The Assam Royal Global University, Guwahati

Royal School of Applied & Pure Sciences

B.Sc. Mathematics 2nd Semester

Semester End Examination, July 2022

Course Title: Ordinary Differential Equations

Course Code: MAT012C203

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt all questions as per instructions given.

The figures in the right-hand margin indicate marks.

Section – A

1. Attempt all questions: **2 x 8 = 16**

a) Check the exactness of the following equation:

$$y \sin 2x\, dx - (1 + y^2 + \cos^2 x)dy = 0$$

b) Solve: $(y - px)(p - 1) = p$, where $p = \frac{dy}{dx}$.

c) Solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y = 0$

d) Test the linear independence of the functions $\sin 2x$ and $\cos 2x$.

e) Solve: $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$

f) Verify the condition of integrability of the following equation:

$$(2x + y^2 + 2xz)dx + 2xydy + x^2dz = 0$$

g) What do you mean by orthogonal trajectory and oblique trajectory?

h) Find the orthogonal trajectories of the family of curves $r = a(1 - \cos\theta)$.

Section – B

2. Attempt any two of the following: **6 x 2=12**

a) (i) Solve: $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

(ii) Solve: $\frac{dy}{dx} - \frac{y}{x} + \tan\frac{y}{x}$

b) (i) Solve: $\frac{dy}{dx} + y\tan x = y^3 \sec x$

(ii) Solve: $(e^x \sin y + e^{-y})dx + (e^x \cos y - xe^{-y})dy = 0$

c) (i) Solve: $y - 2px = \tan^{-1}(xp^2)$

(ii) Solve: $(xy + 2x^2y^2)ydx + (xy - x^2y^2)xdy = 0$

P.T.O.

3. **Attempt any two of the following:** **7 x 2=14**

a) (i) Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-3x}$

(ii) Solve: $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x\log x$

b) Solve by the method of undetermined coefficients: $\frac{d^2y}{dx^2} + y = \sin x$

c) Show that e^x, e^{-x}, e^{2x} are linearly independent solutions of $y''' - 2y'' - y' + 2y = 0$ and hence or, otherwise solve the given equation.

4. **Attempt any two of the following:** **7 x 2=14**

a) Solve: $\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$, $\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$

b) Solve: $\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$

c) Find $f(z)$ such that the total differential equation $\left[\frac{(y^2+z^2-x^2)}{2x}\right]dx - ydy + f(z)dz = 0$ is integrable. Hence solve it.

5. **Attempt any two of the following:** **7 x 2=14**

a) (i) Show that the orthogonal trajectories of the family of curves $y = ax^n$ is $x^2 + ny^2 = c^2$.

(ii) Find the equation of the system of orthogonal trajectories of the parabolas $r = \frac{2a}{1+\cos\theta}$, where a is the parameter.

b) A mass m free to move along the x-axis is attracted towards the origin with a force proportional to its distance from the origin. Find the motion (i) if it starts from rest at $x = x_0$ and (ii) if it starts at $x = x_0$ with initial velocity v_0 moving away from the origin.

c) The population of a city doubles in 50 years. In how many years will it be triple? Assume that the rate of increase is proportional to the number of inhabitants.

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