The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

B.Sc. Mathematics & Computing 2nd Semester

Semester End Examination, July 2022

Course Title: Differential Equations

Course Code: MAC012C202

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt all questions: **2 x 8 = 16**

a) Find the differential equation of all straight lines passing through the origin.

b) Solve: $p = \sin(y - px)$, where $p = \frac{dy}{dx}$.

c) Solve: $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$

d) Test the integrability of the following equation:

$$(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$$

e) Form PDE by eliminating the arbitrary functions:

$$z = f(x + iy) + g(x - iy)$$

f) Solve by direct integration: $\log_e\left[\frac{\partial^2 z}{\partial x \partial y}\right] = x + y.$

g) Solve: $p^2 + q^2 = 1$

h) Show that the PDE $y\frac{\partial^2 z}{\partial x^2} + x\frac{\partial^2 z}{\partial y^2} = 0$ is hyperbolic in 2nd and 4th quadrants.

Section – B

2. Attempt any two of the following: **6 x 2=12**

a) (i) Solve: $\frac{dy}{dx} = xy + x + y + 1$

(ii) Solve: $(x^2 - y^2)dx + 2xydy = 0$

b) (i) Solve: $\frac{dy}{dx} + \frac{y}{x} = x^2,\ y(x) = 1$

(ii) Solve: $(2x^2 + 6xy - y^2)dx + (3x^2 - 2xy + y^2)dy = 0$

c) (i) Solve: $y + px = x^4p^2$

(ii) Solve: $(1 + xy)ydx + (1 - xy)xdy = 0$

P.T.O.

3. Attempt any two of the following: **7 x 2=14**

a) (i) Solve: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2e^x$

(ii) Solve: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2\{\log(1+x)\}$

b) Solve the equation $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by the method of variation of parameters.

c) Solve the following simultaneous equations

$\frac{dy}{dx} + y = z + e^x, \frac{dz}{dx} + z = y + e^x.$

4. Attempt any two of the following: **7 x 2=14**

a) (i) Solve: $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

(ii) Solve: $p^2 - q^2 = x - y$

b) Solve by Charpit's method: $px + qy - pq = 0$

c) Find the equation of surface satisfying $4yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 1,\ x + z = 2.$

5. Attempt any two of the following: **7 x 2=14**

a) (i) Solve: $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$

(ii) Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = e^{2x+3y}$

b) Solve by Monge's method: $r - t\cos^2 x + p\tan x = 0$

c) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\left(\frac{\partial u}{\partial t}\right) + u$, where

$u(x, 0) = 6e^{-3x}.$

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