervention. As previously mentioned, researchers typically aim to measure the optimal control measure in relation to a specific performance index. The researcher creates an optimal control mode for the plant-pest interaction in this present work, along with the control measures.

A mathematical model is one of the most important tools for understanding the mechanisms of persistence and species extinction. The main motive of the epidemic model is to identify the mechanism that results in the infection's extinction. Eco-epidemic models provide the ecosystem's description of interacting species among which a disease propagates. Infectious disease does not affect the host population yet; there are also various species that collaborate with the infected host population. The mathematical modelling of eco-epidemiology involves the population growth and size, the relationship between predator and prey, the spread rules of infectious diseases, and the impact of infections on species mortality.

### 1.1.2 Mathematical modelling

Mathematical modelling aids in modelling real-life problems into mathematical models, and it resolves them consistently. The mathematical modelling technique has gained attention in recent years because it provides a better understanding of disease propagation, from which few decisions can be made to formulate suitable disease control measures [93]. Even though various models define the dynamics of human diseases and vectors, there are only a few models that define the dynamics of vector-borne plant diseases [88].

In the current decade, the investigation of biological problems has heavily relied on mathematical modelling [10]. For the past few years, people have extensively used mathematical modelling to understand practical issues such as:

- Magnetic flows
- Image processing
- Neural networks
- Signal processing
- Optical soliton theory
- Epidemic models
- Edge detecting approaches
- Probability-based analysis
- Discrete dynamics in nature
- Attraction repulsion problems
- Machine learning
- Medical image alignment
- Applied competing
- Biomedical signal Processing

Because of its varied significance, a huge number of effective numerical techniques have been improved for resolving those issues [65]. Mathematical modelling is an experimental technique that resolves issues and continuously refines them over time to ensure precision, effectiveness, and robustness. Described as the procedure of scientific inquiry, it forms an essential part of the applied mathematician's toolkit.

Theoretically and empirically, mathematical models, based on statistical or deterministic principles, have persistently addressed a wide range of issues in technology and natural science. The mathematical models that intend to define social phenomena must take into account various entities. The two kinds of factors involved in those models are subjective factors and objective factors. Additionally, we cannot always progress the study of remarkably social phenomena using mathematical models based on randomness or certainty, as the information present in these phenomena often reflects uncertainty and subjectivity.

Currently, several algorithms and mathematical models have received attention, laying the groundwork for what is being termed non-numerical and numerical mathematics in uncertainty [208]. A principled activity that has both principles beneath it and approaches that can be successfully implemented is called mathematical modelling. These principles are all-encompassing, or meta-principles, quoted as questions regarding the purposes and intentions of mathematical modelling. It was also revealed that these principles are most likely philosophical in nature.

Figure 1.1 displays the visual portrayal of mathematical modelling with respect to its basic philosophical technique [32]. The list of queries and instructions is not an algorithm for constructing an excellent mathematical model. On the other hand, the fundamental concepts serve as the foundation for both mathematical modelling and problem formulation. Therefore, it is crucial to believe that individual questions will frequently resurface during the modelling procedure, and to view the list as a legitimate tool for philosophical approaches to mathematical modelling.

