The Assam Royal Global University, Guwahati
Royal School of Applied and Pure Sciences
B.Sc. Mathematics, 6th Semester
Semester End Examination, August 2021
Course Title: INTRODUCTION TO PROBABILITY & STATISTICS
Course Code: MAT012D601

Time: 3 Hours   Maximum Marks: 70

Note: Attempt all questions as per instructions given.
The figures in the right-hand margin indicate marks.

Section – A

1. Attempt **all questions**. (Maximum word limit 50)   2 x 8

    a. What is classical definition of probability?
    b. Prove that $P(\emptyset) = 0$.
    c. What is meant by random variable?
    d. Write the probability mass function of binomial probability distribution?
    e. What is curve fitting?
    f. Write the properties of correlation coefficient.
    g. Mention the characteristics of point estimation.
    h. Define level of significance.

2. Attempt **any two** of the following: 6 x 2

    a. A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, two officers of the sales department and 1 chartered accountant. Find the probability of forming the committee in the manner: (i) There must be one from each category. (ii) It should have at least one from the purchase department. (iii) The chartered accountant must be in the committee.

    b. In 1989 there were three candidates for the position of principal- Mr. Chatterjee, Mr. Ayangar and Dr. Singh., whose chances of getting the appointment are in the proportion 4:2:3 respectively. The probability that Mr. Chatterjee if selected would introduce co-education in the college is 0·3. The probabilities of Mr. Ayangar and Dr. Singh doing the same are respectively 0-5 and 0·8. What is the probability that there was co-education in the college in 1990?

    c. State and prove Bayes' theorem.

3. Attempt **any two** of the following: 7 x 2

    a. Obtain mean and variance of binomial probability distribution.

b. Seven coins are tossed and number of heads are noted. The experiment is repeated 128 times and the following distribution is obtained:

Number of heads	0	1	2	3	4	5	6	7	Total
Frequencies	7	6	19	35	30	23	7	1	128

Fit a binomial distribution when the coin is unbiased.

c. X is a normal variate with mean 30 and Standard deviation 5. Find tile probabilities that $(i) 26 \leq X \leq 40, (ii) X \geq 45$ and $(iii) |X - 30| > 5$. [Given: $P(0 \leq z \leq 0.8) = 0.2881, P(0 \leq z \leq 1) = 0.3413, P(0 \leq z \leq 2) = 0.4772, P(0 \leq z \leq 3) = 0.4986.$]

The Assam Royal Global University, Guwahati
Royal School of Applied and Pure Sciences
B.Sc. Mathematics, 6th Semester
Semester End Examination, August 2021
Course Title: INTRODUCTION TO PROBABILITY & STATISTICS
Course Code: MAT012D601

Time: 3 Hours   Maximum Marks: 70

Note: Attempt all questions as per instructions given.
The figures in the right-hand margin indicate marks.

Section – B

4. Attempt **any two** of the following: 7 x 2

    a. Fit a straight line to the following data—

X	1	2	3	4	6	8
Y	2.4	3	3.6	4	5	6

    b. Prove that correlation coefficient is independent of change of origin and scale.
    c. The random variables X and Y are jointly normally distributed and U and V are defined by $U = X\cos\alpha + Y\sin\alpha, V = Y\cos\alpha - X\sin\alpha$. Show that U and V will be uncorrelated if $\tan 2\alpha = \frac{2r\sigma_x\sigma_y}{\sigma_x{}^2 - \sigma_y{}^2}$, where $r = Corr\ (X, Y)$, $\sigma_x{}^2 = Var\ (X)\ and\ \sigma_y{}^2 = Var\ (V)$.
    Are U and V then independent?

5. Attempt **any two** of the following: 7 x 2

    a. An M.V.U. is unique in the sense that if $T_1\ and\ T_2$ are M.V.U. estimators of $\gamma(\theta)$, then prove that $T_1 = T_2$ almost surely.
    b. An insurance agent has claimed that the average age of policyholders who insure through him is less than the average for all agents, which is 30·5 years. A random sample of 100 policyholders, who had insured through him gave the following age distribution:

Age last birthday	No. of persons
16--20	12
21--25	22
26--30	20
31--35	30
36--40	16

    Calculate the arithmetic mean and standard deviation of this distribution and use these values to test his claim at the 5% level of significance. [Given: Z (1·645) = 0·95].

    c. The means of two single large samples of 1000 and 2000 members are 67·5 inches and 68·0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2·5 inches? (Test at 5% level of significance).