

The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

B.Sc. (H) Mathematics, 2nd Semester

Semester End Examination, June 2023

Course Title: Analytical Geometry

Course Code: MAT012C202

Time: 3 Hours

Maximum Marks: 70

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

SECTION – A

1. Attempt all questions:

2 x 8 = 16

- a) Transform parallel axes to the point (a, b) of equation $\frac{x}{a} + \frac{y}{b} = 2$.
- b) Prove that the equation $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ represents a pair of straight lines.
- c) Find the centre of the conic given by $3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0$.
- d) Prove that equation of the tangent to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ at the origin $(0,0)$ is $gx + fy + c = 0$.
- e) If the axes are rectangular, find the distance of the plane $6x - 3y + 2z - 14 = 0$ from the origin.
- f) Find the angle between the planes $x - y + z = 1$ and $2x + y + 3z = 4$.
- g) Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$.
- h) Find the equation of the sphere whose centre is $(2, -3, 4)$ and radius 5.

SECTION – B

2. Attempt any two of the following questions:

6 x 2=12

- a. i. Prove that the orthogonal transformation without the change of origin, the quantity $(g^2 + f^2)$ in the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, is an invariant.
 ii. If by any change of axes without the change of origin, the expression $ax^2 + 2hxy + by^2 = 0$ becomes $ax'^2 + 2h'x'y' + by'^2 = 0$, then prove that $a + b = a' + b'$.
- b. Show that the angle between the pair of straight lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ is $\tan^{-1} \left(\frac{2\sqrt{h^2 - ab}}{a+b} \right)$.
- c. Prove that the general equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel straight lines if $h^2 = ab$ and $bg^2 = af^2$.

3. Attempt any four of the following questions:

7 x 2=14

- a. i. Find the equation of the tangent to the conic $4x^2 + 3xy + 2y^2 - 3x + 5y + 7 = 0$ at the point $(1, -2)$.
- ii. Find the equation of the chord of contact $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.
- b. i. Find the equation of the polar of the point $(2,3)$ w.r.t. the conic $x^2 + 3xy + 4y^2 - 5x + 3 = 0$.
- ii. Prove that the points $(1,2)$ and $(-2,3)$ are conjugate w.r.t. the conic $2x^2 + 6xy + y^2 + 4x - 2y + 8 = 0$
- c. i. Deduce the standard equation of the parabola.
- ii. Find the equation of the parabola whose focus is $(-1, 1)$ and whose directrix is $x + y + 1 = 0$.

4. Attempt any two of the following questions:

7 x 2=14

- a) i. Find the equation of the plane $Ax + By + Cz + D = 0$ in the intercept form.
- ii. A plane meets the coordinate axes at A, B, C such that centroid of the ΔABC is the point (a, b, c) ; show that the equation to the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.
- b) Prove that the four points whose coordinates are $(0, -1, 0), (2, 1, -1), (1, 1, 1)$ and $(3, 3, 0)$ are coplanar and find the equation of the plane containing them.
- c) Find the equation of the plane through the three points $(0, 1, 1), (1, 1, 2), (-1, 2, -2)$.

5. Attempt any two of the following questions:

7 x 2=14

- a) (i) Find the equation of the sphere through the four points.
- (ii) Find the equation of the sphere which passes through origin and $(a, 0, 0), (0, b, 0)$ and $(0, 0, c)$.
- b) Find the equation of the sphere passing through the points $(0, 0, 0), (0, 1, -1), (-1, 2, 0)$ and $(1, 2, 3)$.
- c) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C . Show that the locus of the centre of the sphere $OABC$ is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$