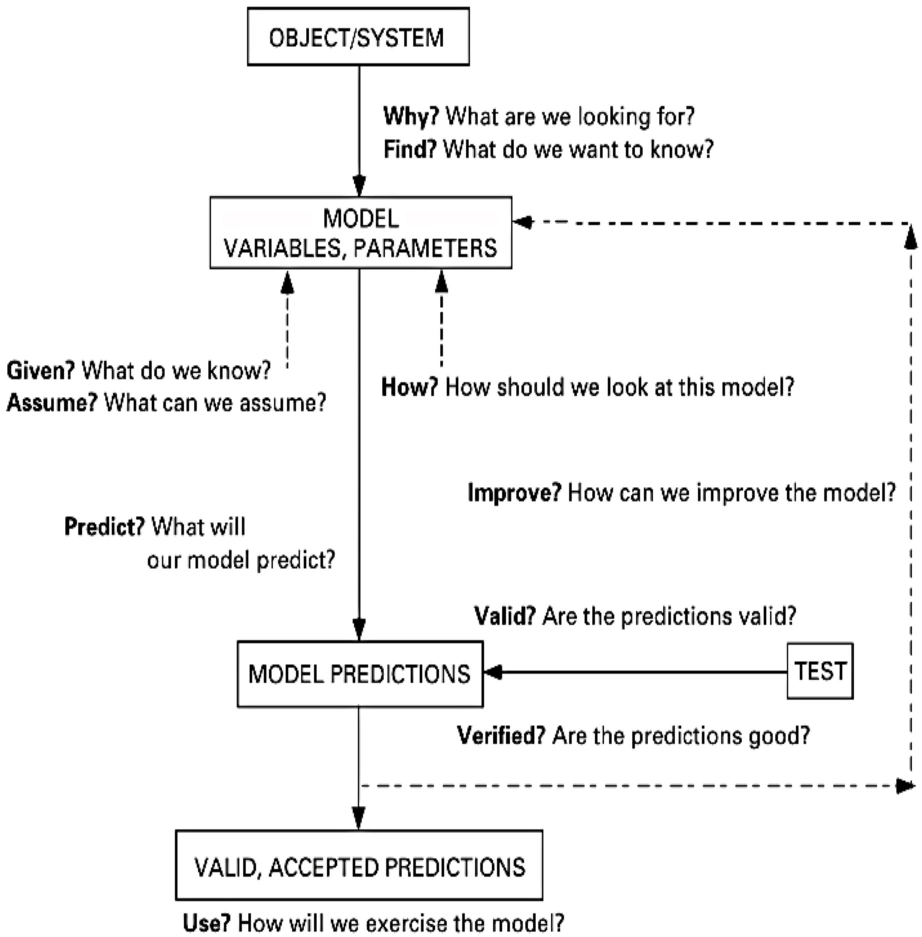


**Figure 1.1. Interpretation of Mathematical modelling**

**(Source: [32])**

Mathematical models have turned out to be the major tools for evaluating the spread of infectious diseases and their control. McKendrick and Kermack established the Susceptible-Infected-Recovered model, which is the ancestor of several infectious disease transmission models. Infection occurs when inclined individuals come into contact with infectious ones. However, these species do not exist in isolation within the ecosphere. The species under consideration not only spreads diseases, but also engages in competition with other species for food or space, or becomes a target for predators. While epidemiology and ecology are undoubtedly the primary areas of study, it is crucial not to ignore diseases within ecological species. The models that address diseases in ecological systems are referred to as eco-epidemiological systems [35].

The investigators were gratified to identify various techniques to control infectious diseases, like culture, epidemiology, pathology, and various other phases. In order to understand the growth of the variables, mathematical modelling is considered the most effective method [33]. Researchers use the mathematical model to uncover the evolution of infectious diseases within species. In 1760, Bernoulli introduced the first mathematical model for epidemiology. It is used to determine the magnitude and immunisation status of smallpox.

### 1.1.3. Eco-Epidemiology system

The two fields, such as ecology and epidemiology, were learned distinctively for a few years. To understand the spread of diseases, Kermack and McKendrick introduced the first mathematical modelling [102]. Simultaneously, Volterra and Lotka initiated an investigation into the biological-species interaction. In the late 1980s and early 1990s, the fields of epidemiology and ecology merged, leading to the development of an advanced area of investigation known as eco-epidemiology. Ever since its creation, a huge amount of work has been executed bearing in mind the connection of ecological and epidemiological problems [69, 207]. Even though these two areas are considered independent, they may have various overlapping features.

Epidemiology is the foundation of public health knowledge. The concept of epidemiology is deficient without the inclusion of ecology; accordingly, it forms eco-epidemiology [175]. The term eco-epidemiology refers to the ecosystem of microbial amplification, the ecology and epidemiology of their diffusion, and the use of eco-epidemiology as a determining factor in disease distribution [29]. This concept is being developed to provide a dynamic and comprehensive model understanding infectious diseases.

Eco-epidemiology is one of the advanced branches in mathematical ecology that considers both epidemiological and ecological issues. Since the introduction of ecological research performed by Volterra [187], Lotka [121], and the innovative work of Kermack and Mckendrick [103] in epidemiology, several investigators have been encouraged to follow quite an excellent number of investigations, both in epidemiology and theoretical ecology. Following various investigations, Chattopadhyay and Arino [35] investigated a 3-dimensional eco-epidemiological system and transformed it to a 2-dimensional one based on a few standard assumptions and properties.

***Eco-epidemiological model***

The subsequent assumptions are made in framing the simple eco-epidemiological model [141],

- The prey population develops logistically in the absence of predation and infection.
- Prey populations are segregated into two separate classes, such as infected populations ($I$) and susceptible populations ($S$), in the presence of infection.
- It is considered that only susceptible populations, $S$, have the capability of reproducing logistically. Additionally, the infective population, $I$, expires before having the opportunity to replicate. Conversely, from the perspective of resounding capacity, the infective population, $I$ continues to decline along with the population growth.
- The disease transmission mode tracks the simple law of mass action. Only the prey population can propagate the disease, as it does not inherit it innately. The infected population does not maintain or develop immunity.
- It is regarded that predators cannot separate between healthy prey and infected prey. They consume infected prey and susceptible prey at the rates of $g(I)$ and $h(S)$, respectively, where $g(I)$ and $h(S)$ are regarded as the functional responses.

## 1.2 Integrated Pest Management

Pest control is important in agriculture because pest outbreaks can cause huge economic and ecological losses. The well-established approach involves drenching a huge amount of chemical pesticides on top of the crops. Even though pesticide pollution has turned out to be the main issue for human beings and valuable insects, it has destructive effects on non-target organisms through the accumulation of hazardous chemicals in the food chain. Furthermore, the overutilisation of chemical pesticides leads to pesticide resistance. As a result, a more robust and secure strategy, integrated pest management, was introduced in the late 1950s and more extensively implemented in the 1970s and 1980s. Experience has revealed that integrated pest management is efficient when compared to traditional techniques like biological control and chemical control [100].

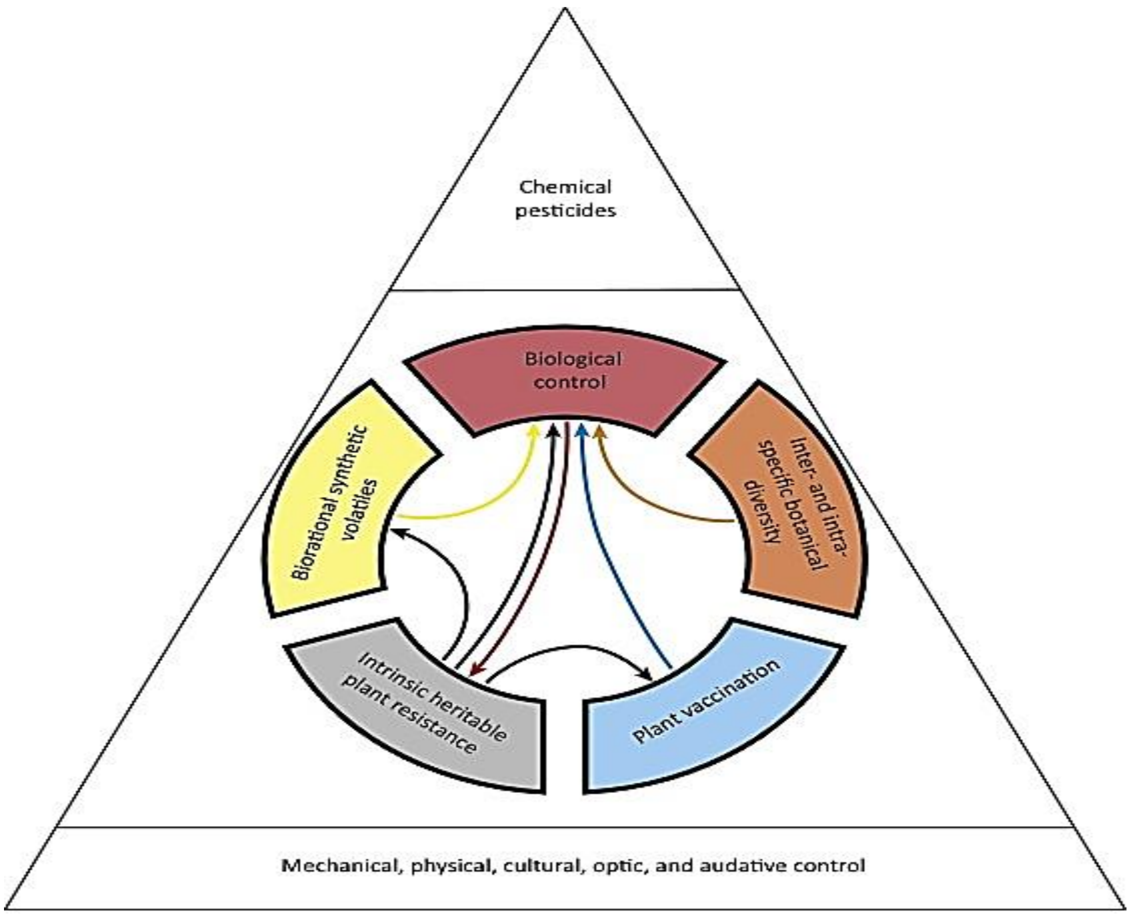


**Figure 1.2. Pyramid of Integrated Pest Management**

**(Source: [173])**

Integrated Pest Management has significantly evolved since the introduction of integrated control. It is now defined as a pest control approach that combines and integrates chemical and biological control methods [172]. Figure 1.2 illustrates the foundational level of the Integrated Pest Management Pyramid, which can be applied either before or immediately after planting and will remain effective until the crops are harvested. Adjacent to it is a circular structure consisting of important components that can be classified as ecological and serve as the basis for Integrated Pest Management based on ecology.

Also, the pyramid's top tier includes chemical pesticides, which must only be smeared whenever needed if several Integrated Pest Management elements fail to keep pest populations below the economic threshold. The arrows represent the specifically significant interactions among the ecological elements. Those seven interactions mentioned in Figure 1.2 denote the investigation areas that require interpretation in order to incorporate the elements and recognise the full competence of integrated pest management.

Integrated pest management is a systematic and long-term tactic that incorporates chemical, biological, and cultural tactics to decrease pest populations below economically detrimental levels. There are various successful instances of integrated pest management applications [85]. The mathematical models of integrated pest management strategies have gained widespread knowledge in recent times.

### 1.3 Problem statement

Plant diseases are caused by pathogens that alter or interfere with important plant functions, resulting in discrepancies in the plant's well-being. Plant disease outbreaks are increasing globally, significantly impacting the environment, food security, and agriculture [174]. Improving our knowledge and organisation of plant diseases is becoming a worldwide issue. Using plant disease epidemiological models can help reduce the need for making tools that are easy for everyone to use to find and control the interference in environments where crop damage will occur.

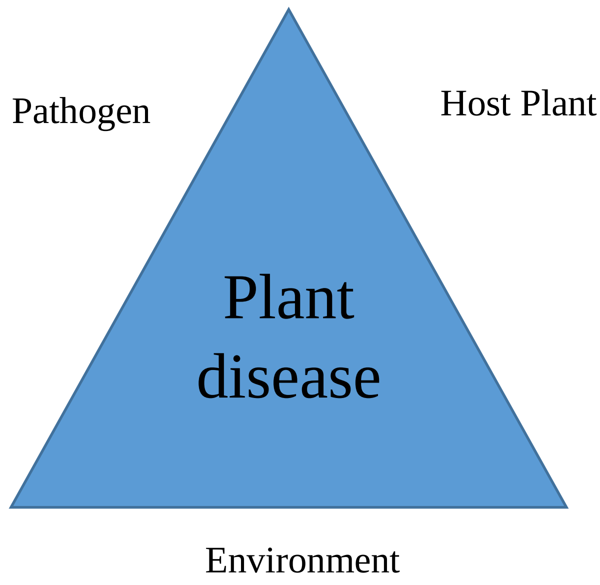


**Figure 1.3 Plant disease triangle**

**(Source:[79])**

Figure 1.3 diagrammatically represents the multifaceted interaction between the disease system constituents: environment, pathogen, and host plant [162], which determines the existence and pervasiveness of plant diseases in time and space. Hence, in order to comprehend the multifaceted and changing aspects of plant diseases, it is significant to embrace a multidisciplinary technique that incorporates ecological, biological, and mathematical expertise. Since pest control is now a global concern because of population growth, suitable approaches are required to manage pest populations, and the agricultural awareness of farmers is correspondingly significant in pest control.

Efficient pest control strategies possess a remarkable influence on society. Therefore, the efficient usage of predators and the suitable usage of pesticides or chemicals are highly endorsed to manage pest populations [90]. Chemical pesticides have been widely used to manage pest populations, despite the fact that their widespread use can have unpleasant side effects on the surrounding area, such as a decrease in the pest's natural enemies and increased pollution.

Furthermore, the improvement of insect resistance to chemicals necessitates the use of stronger and more toxic pesticides to maintain their efficacy. Hence, the widespread usage of pesticides is not an ecological solution for pest control. Continuous efforts are made to decrease pesticide toxicity for consumers and applicators. Additionally, researchers are creating or developing alternative techniques to align with the charter of the Integrated Pest Management program. Mathematical modelling can be very beneficial to gather an excellent understanding of the pest population dynamics, and several control strategies can be learned to enhance the control.

Mathematical models turn out to be the requisite tools for evaluating pest control strategies. On the other hand, in the realm of pest control research, it has been relatively rare to examine how a model that incorporates natural enemies, pest diseases, and pests influences a plant population [148]. Therefore, the current study suggests an eco-epidemic model for the disease-affected plant population. Finally, the study results will provide the optimal control path policy for eradicating plant disease propagation.

### 1.4 Aim and scope of the study

Bacteria, viruses, and fungi recurrently transport several infectious diseases that infect the plant ecological population. These diseases can vary in severity, from minor leaf or fruit damage to death. One of the prominent plant diseases is the algal leaf spot of tea (*Cephaleuros virescens*). Pest damage to crops is the main reason for concern around the world. The United Nations Food and Agriculture Organisation (FAO) reported in 2021 that pests waste around forty percent of the world's agricultural crops annually. Therefore, understanding plant infectious diseases within the ecosystem is crucial, and the study aims to develop an eco-epidemic model to enhance our understanding of plant diseases. Finally, this study provides techniques to manage this prominent plant disease.

### 1.5 Significance of study

The ecological system’s mathematical modelling is gradually turning into a significant research topic because of its significance and universality. Lotka developed a fundamental mathematical model to understand the evolving aspects of predator-prey interactions. Following this, various mathematical models were produced to understand the exact tendencies of several predator-prey systems. It is a well-established fact that several types of vegetables and crops are the main sources of food for people across the globe.

Several billions of common people are employed in the agricultural economy in order to fulfil their basic necessities. Also, a pest population or large insect demolishes the crops by consuming plant fruitlets, leaves, or various fragments. Therefore, there's a high likelihood that pests will infect crops and cause harm. Conversely, there are various creatures in the natural atmosphere that consume pests, such as frogs, birds, and so on. Entomologists primarily established the integrated pest management concept after encountering indiscriminate broad-spectrum insecticide use, insect outbreaks caused by eliminating natural enemies, and the evolution of pesticide resistance. Today, integrated pest management realises its potential in every aspect of plant safety and protection.

In the model, the predator forages for pests without causing damage to the produce. The predators also naturally interact with pests to keep them under their control. While pesticides can be used to control pests in food production, overuse of these chemicals can harm both human populations and crops. Therefore, understanding the pest-predator model is essential for protecting crops.

A multi-faceted ecological procedure involving complicated interactions among natural enemies, plants, pests, various organisms, and the environment is pest control. Several considerations must be encountered prior to control actions, and they must be executed. Researchers in this field typically develop mathematical models to identify and, in certain situations, simplify this challenge, using integrated pest management as a reference framework. These models are beneficial as they help develop an understanding of the intricate interactions, and they facilitate process interventions by combining existing measures to combat pest density.

In spite of the fact that it is less expensive and destroys pests in a short amount of time, the use of chemical pesticides for pest control is associated with a significant amount of environmental degradation. The use of biological control, on the other hand, is a technique that is both extensive and expensive to implement, but it results in less damage to the environment. The use of biological and chemical agents will yield favourable outcomes in terms of environmental loss as well as economic cost of controls. It is critical to optimise the system in order to minimise the impact of any undesirable consequences and ensure that the method is as cost-effective as possible. In order to increase crop production, a significant number of stakeholders in agricultural policy and industrial producers have made substantial use of epidemiological models of plant health.

### 1.6 Conclusion / Summary

This chapter presents an overview of the conducted research. In the next paragraphs, the history of the investigation is discussed in detail. In the following step, the problem statement of the study is elaborated upon. This is followed by an explanation of the objectives and parameters of the investigation. Following that, the aims of the study are discussed in further detail. After that, an explanation of the significance of the research is provided. This chapter comes to a close with a section that provides a summary.